Physics 116

Session 25
Diffraction and resolution
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Announcements

• Posted exam score = (6 pts x number correct) + 10 pts
• Scores for all 3 midterm exams will be normalized to a common average to minimize differences
  • Class average final grade will be 2.9
  • Only your 2 best exam scores are used
    • Of course, if you get a perfect score on everything (exams, homeworks, quizzes), you get a 4.0, regardless of which exam was dropped!

• Don’t forget: UW is closed tomorrow - no class!!
# Lecture Schedule
*(up to exam 3)*

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Diffraction

• Everyday experience: light “gets around corners”
  - Shadows are not usually sharp-edged
  - Analogy: you can hear sound waves around the corner of a building, even if source of sound is not in your line of sight

• Apply Huygens’ Principle to a single narrow slit
  - Picture tells us two things:
    1. Spherical wavelets - some light will be seen at large angles to axis
    2. Light from different parts of slit area will interfere

So we expect to see fringes on a distant screen, including some at large angles:
This kind of interference is called DIFFRACTION

We see diffraction effects near any obstacle, IF we look closely enough (on a scale comparable to light wavelengths)
Diffraction effects

- Also see diffraction around knife-edge, needle point, etc
  - Shadow of knife or needle is sharp-edged only if you don’t look too closely (and use *coherent* or at least “monochromatic” light)
  - On a microscopic scale you see diffraction fringe patterns
- But shadows created by a “distant” light source (parallel rays) should have sharp edges…

Completely inexplicable if light = particles; easily explained by wave theory
What’s a “coherent” light source?

- Ordinary light (Sun, light bulb, fluorescent tube, or neon tube) is made by billions of atoms radiating *independently*
  - Waves from individual atoms don’t interfere: have *random* phases
    - “Incoherent” light (“natural” light)

- Laser = device to make atoms radiate *in unison*
  - Individual atoms’ contributions add up *constructively*
  - “coherent” light source
  - Laser acts like one giant atom!
  - **How’s it done?**
    - Use carefully adjusted mirrors to make neon tube a *resonant cavity* for light (even though it is millions of \( \lambda \) long!)
    - “Pump” atoms into a high-energy state (electrical discharge)
    - Standing waves in cavity *stimulate* atoms to emit together
    - LASEr = Light Amplification by Stimulated Emission (Einstein again!)

- We can use lasers to make the 2-slit experiment *easy*
  - How did Thomas Young manage in 1804?
    - Used a *pinhole* to select a tiny region of lamp surface
    - “Partially coherent light” – pattern is partially washed out
We can picture a single slit as 2 slits but with no gap between:

- Single slit width $W = \text{two adjacent slits of width } W/2$
- Consider ray of light from top of slit, and center of slit
  - Meaning: Top of half-slit 1 and top of half-slit 2
- There will be a bright fringe on the axis (angle $= 0$)
  - Equal path lengths: constructive interference
- Calculate the angle to the first off-axis dark fringe:
  - Find angle to get destructive interference: half-wavelength path difference
  - Bright fringes occur approx halfway between dark fringes (exact calculation is more complicated – we’ll skip)

Screen is far away – many slit-widths!

Central fringe for $m=0$, the next fringe on either side for $\pm 1$, etc
Single slit diffraction: in detail

- 2-slit experiment: recall our picture of interference between separate rays from spaced slits

- Remove center part of slit mask: single slit of width w
  - interference between rays from different parts of slit
  - Rays 1, 2 and 3 are from top of slit, axis, and bottom of slit
  - for \( r >> w \), \( \theta_1 \sim \theta_2 \sim \theta_3 = 0 \)

- Each ray between 1 and 2 has a partner between 2 and 3 (distance \( w/2 \) at slit) with \( \Delta = \lambda/2 \)

- Order number \( m = \pm 1, \pm 2, \pm 3 \ldots \)
  - Negative \( m = \) below axis

\[ \Delta_{12} \approx \Delta_{23} = \Delta = \frac{w}{2} \sin \theta \]

minimum when \( \Delta = \frac{\lambda}{2} \)

So \( w \sin \theta = \lambda \) (or \( w \sin \theta = m\lambda \))

Condition to get a dark fringe at location y on screen
**Single slit diffraction patterns**

- Fringe pattern of single slit has
  - bright central peak
    - We can calculate its half-width: just distance to first dark fringe location
  - Much dimmer higher-order \((m>1)\) bright fringes
  - Dark fringes are equally spaced, but...
  - Bright fringes are not exactly halfway between
    - Slightly offset toward center

\[
\beta = \frac{kw}{2} \sin \theta \quad \text{or} \quad \sin \theta = \frac{2\beta}{kw}
\]

\[
\sin \theta \sim \theta \rightarrow \Delta \theta_1 = \frac{2\pi}{kW} = \frac{\lambda}{w} \quad \text{(halfwidth of central peak)}
\]

95% of energy in central peak
Interference and diffraction in everyday life

• We don’t usually notice diffraction fringes
  – Incoherent light: fringes are smeared
  – Need to look very closely at edge of an object’s shadow (few wavelengths distance scale)

Diffraction effects we can see directly:

• “Floaters” in your eyeballs
  – Look at bright, uniform source through tiniest pinhole you can make—you’ll see slowly moving specks with rings around them—diffraction rings around tiny particles in your eye fluid

• Shadow between pinched thumb and forefinger
  – Appears to connect before they actually touch

• Streaked street-lights through gauze curtain
  – Fabric forms coarse diffraction grating

Notice rainbow effect: fringe angles depend on wavelength

How does diffraction differ from interference?
Interference = light from multiple sources (e.g., separate slits)
Diffraction = interference between waves from different parts of one slit (or knife-edge, or hole in a screen, etc)
Diffraction for a circular aperture: resolution

- **Pinholes** also show diffraction fringes
  - Similar to single slit pattern, but with circular symmetry
  - Mathematical form is called the *Airy function*

- Can just **resolve** 2 pinholes if their 1st minima overlap:

  (a): One pinhole  
  (b): Two, just separable  
  (c): Two, not separable!

- Airy function says:
  Angle to first dark fringe for a pinhole is
  \[
  \sin \theta = \frac{3.83 \lambda}{2\pi R} = 1.22 \frac{\lambda}{D} \approx \theta
  \]

- **Rayleigh Criterion:** resolution for aperture of diameter D

  telescope, camera, binoculars and human eye = circular apertures!  
  Rayleigh criterion lets us estimate resolution limits for optical devices
Resolution: Example

- Alpha Centauri is a nearby double star
  - Centaurus A and Centaurus B
  - Distance = 1.34 parsecs*
  - Angular separation = 19"

(" = 1 second of arc = 1/60 of 1 minute = 1/3600 degree)

So 19 sec = (19/3600 deg)*(0.017 radian/deg) = 9x10^{-5} rad

- What is the smallest diameter telescope that can resolve Cent A from Cent B?

\[ D_{MIN} = \frac{\lambda}{\theta_{MIN}} = \frac{\lambda}{9 \times 10^{-5} \text{ rad}} = \frac{555 \times 10^{-9} \text{ m}}{9 \times 10^{-5} \text{ rad}} = 0.0075 \text{ m} \]

- So a 7.5 mm aperture would be minimum - a 1" telescope (small binoculars) should be more than enough!

*parsec = "professional" astronomy distance unit = 3.26 light-years
(more about parsecs soon)
Quiz #9

In physics, the term “diffraction” refers to

A. X Interference effects seen when you look very close to the shadow of a pinhole or knife-edge

B. The bending of light as it enters a slab of glass

C. The orientation of the electric field in a light wave

D. The phenomenon that causes regularly-spaced bright fringes in a 2-slit experiment