Physics 116

Session 35
Review of Chapters 28-30
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Announcements

• Exam 3 tomorrow: Material covered in class from Chapters 28, 29, 30

• Usual exam rules and arrangements: please take alternate seats, bring your own mark-sense bubble sheet and pencils, calculators only – no laptops

• Kyle Armour will hold a special office hour Tuesday 11:30, in B442 PAB
# Lecture Schedule  
(up to exam 3)

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Today is circled in red.
Constructive interference: \[ \Delta l = m\lambda, \quad m = 0,1,2 \]

2-slit pattern, slit spacing \( d \), bright fringes: \( \sin \theta = m\lambda/d \)

Fringe position on screen at distance \( L \): \( y_m = L \tan \theta_m \)

Path diff for constructive: \( \Delta l = m\lambda_{VACUUM} = 2nt \) \( \text{(or} \Delta l = m\lambda_n = 2t \text{ where} \lambda_n = \lambda_{VACUUM}/n \) \n
\( n = \) film index, \( t = \) film thickness, \( m = \) integer 0, 1, 2...

Use \((m + 1/2)\) for destructive.

**Note:** Add \((\lambda/2)\) to \( \Delta l \) for a reflection with phase change, e.g. thin film:

So in eqns above, use \( m = \) integer for destructive, \((m + 1/2)\) for constructive

Single slit of width \( W \), angle \( \theta_m \) to order-\( m \) minimum: \( W = \frac{m\lambda}{\sin \theta_m} \)

Rayleigh criterion: \( \theta_{\min} = 1.22 \frac{\lambda}{D} \)

Separation at distance \( L \): \( y = L \tan \theta_{\min} \approx L \theta_{\min} \)

Angle of \( m \)th bright fringe for grating with \( N \) lines/unit length: \( \lambda = \frac{\sin \theta}{mN} \)

\[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} \quad L = L_0\sqrt{1 - v^2/c^2} \]

\[ p = \frac{mv}{\sqrt{1 - v^2/c^2}} \quad m = \frac{m_0}{\sqrt{1 - v^2/c^2}}, \quad E = mc^2 \quad p = \frac{h}{\lambda} \]

\[ K = m_0c^2\left( \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) \]

\[ K_{\text{max}} = hf - W_0 \quad K = \frac{p^2}{2m} \]

\( f_{\text{peak}} = (5.88 \times 10^{10} \text{ s}^{-1} \cdot \text{K}^{-1})T \)

\( E = hf = \frac{hc}{\lambda} \)

\( \Delta x \Delta p \geq \frac{h}{2\pi} \quad \Delta E \Delta t \geq \frac{h}{2\pi} \)

\( c = 3 \times 10^8 \text{ m s}^{-1} \)

\( h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \)

\( 4.135 \times 10^{-15} \text{ eV} \cdot \text{s} \)

\[ 1 \text{ rad} = 57.296^\circ \quad c = f\lambda \]

\( m_{\text{ELECTRON}} = 9.11 \times 10^{-31} \text{ kg} \)

\( m_{\text{PROTON}} = 1.67 \times 10^{-27} \text{ kg} \)
Review: what you’ve learned since exam 2

- Superposition and Interference: overlapping waves’ amplitudes add
- Young's Two-Slit Experiment: proof that light = waves; regularly spaced bright and dark fringes
- Interference in Reflected Waves: phase shift in reflected wave when going from lower to higher index of refraction; fringes, filtering, and color effects in thin films
- Diffraction: interference between wavefronts from different parts of a single aperture; regularly spaced dark fringes; bright fringes have asymmetrical shape and spacing but approx halfway between dark
- Resolution: diffraction limits our ability to distinguish point sources separated by small angle; Rayleigh Criterion is commonly used standard
- Diffraction Gratings: many-slit interference patterns are characterized by sharp, clearly separated fringes; spacing depends on wavelength; application to spectroscopy
- Postulates of Special Relativity: speed of light is the same for any inertial frame; all inertial frames are equally valid; time is just another coordinate; spacetime and worldlines
- Time and Time Dilation: the other guy’s clock seems slow!
- Length and Length Contraction: the other guy’s meter stick seems short!
- Relativistic Momentum, Energy, and E=mc²
Review: what you’ve learned since exam 2

- Blackbody Radiation and Planck’s Quantum Hypothesis: energy is transferred in bundles
- Photons and the Photoelectric Effect: the particle nature of light
- The Momentum of a Photon, Photon Scattering, and the Compton Effect: photons act like particles when interacting with matter
- de Broglie’s Matter Wave Hypothesis and Wave-Particle Duality: wave nature of “matter” at subatomic scales
- Heisenberg’s Uncertainty Principle: limit our knowledge of position AND momentum of a given object; better-defined position means less-defined momentum and vice-versa – “granularity” of nature at subatomic scales

Note: Textbook sections not covered / not on exam
- 29-4 Relativistic addition of velocities
- 29-8 General relativity (recommended reading!)
- 30–7 Quantum Tunneling
Review questions

1) Two beams of light are coherent if they
A) have the same amplitude.
B) have the same intensity.
C) have the same velocity.
D) have the same wavelength and maintain a constant phase difference for equal paths.
E) none of the above describes coherent light.
Answer: D

2. When a beam of light, which is traveling in glass, strikes an air boundary, there is
A) a 90° phase change in the reflected beam.
B) no phase change in the reflected beam.
C) a 180° phase change in the reflected beam.
D) a 60° phase change in the reflected beam.
E) a 45° phase change in the reflected beam.
Answer: B
No phase change on reflection when going from higher to lower index of refraction.

3. A single-slit diffraction pattern is formed on a distant screen. Assuming the angles involved are small, by what factor will the width of the central bright spot on the screen change if the wavelength is doubled?
A) It will be cut to one-quarter its original size.
B) It will be cut in half.
C) It will double.
D) It will become four times as large.
E) It will become eight times as large.
Answer: C
Single slit of width $W$, angle $\theta_m$ to order-$m$ minimum: $W = \frac{m\lambda}{\sin \theta_m} \rightarrow \sin \theta_m = \frac{m\lambda}{W}$
Width is proportional to angle to first dark fringe, minimum $m=1$: 
Review questions

4. In Young's two-slit experiment, the distance between the slits and the screen is 1.10 m and the distance between the slits is 0.0400 mm. If the second order bright fringe is measured to be 4.20 cm from the centerline, what is the wavelength of light?
A) 200 nm
B) 381 nm
C) 401 nm
D) 620 nm
E) 763 nm
Answer: E

For m=2: 2-slit pattern, slit spacing \( d \), mth bright fringe:

\[
\sin \theta_m = \frac{m\lambda}{d} \quad \text{Note: } \tan \theta \approx \sin \theta \approx \theta \quad \text{for small angles}
\]

Fringe position on screen at distance \( L \):

\[
y_m = L \tan \theta_m \approx L \left( \frac{m\lambda}{d} \right)
\]

\[
y_m = 1.10 m \left( \frac{2\lambda}{0.000040m} \right) \rightarrow \lambda = \left( \frac{0.042m}{2 \cdot (1.10m)} \right) = 7.63 \times 10^{-7} m
\]

5. What is the limiting angle of resolution for a camera with lens diameter 4.0 mm, at wavelength of light of 600 nm?
A) 4.2 \times 10^{-4} \text{ rad}
B) 2.1 \times 10^{-4} \text{ rad}
C) 1.8 \times 10^{-4} \text{ rad}
D) 1.5 \times 10^{-4} \text{ rad}
E) 2.6 \times 10^{-4} \text{ rad}
Answer: C

Rayleigh criterion:

\[
\theta_{\text{min}} = 1.22 \frac{\lambda}{D} = 1.22 \left( \frac{6 \times 10^{-7} m}{0.004m} \right) = 1.8 \times 10^{-4} \text{ rad}
\]
Review questions

6. A sodium light ($\lambda = 589.3$ nm) is used to view a soap film to make it look black when directed perpendicular to the film. What is the minimum thickness of the soap film if the index of refraction of soap solution is 1.38?
A) 114 nm
B) 194 nm
C) 294 nm
D) 314 nm
E) 214 nm
Answer: E
Minimum thickness corresponds to $m=1$:
Path diff for destructive in thin film: $m\lambda_{\text{VACUUM}} = 2nt$

$$t_{\text{MIN}} = (m = 1)\frac{\lambda_{\text{VACUUM}}}{2n} = \frac{589.3 \text{ nm}}{2 \cdot 1.38} = 213.5 \text{ nm}$$

7. Observer A sees a pendulum oscillating back and forth in a relativistic train as it passes him, and measures its period to be $T_A$. Observer B is on the train, and measures the period of the pendulum to be $T_B$. These two results will be such that
A) $T_A > T_B$.
B) $T_A = T_B$.
C) $T_A < T_B$.
D) $T_A$ could be greater or smaller than $T_B$ depending on the direction of the motion.
Answer: A
Train’s clock seems slower to stationary observer, so period of pendulum seems longer.
8. At what speed, relative to the frame of the observer making the measurement, is the length of a meter stick 0.60 m?
A) 0.40c
B) 0.60c
C) 0.70c
D) 0.80c
E) 0.90c
Answer: D
\[ L = L_0 \sqrt{1 - \frac{v^2}{c^2}} \] \[ v = c \left[ 1 - \left( \frac{L}{L_0} \right)^2 \right]^{1/2} = c \sqrt{1 - \left( \frac{0.6}{1} \right)^2} = 0.8c \]

9. 17. The work function of a certain metal is 1.90 eV. What is the longest wavelength of light that can cause photoelectron emission from this metal?
A) 231 nm
B) 14.0 nm
C) 62.4 nm
D) 344 nm
E) 653 nm
Answer: E
Least energy to emit electron is when \( E = hf = W \)
Review questions

10. An electron is moving with the speed of 1780 m/s. What is its de Broglie wavelength?
   A) 409 nm
   B) 302 nm
   C) 205 nm
   D) 420 nm
   E) 502 nm
   ANSWER: A

\[
p = \frac{h}{\lambda} \rightarrow \lambda = \frac{h}{p}
\]

\[
p = \frac{mv}{\sqrt{1 - v^2/c^2}} = \frac{(9.11 \times 10^{-31} \text{ kg})(1780 \text{ m/s})}{\sqrt{1 - (1780 \text{ m/s})^2/(3 \times 10^8 \text{ m/s})^2}} = \frac{1.62 \times 10^{-27} \text{ kg} \cdot \text{m/s}}{1} \\
= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s} = \text{kg} \cdot \text{m}^2/\text{s})}{1.62 \times 10^{-27} \text{ kg} \cdot \text{m/s}} = 409 \text{ nm}
\]
Review questions

11. What is the minimum uncertainty in determining the position of an electron if the speed of the electron is 6000 m/s within an accuracy of 0.004%?
A) 0.105 mm  
B) 0.153 mm  
C) 0.241 mm  
D) 0.335 mm  
E) 0.483 mm  
Answer: E

\[
p = \frac{mv}{\sqrt{1-v^2/c^2}} = \frac{(9.11 \times 10^{-31} \text{kg})(6000 \text{ m/s})}{\sqrt{1-(6000 \text{ m/s})^2/(3 \times 10^8 \text{ m/s})^2}} = \frac{5.47 \times 10^{-27} \text{ kg} \cdot \text{m/s}}{1}
\]

\[
\Delta p = 0.00004 \times 5.47 \times 10^{-27} \text{ kg} \cdot \text{m/s}
\]

\[
\Delta x \geq \frac{h}{2\pi \Delta p} = 4.82 \times 10^{-5} \text{ m}
\]

12. A proton and an electron are both accelerated to the same final kinetic energy. If \( \lambda_p \) is the de Broglie wavelength of the proton and \( \lambda_e \) is the de Broglie wavelength of the electron, then
A) \( \lambda_p > \lambda_e \)
B) \( \lambda_p = \lambda_e \)
C) \( \lambda_p < \lambda_e \)
D) Not enough data to answer this question.
Answer: C

\[
K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}
\]

Wavelengths inversely proportional to m’s:

\[
K_p = \frac{h^2}{2m_p \lambda_p^2} = K_e = \frac{h^2}{2m_e \lambda_e^2} \rightarrow \frac{\lambda_p^2}{\lambda_e^2} = \frac{m_e}{m_p} \cdot \frac{9.11 \times 10^{-31} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} = 0.00055
\]

so \( \lambda_p << \lambda_e \)
Quantum “Tunneling”

- Gravity analogy: ball rolls downhill, toward a potential barrier $h$
  - If $E > mgh$ it goes past the barrier
  - If $E < mgh$ it is repelled and goes backward

- Particle picture of an electron approaching a potential $V$
  - If $E > V$ it goes past the barrier
  - If $E < V$ it is repelled and goes backward

- Wave picture of an electron approaching a potential $V$
  - If $E > V$ it goes past the barrier
  - If $E < V$ some of the wave gets through anyway!
    - Like a light wave striking a glass plate

Alpha particles can “tunnel out” of nuclei

Particle (classical) picture

Wave (quantum) picture

Not On Test
Heisenberg principle

- What does this mean?
  \[ \Delta p_y \Delta y = h \] (=very small number!)

- If you measure position of an electron very precisely, you cannot measure its momentum very precisely
  - You know where the electron is, but not where it is going

- If you measure momentum of an electron very precisely, you cannot measure its position very precisely
  - You know where the electron is going, but not where it is now

- You never notice this limitation, unless you are looking at very tiny objects or effects
  - Try calculating uncertainty limits for a baseball:
    - Say \( p = 100 \text{ kg-m/s} \), and you measure it to \( \pm 1\% \)
    - What is Heisenberg limit on how well you can find its position?

This is related to the wave function description of particles
Revisiting uncertainty: what does \emph{bandwidth} mean?

- Another “cultural” digression: more math detail than you need to know
- \emph{Fourier} analysis: describe waves as sums of sines/cosines
  - \textit{Any} wave (or periodic electrical signal) can be described as a sum of sine and cosine waves at frequencies which are \emph{harmonics} (integer multiples of some f: 1f 2f, 3f, 4f…)
    - For complicated shape, may need huge number of harmonics
    - Truncated set of harmonics may be \textbf{adequate approximation}
  - Example: computers use “square waves” as clock ticks:
    - Square-wave = sum of \emph{odd-# harmonics} of sine waves

\[
V(t) = \frac{4}{\pi} \left( \sin(kt) + \frac{1}{3} \sin(3kt) + \frac{1}{5} \sin(5kt) + \cdots \right)
\]

- \textbf{Approximation to square shape gets better and better} as more harmonics are added to the sum
- Send over a poor \textit{transmission line} with small \textit{bandwidth} (cannot pass the higher frequencies): the wave shape gets \textbf{degraded}!
An element of uncertainty…

- W. Heisenberg (1927): “Heisenberg uncertainty principle”
  - If a particle is confined to a small region of space, its wave function must be a wave packet which =0 outside that small region
  - ...because $\Psi^2$ gives probability of finding particle at $x$
  - Narrowly confined particle wave function must contain many different wavelengths (Fourier analysis: large bandwidth is needed to describe sharp edges, so $\Psi$ must be a sum of many different frequencies!)
    - Each wavelength corresponds to a different momentum ($p=h/\lambda$)
  - So: If particle’s position is sharply defined, its momentum is poorly defined (acts like a mixture of many different momenta), and vice versa

\[ \Delta x \Delta p \sim h \]

This conclusion falls out of the math: wave concept is the only physics involved!