Economic Analysis of Decentralized Exchange Market with Transaction Fee Mining

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ABSTRACT

Blockchain-based Web 3.0, denoting the next-generation Internet, has attracted attention from academia and industry. However, the open-source application and the decentralized storage of users' interaction data break down the algorithm and data barriers, resulting in more fierce competition among service providers. To cope with the competition, decentralized exchanges (DEXs), the financial infrastructures of Web 3.0, adopt the *transaction fee mining* mechanism, which refunds the transaction fees to users in the form of governance tokens. However, the ratio of governance tokens a DEX decides to give to users would affect the enthusiasm of users to participate, which has not been discussed yet. In this paper, we establish the DEX market with transaction fee mining and formulate our model based on the Hotelling model. Besides, we propose a two-stage game to formulate the interaction between DEXs and users and derive the equilibriums under different conditions of two parameters: the transaction cost difference and users' stickiness. We show that though the service provider with a lower transaction cost can win the market, users' stickiness can offset the market advantage. Thus, incentivizing users with transaction fee mining has become a crucial strategy in the duopoly competition.

CCS CONCEPTS

• **Networks** → **Network economics**.

KEYWORDS

Blockchain, decentralized exchanges, network effect, economic analysis

ACM Reference Format:
Hongbo Zhang, Sizheng Fan, Zhixuan Fang, and Wei Cai. 2022. Economic Analysis of Decentralized Exchange Market with Transaction Fee Mining. In *Proceedings of the Fourth ACM International Symposium on Blockchain and Secure Critical Infrastructure (BSCI '22), May 30, 2022, Nagasaki, Japan.* ACM, New York, NY, USA, 12 pages. https://doi.org/10.1145/3494106.3528677

1 INTRODUCTION

Blockchain-based Web 3.0 is regarded as the next-generation Internet, providing users with decentralized services running on smart contracts [5]. Smart contracts that provide services in Web 3.0 are open-source, ensuring that these smart contracts cannot be tampered [19] [8]. Besides, users' interaction data is stored on the blockchain and can be accessed by everyone.

The open-source application and the decentralized storage of users' interaction data break down the algorithm and data barriers established in Web 2.0 enterprises, resulting in more fierce competition among Web 3.0 service providers. For example, Uniswap[1], one of the most popular financial infrastructure providers for Web 3.0, lost more than 50% of its liquidity in just a few days after SushiSwap[2] forked its code and launched a vampire attack. Due to the loss of data barriers, the value of users' interaction data no longer belongs to service providers but users themselves, which significantly reduces users' dependence on service providers. Besides, the value of Web 3.0 applications is provided by users' interaction data other than the applications themselves. Users can fork a new application by themselves and continue to use the interaction data in the previous application. Hence, service providers in Web 3.0 reward users who have used their applications with governance tokens, which are tokens that have both governance rights and economic values. Otherwise, users can get back their value by establishing a decentralized autonomous organization (DAO) and

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[1]https://uniswap.org/
[2]https://sushi.com/

BSCI '22, May 30, 2022, Nagasaki, Japan.

ACM ISBN 978-1-4503-9175-7/22/05…$15.00
https://doi.org/10.1145/3494106.3528677

issuing governance tokens by themselves. For example, OpenSea[3], the NFT exchange of Web 3.0, intends to offer shares through an initial public offering (IPO), in which case users cannot get the value they deserve. It triggers its users to establish OpenDAO[4] and issue governance tokens called SOS to users traded in Opensea. Therefore, in the fierce competition of the Web 3.0 era, competing for and retaining users is a problem that all service providers of Web 3.0 applications have to consider.

In order to cope with the fierce competition, decentralized exchanges (DEXs), one of the most influential applications in Web 3.0, adopt the *transaction fee mining* mechanism. When users swap tokens in the DEXs, they need to pay the transaction fee, which normally takes up 0.3% of the transaction volume. With the transaction fee mining mechanism, the DEX refunds the transaction fee to users in the form of governance tokens, which have attracted many cost-sensitive users. After dYdX[5] launched its governance tokens DYDX and announced it would offer the transaction fee mining, its trading volume skyrocketed in August 2021. According to CoinGecko, the total trading volume of dYdX reached \$9.8 billion in a month, with daily trading volume exceeding \$2.8 billion at one point[6].

As more and more DEXs offer the transaction fee mining (MDEX[7], IDEX[8], etc.), competition among different DEXs become increasingly fierce. Unlike competition in other markets, users in the DEX market not only compare transaction costs but also consider the rewards of the transaction fee mining. The transaction costs in DEXs mainly consist of the transaction fees and the gas fees. As mentioned before, the transaction fees are the costs for swapping tokens in DEXs. Meanwhile, the gas fees are the costs for running smart contracts on the blockchain, and smart contracts of different DEXs consume various gas fees[9]. Users prefer to choose the DEX with less transaction cost. Besides, the rewards of the transaction fee mining depend on both the number and value of the received governance tokens. The number of governance tokens users receive is determined by the *incentive level*, which is the proportion of the governance tokens that DEXs decided to give back to users as rewards. As for the governance tokens' value, Stylianou et al. [17] and Gandal et al. [11] proposed that the governance tokens have network effects, and their value is determined by their user base. A higher incentive level can attract more users and thus higher value of governance tokens. Therefore, the competition in the DEX market is a competition of both service providers' transaction costs and provided incentive levels.

At the same time, the transaction fee mining affects the supply of governance tokens, leading to price changes in the secondary market. According to the analysis from HiveTech[10], transaction fee mining increases selling pressure on DYDX and leads to its price decrease. As its price decreases, so do users' enthusiasm to engage in transaction fee mining, leading to an overall decline in dYdX's trading volume. We do not consider the price changes of the governance tokens in this work. Instead, we emphasize the importance of determining the incentive level for DEXs.

[3]https://opensea.io/
[4]https://www.theopendao.com/
[5]https://dydx.exchange/
[6]https://www.coingecko.com/en/exchanges/dydx_perpetual#statistics
[7]https://mdex.com/
[8]https://idex.io/
[9]https://debank.com/
[10]https://mirror.xyz/0x633653A579959D7e2C0331A4d0Ef0D114Fd47aA4/BG2zBm-Bo5p51bDmyslqzDgKQ34xMCi2hz7g7nbO7xI

In this paper, we establish the DEX market with transaction fee mining and formulate competition among different DEXs based on the Hotelling model. We consider the DEX market that consists of two service providers and assume that they have different transaction costs without loss of generality. Meanwhile, users have different preferences for service providers, and users' stickiness measures the cost for switching platforms. To analyze the competition between service providers, we consider the incentive levels as the decisions of service providers in our model, which is different from the Hotelling model. Besides, we propose a two-stage game to formulate the interactions between service providers and users, and derive the equilibriums under different conditions of two parameters: the transaction cost difference and users' stickiness. In Stage I, service providers optimize their incentive levels to maximize their profits. After service providers decide their incentive levels (Stage II), each user chooses the service provider who provides a higher payoff.

In summary, our key contributions are as follows.

- To the best of our knowledge, we are the first to formulate the DEX market competition with transaction fee mining. Besides, our model derives service providers' optimal incentive levels, which have not been discussed in the context of platform competition.
- We derive the equilibriums whose objective function involves piece-wise functions that vary under different conditions of the transaction cost difference and users' stickiness and analyze the equilibrium results under different parameter conditions.
- We provide insight for DEXs competition by revealing the equilibriums results. We show that though the service provider with a lower transaction cost can win the market, users' stickiness can offset the market advantage. In the most intense competition cases, service providers compete with each other, and both of them incentivize users with transaction fee mining.

The remainder of this paper is organized as follows: Section 2 presents the related works and analyzes the difference between our work and the state-of-the-ark work. The system model is presented in Section 3. We analyze users' decisions in Stage II and service providers' decisions in Stage I in Sections 4 and 5 separately. Finally, we conduct simulations to verify the effectiveness of the results in Section 6. Lastly, we conclude this paper in Section 7 .

2 RELATED WORK

2.1 DEXs

The popularity of the blockchain-based Web 3.0 has attracted the attention of academia. Many scholars attempt to analyze the DEX market and the blockchain-driven Metaverse (e.g., Duan et al. [7]). In the state-of-the-art literature, related studies in the DEX market mainly focus on the problem of impermanent loss (e.g., Loesch et al. [15]), cyclic arbitrage (e.g., Wang et al. [18]) and multi-asset trading (e.g., Angeris et al. [2], Angeris et al. [3]), etc. Especially, Angeris et al. [4] give a formal analysis on the market model of a DEX called

Uniswap and present optimal arbitrage actions. To the best of our knowledge, our paper is the first paper to analyze the platform competition in DEX market.

2.2 Platform Competition

Market competition has always been a subject of extensive research. In Hotelling's pioneering paper [12], he considered the price competition between two sellers in a one-dimensional market with linear transportation costs. Based on the model proposed by Hotelling, many scholars have studied the platform competition in different markets, such as the competition among media platforms (e.g., Anderson et al. [1]; Reisinger [16]), competition among P2P lending platforms (e.g., Liu et al. [14]) and competition among different blockchains (e.g., Jiang et al. [13]). However, the research on the competition between DEXs is still blank at present. Unlike other markets, there exists a new mechanism called transaction fee mining in the DEX market. Specifically, the transaction fees paid by users when they trade in DEX are returned to the users in the form of governance tokens. Therefore, the competition between different DEXs under the mechanism of transaction fee mining is worth discussing. Inspired by the model proposed by Fang et al. [9], we formulate the competition among different DEXs under the transaction fee mining.

3 SYSTEM MODEL

In this section, we first establish the DEX market and briefly introduce the models in Section 3.1. Then, we study the strategies of service providers and users in Section 3.2. Then, we establish the models of users' payoff and service providers' profits in Section 3.3 and Section 3.4, respectively. Finally, we formulate the interactions between DEXs and users in a two-stage game in Section 3.5. The list of key notations is shown in Table 1.

3.1 Preliminary

The system architecture of the DEX market is shown in Fig.1. The service providers (e.g., Uniswap, dYdX) provide smart contracts, and users can swap tokens by calling smart contracts with their blockchain addresses. Since these services are running on the blockchain, the interaction data is open and can be accessed by everyone.

In our model, we consider the DEX market that consists of two service providers (1 and 2) and a set of users $\mathcal{M} = \{1, 2, ..., M\}$. Without loss of generality, we normalize the number of users as one. The number of users who choose service provider i is denoted by N_i, and we can achieve that sum of users choosing two service providers is one, i.e., $N_1 + N_2 = 1$. Meanwhile, we refer to the transaction cost of service provider i as P_i. We assume service provider 1 has a higher transaction cost without loss of generality, i.e., $P_1 \geq P_2 > 0$. Note that such a duopoly model is widely adopted by literature in platform competition [6] and blockchain competition [13], etc.

In the DEX market, there is a new type of incentive mechanism called transaction fee mining, which is different from the previous economic model. The platforms in the sharing economy (e.g., Uber and Didi) provide users with subsidies in order to win a larger market share[10]. Meanwhile, the service providers in the DEX market issue governance tokens to users as subsidies for transaction

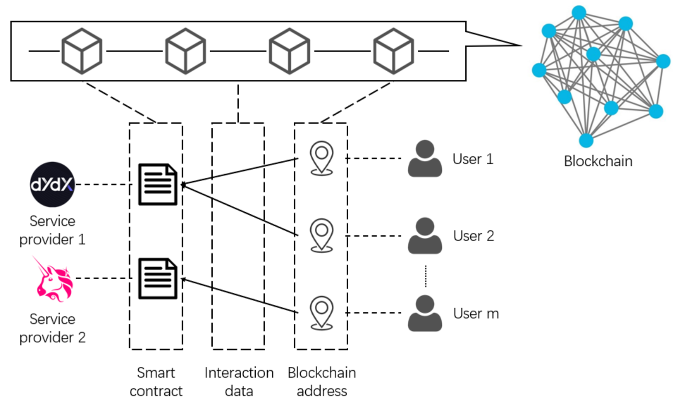


Figure 1: System Architecture

fees. The proportion of the governance tokens that DEXs decide to give to users is determined by the incentive level. The incentive level of service provider i is denoted by β_i. The value of governance tokens is proportional to the user bases of service providers, and its value varies as the market share of the service provider changes. Users in the DEX market can achieve desired services and obtain profits from the network effect of the selected service providers.

As shown in Fig.2, we establish our model based on the Hotelling Model [12]. We assume that users are uniformly distributed on the line segment $[0, 1]$, and two service providers are located on the endpoints, where users' locations characterize their preferences on service providers. Meanwhile, users' stickiness measures the cost for switching platforms and we denote users' stickiness by λ. Different from the Hotelling Model, our model focus on discussing the incentive level of service providers.

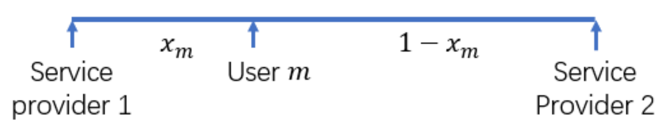


Figure 2: Hotelling Line

3.2 Strategies of Service Providers and Users

3.2.1 Service Providers' Strategies. For service provider $i \in \{1, 2\}$, he needs to decide his incentive level $\beta_i \in [0, 1]$. When making decisions on the incentive level, service providers have a trade-off between the user base and the loss of issuing governance tokens to users. The service provider who sets a high incentive level can win a larger user base. Due to the network effect, the value of its governance tokens becomes higher. However, its loss is also larger since more governance tokens are given to users.

3.2.2 Users' Strategies. For user $m \in \mathcal{M}$, he needs to choose a service provider $i \in \{1, 2\}$. User m's location on the line segment characterizes his preference. Besides, he compares the transaction cost and the reward offered by transaction fee mining. Considering all the above factors, he chooses the service provider who provides a higher payoff.

3.3 Users' Payoff

Users get different payoffs when choosing different service providers. As mentioned above, users' payoff includes three parts: transaction cost P, personal preference, and the reward offered by transaction fee mining.

We refer user m's location on the line segment as x_m and $|x_m - x_i|$ is the distance between user m and service provider i. Since two service providers are located on the endpoint, we can get user m's distances to service providers 1 and 2 are x_m and $1 - x_m$, respectively. In the Hotelling model, λ is the travel cost, while λ represents a user's stickiness towards the service provider in our case. The larger λ is, the higher user's cost is to switch from his preferred service provider to another.

The reward offered by transaction fee mining is the product of the number and value of received governance tokens. The number of received governance tokens is determined by service provider i's incentive level β_i. As for the value, it is determined by its user base N_i. Hence, the reward users receive becomes $\beta_i N_i$.

Hence, when user m choose service provider i, the user's payoff is summarized as follows:

$$U_m^i = -P_i + \beta_i N_i - \lambda |x - x_i|, i \in \{1, 2\}. \tag{1}$$

3.4 Service Providers' Profits

Unlike other markets, the transaction fee in the DEX market is not the service providers' profits. The profit of service provider i comes from the network effect of the governance tokens he owns, which is determined by their user base N_i. Meanwhile, $\beta_i N_i$ is the proportion of governance tokens that are given to users who choose service provider i.

Though service providers can attract users by providing transaction fee mining, they need to consider the trade-off between the loss of issuing away governance tokens and the profits gained from the network effect of their remaining governance tokens.

Hence, the profits of service provider i is summarized as follows:

$$\Pi_i(\beta_i) = (1 - \beta_i N_i) N_i, i \in \{1, 2\}. \tag{2}$$

3.5 The Two-stage Game

As shown in Fig.3, we formulate the interactions between service providers and users as a two-stage game. In Stage I, service providers optimize their incentive levels to maximize their profits. After service providers decide their incentive levels (Stage II), each user chooses the service provider who provides a higher payoff.

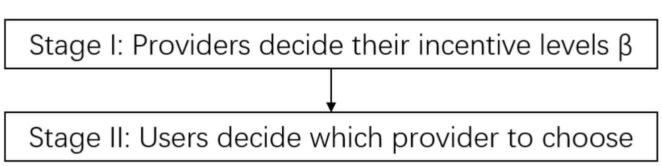


Figure 3: The Two-stage Game

We analyze the Nash equilibrium of the two-stage game by backward induction. We analyze users' decisions in Section 4 and derive the equilibriums of service providers' decisions in Section 5.

Table 1: Key Notations

Symbol	Definition
$\mathcal{M}$	The set of users
x_m	The location of user m
λ	Users' stickiness
P_i	The cost of users for using service provider i's service
N_i	The number of users who choose service provider i
β_i	The incentive level provided by service provider i
$\Pi_i(\beta_i)$	The profit of service provider i with his strategy β_i
U_m^i	The payoff of user m who choose service provider i

4 USERS' DECISIONS IN STAGE II

In this section, we analyze users' decisions in Stage II. Specifically, users compare payoffs provided by two service providers and choose service provider $i \in \{1, 2\}$ who offers the higher payoff. Recall that payoff of user m defined in Eq.1, user m's optimal service provider i^* is:

$$i^* = \arg\max_{i \in \{1,2\}} -P_i + \beta_i N_i - \lambda |x_m - x_i|. \tag{3}$$

We can define users' decisions game in Stage II as follows:

Definition 4.1. (Users' Decisions Game in Stage II).

- Players: The set of users $\mathcal{M} = \{1, 2, ..., M\}$
- Strategies: Each user $m \in \mathcal{M}$ chooses service provider $i \in \{1, 2\}$ from the DEX market.
- Payoff: U_m^i, the payoff of user $m \in \mathcal{M}$ choose service provider i, which is defined in Eq.1.

The number of users who choose service provider i depends on the strategies of both service provider i and j, where $i, j \in \{1, 2\}$, $j \neq i$. Next, we give the number of users choosing two service providers in Lemma 4.2, where $\Delta P = P_1 - P_2 \geq 0$.

LEMMA 4.2. *Given the strategies of two service providers, the numbers of users who choose service providers 1 and 2 are:*

- *If $\beta_1 > \lambda + \Delta P$ and $\beta_2 < \lambda - \Delta P$, all the users in the DEX market choose service provider 1, i.e., $N_1 = 1$ and $N_2 = 0$.*
- *If $\beta_1 < \lambda + \Delta P$ and $\beta_2 > \lambda - \Delta P$, all the users in the DEX market choose service provider 2, i.e., $N_1 = 0$ and $N_2 = 1$.*
- *If $\beta_1 = \lambda + \Delta P$ and $\beta_2 = \lambda - \Delta P$, half of the users in the DEX market choose service provider 1, while the other half choose service provider 2, i.e., $N_1 = \frac{1}{2}$ and $N_2 = \frac{1}{2}$.*
- *In other cases, we can derive the numbers of users who choose service provider 1 and service provider 2 in the DEX market, i.e., $N_1 = \frac{\lambda - \beta_2 - \Delta P}{2\lambda - \beta_1 - \beta_2}$ and $N_2 = \frac{\lambda - \beta_1 + \Delta P}{2\lambda - \beta_1 - \beta_2}$.*

Proof of Lemma 4.2 is given in Appendix A. Service provider 2 with a lower transaction cost win more users when both service providers provide a same incentive level. Besides, we find that service provider 2 can win the market when $\lambda - \Delta P \leq 1$ and $\lambda + \Delta P > 1$, which take up half of all cases. Though service provider 1 has a higher transaction cost, he can win all users when service provider 2 provides a relative low incentive level.

5 SERVICE PROVIDERS' DECISIONS IN STAGE I

In this section, we discuss service providers' decisions in Stage I. From Stage II, we find that the number of users choose service provider i depends on the incentive level of service provider i and j, where $i, j \in \{1, 2\}, i \neq j$. Hence, we denote it as $N_i(\beta_i, \beta_j)$. Recall that the service provider i's profit defined in Eq.2, service provider i choose an optimal incentive level $\beta_i \in [0, 1]$ to maximize his profit, which is shown as follows:

$$\beta_i^* = \underset{\beta_i \in [0,1]}{\arg\max}\,(1 - \beta_i N_i(\beta_i, \beta_j))N_i(\beta_i, \beta_j), i \neq j. \tag{4}$$

Service providers' decisions game in Stage I is defined as follows:

Definition 5.1. (Service Providers' Decisions Game in Stage I).

- Players: Service providers 1 and 2.
- Strategies: Each service provider $i \in \{1, 2\}$ choose an incentive level β_i. The feasible set of incentive level is $\beta_i \in [0, 1]$.
- Payoff: $\Pi_i(\beta_i, \beta_j), i \neq j$. The payoff is affected by the strategies of both service provider i and j.

In the following, we discuss the optimal strategies of service providers 1 and 2 under different conditions of transaction cost difference and users' stickiness in Lemma 5.2 and Lemma 5.3.

Lemma 5.2. *Given the number of users who choose service provider 1, his optimal strategies under different λ and ΔP are summarized as follows:*

- *If* $\lambda + \Delta P < 1, \beta_1^* =$

$$\begin{cases} 0 & ,\beta_2 \leq \lambda - \Delta P - 1 \\ \min\{\widehat{\beta_1}, \lambda + \Delta P\} & ,\lambda - \Delta P - 1 < \beta_2 < \lambda - \Delta P \\ \lambda + \Delta P & ,\beta_2 = \lambda - \Delta P \\ \min\{\max\{\lambda + \Delta P, \widehat{\beta_1}\}, 1\} & ,\lambda - \Delta P < \beta_2 < 2\lambda \\ \lambda + \Delta P & ,2\lambda \leq \beta_2 \leq \lambda - \Delta P + 1 \\ \underset{\beta_1 = \{\lambda + \Delta P, 1\}}{\arg\max}\ \{\Pi_1(\beta_1, \beta_2)\} & ,\beta_2 > \lambda - \Delta P + 1 \end{cases} \tag{5}$$

- *If* $\lambda + \Delta P = 1$,

$$\beta_1^* = \begin{cases} 0, & \beta_2 \leq \lambda - \Delta P - 1 \\ \min\{\widehat{\beta_1}, 1\}, & \lambda - \Delta P - 1 < \beta_2 < \lambda - \Delta P \\ \lambda + \Delta P, & \beta_2 = \lambda - \Delta P \\ 0, & \beta_2 > \lambda - \Delta P \end{cases} \tag{6}$$

- *If* $\lambda + \Delta P > 1$,

$$\beta_1^* = \begin{cases} 0, & \beta_2 \leq \lambda - \Delta P - 1 \\ \min\{\widehat{\beta_1}, 1\}, & \lambda - \Delta P - 1 < \beta_2 < \lambda - \Delta P \\ 0, & \beta_2 = \lambda - \Delta P \\ 0, & \beta_2 > \lambda - \Delta P \end{cases} \tag{7}$$

Proof of Lemma 5.2 is shown in Section 5.3. Recall that $\widehat{\beta_1}$ and $\widehat{\beta_2}$ are defined in Eq.20 and Eq.27, respectively. When the users' stickiness is much larger than the transaction cost difference, service provider 1 does not provide incentives because the cost for users to switch to another DEX platform is too high. Users will not switch unless service provider 1 provides an extremely high incentive level.

Lemma 5.3. *Given the number of users who choose service provider 2, his optimal strategies under different λ and ΔP are summarized as follows:*

- *If* $\lambda - \Delta P < 0$,

$$\beta_2^* = \begin{cases} 0 & ,\beta_1 < \lambda + \Delta P \\ 0 & ,\beta_1 = \lambda + \Delta P \\ 0 & ,\beta_1 > \lambda + \Delta P \end{cases} \tag{8}$$

- *If* $0 \leq \lambda - \Delta P < 1$,

$$\beta_2^* = \begin{cases} 0 & ,\beta_1 \leq \lambda + \Delta P - 1 \\ \min\{\widehat{\beta_2}, \lambda - \Delta P\} & ,\lambda + \Delta P - 1 < \beta_1 < \lambda + \Delta P \\ \lambda - \Delta P & ,\beta_1 = \lambda + \Delta P \\ \max\{\widehat{\beta_2}, \lambda - \Delta P\} & ,\lambda + \Delta P < \beta_1 < 2\lambda \\ \lambda - \Delta P & ,\beta_1 \geq 2\lambda \end{cases} \tag{9}$$

- *If* $\lambda - \Delta P = 1$,

$$\beta_2^* = \begin{cases} 0, & \beta_1 \leq \lambda + \Delta P - 1 \\ \widehat{\beta_2}, & \lambda + \Delta P - 1 < \beta_1 < \lambda + \Delta P \\ \lambda - \Delta P, & \beta_1 = \lambda + \Delta P \\ 0, & \beta_1 > \lambda + \Delta P \end{cases} \tag{10}$$

- *If* $\lambda - \Delta P > 1$,

$$\beta_2^* = \begin{cases} 0, & \beta_1 \leq \lambda + \Delta P - 1 \\ \widehat{\beta_2}, & \lambda + \Delta P - 1 < \beta_1 < \lambda + \Delta P \\ 0, & \beta_1 = \lambda + \Delta P \\ 0, & \beta_1 > \lambda + \Delta P \end{cases} \tag{11}$$

Proof of Lemma 5.3 is shown in Section 5.4. When the transaction cost difference is larger than the users' stickiness, service provider 2 does not need to provide incentives. Even service provider 1 provides the maximum incentive level, i.e., $\beta_1 = 1$, he can still win large number of users and the governance tokens have high value because of the network effect.

5.1 Nash Equilibrium Analysis

According to the previous analysis, we have derived the optimal strategies of service providers 1 and 2. Each service provider $i \in \{1, 2\}$ chooses his incentive level $\beta_i \in [0, 1]$ to maximize his profit $\Pi_i(\beta_i, \beta_j)$, where $j \in \{1, 2\}, j \neq i$. The equilibrium of service providers' incentive level game is defined as follows:

Definition 5.4. (Incentive Level Equilibrium in Stage I). The equilibrium of the service providers' incentive level game is (β_1^*, β_2^*) such that for each service provider $i \in \{1, 2\}$,

$$\Pi_i(\beta_i^*, \beta_j^*) \geq \Pi_i(\beta_i, \beta_j^*), \forall \beta_i \in [0, 1] \tag{12}$$

The analysis of the incentive level equilibrium in Stage I is extremely challenging since the objective functions of service providers 1 and 2 are piece-wise functions, which varies under different conditions of $\lambda - \Delta P$ and $\lambda + \Delta P$. We solve the problem by decomposing it into several cases. We first consider the case when service providers have price differences.

Lemma 5.5. *Given $\Delta P = P_1 - P_2 > 0$, the equilibriums of service providers' decisions game under different λ and ΔP (Stage I) are:*

- *When $\lambda - \Delta P < 0$ and $\lambda + \Delta P < 1$, there exist three possible equilibriums*

$$(\beta_1^*, \beta_2^*) = \begin{cases} (\lambda + \Delta P, 0), & if \widehat{\beta_1} \leq \lambda + \Delta P \\ (\widehat{\beta_1}, 0), & if \lambda + \Delta P < \widehat{\beta_1} < 1 \\ (1, 0), & if \widehat{\beta_1} \geq 1 \end{cases} \tag{13}$$

- *When $\lambda - \Delta P < 0$ and $\lambda + \Delta P \geq 1$, there exists a unique equilibrium $(\beta_1^*, \beta_2^*) = (0, 0)$.*
- *When $0 \leq \lambda - \Delta P < 1$ and $\lambda + \Delta P \leq 1$, there exists a unique equilibrium $(\beta_1^*, \beta_2^*) = (\lambda + \Delta P, \lambda - \Delta P)$.*
- *When $0 \leq \lambda - \Delta P < 1$ and $1 < \lambda + \Delta P < 2$, there exist two possible equilibriums*

$$(\beta_1^*, \beta_2^*) = \begin{cases} (\widehat{\beta_1}, 0), & if \widehat{\beta_1} \leq \lambda + \Delta P - 1 \\ (1, \widehat{\beta_2}), & if \widehat{\beta_1} > 1, \widehat{\beta_2} < \lambda - \Delta P \end{cases} \tag{14}$$

  Meanwhile, if the above conditions are both satisfied, there exist two co-existing equilibriums $(\beta_1^*, \beta_2^*) = (\widehat{\beta_1}, 0)$ and $(\beta_1^*, \beta_2^*) = (1, \widehat{\beta_2})$. However, there exists no equilibrium in other conditions.
- *When $0 \leq \lambda - \Delta P < 1$ and $\lambda + \Delta P \geq 2$, there exist two possible equilibriums*

$$(\beta_1^*, \beta_2^*) = \begin{cases} (\widehat{\beta_1}, 0), & if \widehat{\beta_1} < 1 \\ (1, 0), & if \widehat{\beta_1} \geq 1 \end{cases} \tag{15}$$

- *When $\lambda - \Delta P \geq 1$ and $\lambda + \Delta P > 1$, there exists a unique equilibrium $(\beta_1^*, \beta_2^*) = (0, 0)$.*

Proof of Lemma 5.5 is given in Appendix B. Recall the optimal strategies of service providers 1 and 2 that are derived in Lemma 5.2 and Lemma 5.3, respectively. Due to the piece-wise objective function of service providers, the equilibrium results are very complex. In most cases, there exists a unique equilibrium. We list some interesting observations when $0 \leq \lambda - \Delta P < 1$ and $1 < \lambda + \Delta P < 2$ as follows:

- When $\lambda < \frac{\Delta P + 2}{3}$, there exists no equilibrium. If service provider 2 provides a high incentive level, i.e., $\beta_2 = \lambda - \Delta P$, no user chooses service provider 1 and thus $\beta_1^*(\beta_2) = 0$. However, service provider 2 will not provide such a high incentive level if service provider 1 does not provide incentives. Meanwhile, service provider 1 provides $\beta_1^*(\beta_2) = 1$ to attract users when service provider 2 does not provide incentive, i.e., $\beta_2 = 0$. However, the high incentive level of service provider 1 leads to service provider 2 will provide $\beta_2 = \lambda - \Delta P$ to attract users. Hence, two service providers will keep competing and can not achieve equilibrium.
- When $\frac{2\sqrt{\Delta P^2 - \Delta P + 1} + \Delta P + 1}{3} < \lambda < f(\Delta P)$[11], there exist two co-existing equilibriums $(\beta_1^*, \beta_2^*) = (\widehat{\beta_1}, 0)$ and $(\beta_1^*, \beta_2^*) = (1, \widehat{\beta_2})$. We find that service provider 1 can win a larger user base in the equilibrium when he chooses to provide the incentive level with the lower incentive level, i.e., $\beta_1^* = \widehat{\beta_1}$. Since service provider 2 does not provide incentives, he can win a larger user base. Service provider 1 can win a larger user base by providing a lower incentive level, and thus his profit is also larger in this equilibrium.

In the following, we discuss the equilibriums when service providers do not have price differences, i.e., $\Delta P = P_1 - P_2 = 0$.

Lemma 5.6. *Given $P_1 = P_2$, the equilibriums of service providers' decisions game under different λ are:*

- *When $0 < \lambda < 1$, there exists a unique equilibrium $(\beta_1^*, \beta_2^*) = (\lambda, \lambda)$.*
- *When $\lambda = 1$, there exist various equilibriums $(\beta_1^*, \beta_2^*) = (\widehat{\beta_1}, \widehat{\beta_2}) = (\beta_2, \beta_1)$.*
- *When $\lambda > 1$, there exists a unique equilibrium $(\beta_1^*, \beta_2^*) = (0, 0)$.*

Proof of Lemma 5.6 is given in Appendix B. When $\lambda = 1$ and service providers provide the same incentive level, the reward received from transaction fee mining are equal to the cost induced by users' stickiness. Meanwhile, transaction costs of service providers are the same. Hence, users get same payoff when choosing service providers, and both service providers can win half of the users in the market.

5.2 Equilibrium Characterization

To present insights, we divide the domain of λ and ΔP into four regions to show how $\lambda + \Delta P$ and $\lambda - \Delta P$ affect the behaviors. Moreover, we define $\lambda + \Delta P$ as the impact level of users' stickiness and transaction cost difference and divide it into two levels. We define the impact level to be "*large*" when $\lambda + \Delta P > 1$ and "*small*" when $0 < \lambda + \Delta P < 1$. Then, we define the market to be "*cost dominating*" when $\lambda - \Delta P < 0$ and "*stickiness dominating*" when $\lambda - \Delta P > 0$.

Theorem 5.7. *There exists four equilibrium structures, which are decided by $\lambda + \Delta P$ and $\lambda - \Delta P$:*

[11] where $f(\Delta P) = \frac{g(\Delta P)}{9\sqrt[3]{2}} - \frac{\sqrt[3]{2}(6\Delta P - 13(\Delta P)^2)}{9g(\Delta P)} - \frac{2(\Delta P - 3)}{9}$,
$g(\Delta P) = \sqrt[3]{-70(\Delta P)^3 + 198(\Delta P)^2 + 9h(\Delta P) + 108(\Delta P) + 27}$,
$h(\Delta P) = \sqrt{-48(\Delta P)^6 - 192(\Delta P)^5 + 228(\Delta P)^4 + 492(\Delta P)^3 + 276(\Delta P)^2 + 72(\Delta P) + 9}$.

- *"Impact level is small and it is stickiness dominating", i.e., $0 < \lambda + \Delta P < 1$ and $\lambda - \Delta P > 0$: there exists a unique equilibrium $(\beta_1^*, \beta_2^*) = (\lambda+\Delta P, \lambda-\Delta P)$. Both service providers provide incentives.*
- *"Impact level is small and it is cost dominating", i.e., $0 < \lambda + \Delta P < 1$ and $\lambda - \Delta P < 0$: there exists a unique equilibrium $(\beta_1^*, \beta_2^*) = (\min\{\max\{\lambda + \Delta P, \widehat{\beta_1}\}, 1\}, 0)$. Service provider 2, who has a lower cost, does not provide incentives, while service provider 1 provides incentives depending on λ and ΔP.*
- *"Impact level is large and it is cost dominating", i.e., $\lambda+\Delta P > 1$ and $\lambda - \Delta P < 0$: there exists a unique equilibrium $(\beta_1^*, \beta_2^*) = (0, 0)$. Service provider 1 quits the DEX market, and service provider 2 does not provide incentive.*
- *"Impact level is large and it is stickiness dominating", i.e., $\lambda + \Delta P > 1$ and $\lambda - \Delta P > 0$: there exists various unique equilibriums $(\beta_1^*, \beta_2^*) = (\min\{\max\{0, \widehat{\beta_1}\}, 1\}, 0)$, $(\beta_1^*, \beta_2^*) = (1, \widehat{\beta_2})$. Menawhile, there also exist two co-existing equilibriums $(\beta_1^*, \beta_2^*) = (\widehat{\beta_1}, 0)$ and $(\beta_1^*, \beta_2^*) = (1, \widehat{\beta_2})$. Besides, there might not exists any equilibrium. Both service providers are possible to provide incentives.*

When the impact level is small and it is stickiness dominating, both service providers 1 and 2 can take over the whole market by providing incentive levels $\beta_1 = \lambda+\Delta P$ and $\beta_2 = \lambda-\Delta P$, respectively. Hence, we can achieve the equilibrium $(\beta_1^*, \beta_2^*) = (\lambda + \Delta P, \lambda - \Delta P)$ and both of them win half of the users in the DEX market.

When the impact level is small and it is cost dominating, service provider 2 is more competitive. All users choose service provider 2 unless service provider 1 provides an incentive level $\beta_1 \geq \lambda + \Delta P$. Due to the cost advantage, service provider 2 has a large user base. The loss of issuing governance tokens is large enough such that he does not provide incentive, i.e., $\beta_2^* = 0$. Given $\beta_2 = 0$, we derive $\beta_1^*(\beta_2) = \min\{\max\{\lambda + \Delta P, \widehat{\beta_1}\}, 1\}$. Hence, we can achieve the equilibrium $(\beta_1^*, \beta_2^*) = (\min\{\max\{\lambda + \Delta P, \widehat{\beta_1}\}, 1\}, 0)$.

When the impact level is large and it is cost dominating, service provider 2 take over the DEX market even when he does not provide incentives, i.e.$\beta_2 = 0$. Hence, service provider 1 quits the market. Hence, we can achieve the equilibrium $(\beta_1^*, \beta_2^*) = (0, 0)$.

When the impact level is large and it is stickiness dominating, the problem is complicated and we decompose the problem into several cases. First, when $\lambda - \Delta P > 1$, users' stickiness is large enough such that both of the service providers have a positive user base. Due to the cost advantage of service provider 2, the optimal strategy of service provider 1 is not to provide incentives since he cannot achieve a higher payoff. Second, when $0 < \lambda - \Delta P < 1$, service provider 1 is competitive. He can win more users by providing a high incentive level and can let service provider 2 provides a positive incentive level. When $\lambda < \frac{\Delta P+2}{3}$, He can even let service provider 2 provides an incentive level $\beta_2 = \lambda - \Delta P$ to take over the market.

So far, we have completed the discussion in Theorem 5.7 and we mark the following insights:

- The service provider with a less transaction cost can win the market without providing governance tokens. Even can let the service provider with higher transaction cost quit the market in some cases.
- Though the service provider with a lower transaction cost can win the market, users' stickiness can offset the market advantage. In the most intense competition cases, i.e., $0 < \lambda - \Delta P < 1$, service providers compete with each other, and both of them incentivize users with transaction fee mining.

5.3 Proof of Lemma 5.2

According to backward induction, we can get the number of users choose service provider 1 from Stage II:

- When $\beta_2 < \lambda - \Delta P$,

$$N_1 = \begin{cases} \frac{\lambda-\beta_2-\Delta P}{2\lambda-\beta_1-\beta_2} & ,\beta_1 \leq \lambda + \Delta P \\ 1 & ,\beta_1 > \lambda + \Delta P \end{cases} \tag{16}$$

- When $\beta_2 > \lambda - \Delta P$,

$$N_1 = \begin{cases} 0 & ,\beta_1 < \lambda + \Delta P \\ \frac{\lambda-\beta_2-\Delta P}{2\lambda-\beta_1-\beta_2} & ,\beta_1 \geq \lambda + \Delta P \end{cases} \tag{17}$$

- When $\beta_2 = \lambda - \Delta P$,

$$N_1 = \begin{cases} 0 & ,\beta_1 < \lambda + \Delta P \\ \frac{1}{2} & ,\beta_1 = \lambda + \Delta P \\ 1 & ,\beta_1 > \lambda + \Delta P \end{cases} \tag{18}$$

By substituting N_1 into Eq.2, we can get the profit function of service provider 1. Since service provider 1's profit function is different when the strategies of service provider 2 is in different cases, we discuss the optimal strategies of service provider 1 separately.

When $\beta_2 < \lambda - \Delta P$, service provider 1's profit is reorganized as follows:

$\Pi_1(\beta_1, \beta_2)$

$$= \begin{cases} \frac{\lambda-\beta_2+\Delta P}{2\lambda-\beta_1-\beta_2} - \beta_1(\frac{\lambda-\beta_2+\Delta P}{2\lambda-\beta_1-\beta_2})^2 & ,\beta_1 \leq \lambda + \Delta P \\ 1 - \beta_1 & ,\beta_1 > \lambda + \Delta P \end{cases} \tag{19}$$

To find out the optimal strategy in this case, we need to find out whether the function in the first section is convex or concave by taking the first and second order derivative. Let the first order derivative equals to 0, we can obtain:

$$\widehat{\beta_1} = \frac{(2\lambda - \beta_2)(1 - \lambda + \beta_2 + \Delta P)}{1 + \lambda - \beta_2 - \Delta P} \tag{20}$$

We find that the curve is different in the following three cases. Hence, we discuss the optimal strategies separately:

- If $\beta_2 \leq \lambda - 1 - \Delta P$, the profit of service provider 1 keeps decreasing as β_1 increases. Hence, we can get $\beta_1^* = 0$.
- If $\lambda - 1 - \Delta P < \beta_2 \leq \lambda - \frac{1}{2} - \Delta P$, we can get $\beta_1^* = \widehat{\beta_1}$. However, we need to consider the domain of the function in the first section and the domain of service provider 1's strategy. Hence, the optimal strategy becomes $\beta_1^* = \min\{\widehat{\beta_1}, \lambda+\Delta P, 1\}$.
- If $\lambda - \frac{1}{2} - \Delta P < \beta_2 < \lambda - \Delta P$, we can get $\beta_1^* = \widehat{\beta_1}$. Similar to the reasons explained in previous case, the optimal strategy becomes $\beta_1^* = \min\{\widehat{\beta_1}, \lambda + \Delta P, 1\}$.

When $\beta_2 > \lambda - \Delta P$, service provider 1's profit is reorganized as follows:

$\Pi_1(\beta_1, \beta_2)$

$$= \begin{cases} 0 & ,\beta_1 < \lambda + \Delta P \\ \frac{\lambda-\beta_2+\Delta P}{2\lambda-\beta_1-\beta_2} - \beta_1(\frac{\lambda-\beta_2+\Delta P}{2\lambda-\beta_1-\beta_2})^2 & ,\beta_1 \geq \lambda + \Delta P \end{cases} \quad (21)$$

We can find that service provider 1's profits is always 0 when $\lambda + \Delta P \geq 1$. Hence, we let the optimal strategy of service provider 1 to be 0. As for the cases when $\lambda + \Delta P < 1$. Similarly, the curve is different in three cases. Hence, we discuss the optimal strategies separately:

- If $\lambda - \Delta P < \beta_2 < 2\lambda$, we can get $\beta_1^* = \widehat{\beta_1}$. However, we need to consider the domain of the function in the second section and the domain of service provider 1's strategy. Hence, the optimal strategy becomes $\beta_1^* = \min\{\max\{\lambda + \Delta P, \widehat{\beta_1}\}, 1\}$.
- If $2\lambda \leq \beta_2 \leq 1 + \lambda - \Delta P$, the profit of service provider 1 keeps decreasing as β_1 increases in the function in the second section. Hence, we can get $\beta_1^* = \lambda + \Delta P$.
- If $1 + \lambda - \Delta P < \beta_2$, β_1^* has two possible values: $\lambda + \Delta P, 1$. We need to compare $\Pi_1(\lambda + \Delta P, \beta_2)$ and $\Pi_1(1, \beta_2)$. Hence, we can get $\beta_1^* = \underset{\beta_1=\{\lambda+\Delta P,1\}}{\arg\max} \{\Pi_1(\beta_1, \beta_2)\}$.

When $\beta_2 = \lambda - \Delta P$, service provider 1's profit is reorganized as follows:

$$\Pi_1(\beta_1, \beta_2) = \begin{cases} 0 & ,\beta_1 < \lambda + \Delta P \\ \frac{1}{2} - \frac{\beta_1}{4} & ,\beta_1 = \lambda + \Delta P \\ 0 & ,\beta_1 > \lambda + \Delta P \end{cases} \quad (22)$$

We can find that service provider 1 has a positive profits only when $\beta_1 = \lambda + \Delta P$. If $\lambda + \Delta P \leq 1$, we can get $\beta_1^* = \lambda + \Delta P$. Otherwise, service provider 1's profits is always 0 and we let the optimal strategy of service provider 1 to be 0.

5.4 Proof of Lemma 5.3

Through backward induction, the number of users who choose service provider 2 from Stage II:

- When $\beta_1 < \lambda + \Delta P$,

$$N_2 = \begin{cases} \frac{\lambda-\beta_1-\Delta P}{2\lambda-\beta_1-\beta_2} & ,\beta_2 \leq \lambda - \Delta P \\ 1 & ,\beta_2 > \lambda - \Delta P \end{cases} \quad (23)$$

- When $\beta_1 > \lambda + \Delta P$,

$$N_2 = \begin{cases} 0 & ,\beta_2 < \lambda - \Delta P \\ \frac{\lambda-\beta_1-\Delta P}{2\lambda-\beta_1-\beta_2} & ,\beta_2 \geq \lambda - \Delta P \end{cases} \quad (24)$$

- When $\beta_1 = \lambda + \Delta P$,

$$N_2 = \begin{cases} 0 & ,\beta_2 < \lambda - \Delta P \\ \frac{1}{2} & ,\beta_2 = \lambda - \Delta P \\ 1 & ,\beta_2 > \lambda - \Delta P \end{cases} \quad (25)$$

With the number of users choosing service provider 2, we can derive the profit function of service provider 2, which is separated into three cases. For each cases, we can find out the optimal strategy of service provider 1.

When $\beta_1 < \lambda + \Delta P$, service provider 2's profit is reorganized as follows:

$\Pi_2(\beta_1, \beta_2)$

$$= \begin{cases} \frac{\lambda-\beta_1+\Delta P}{2\lambda-\beta_1-\beta_2} - \beta_2(\frac{\lambda-\beta_1+\Delta P}{2\lambda-\beta_1-\beta_2})^2, & \beta_2 \leq \lambda - \Delta P \\ 1 - \beta_2, & \beta_2 > \lambda - \Delta P \end{cases} \quad (26)$$

Obviously, the function in the second section is a downward straight line. If $\lambda - \Delta P \leq 0$, the optimal strategy of service provider 2 is $\beta_2^* = 0$. While in other cases, we need to consider whether the function in the first section is convex or concave to find out the optimal strategy. Let the first order derivative equals to 0, we can obtain:

$$\widehat{\beta_2} = \frac{(2\lambda - \beta_1)(1 - \lambda + \beta_1 - \Delta P)}{1 + \lambda - \beta_1 + \Delta P} \quad (27)$$

We find that $\widehat{\beta_2} \leq 1$ when $\beta_2 \in [0, 1]$. Besides, the curve is different in the following three cases. Hence, we discuss the optimal strategies separately:

- If $\beta_1 \leq \lambda - 1 + \Delta P$, the profit of service provider 2 goes down as β_2 goes up. Hence, we can get $\beta_2^* = 0$.
- If $\lambda - 1 + \Delta P < \beta_1 \leq \lambda - \frac{1}{2} + \Delta P$, we can get $\beta_2^* = \widehat{\beta_2}$. Besides, we find that $\widehat{\beta_2}$ is always less than 1 when $\beta_1 \in [0, 1]$. However, we still need to consider the domain of the function in the first section. Hence, we can get $\beta_2^* = \min\{\widehat{\beta_2}, \lambda - \Delta P\}$.
- If $\lambda - \frac{1}{2} + \Delta P < \beta_1 < \lambda + \Delta P$, we can get $\beta_2^* = \widehat{\beta_2}$. Similar to the reasons explained in previous case, we can get $\beta_2^* = \min\{\widehat{\beta_2}, \lambda - \Delta P\}$.

When $\beta_1 > \lambda + \Delta P$, service provider 2's profit is reorganized as follows:

$\Pi_2(\beta_1, \beta_2)$

$$= \begin{cases} 0, & \beta_2 < \lambda - \Delta P \\ \frac{\lambda-\beta_1+\Delta P}{2\lambda-\beta_1-\beta_2} - \beta_2(\frac{\lambda-\beta_1+\Delta P}{2\lambda-\beta_1-\beta_2})^2, & \beta_2 \geq \lambda - \Delta P \end{cases} \quad (28)$$

We can find that service provider 2's profits is always 0 when $\lambda - \Delta P \geq 1$. Hence, we let the optimal strategy of service provider 1 to be 0. As for the case when $\lambda - \Delta P \leq 0$, service provider 2's profits is the function in the second section. The profits goes down as β_2 increases. Hence, we can get $\beta_2^* = 0$. When $0 < \lambda + \Delta P < 1$, the curve is different in two cases. Hence, we discuss the optimal strategies separately:

- If $\lambda + \Delta P < \beta_1 < 2\lambda$, we can get $\beta_2^* = \widehat{\beta_2}$. Meanwhile, $\widehat{\beta_2} < 1$ when $\beta_1 \in [0, 1]$. Considering the domain of the function in the second section, we can get $\beta_2^* = \max\{\widehat{\beta_2}, \lambda - \Delta P\}$.
- If $\beta_1 \geq 2\lambda$, the profit of service provider 2 keeps decreasing when $\beta_2 \geq \lambda - \Delta P$. Hence, we can get $\beta_2^* = \lambda - \Delta P$.

When $\beta_1 = \lambda + \Delta P$, service provider 2's profit is reorganized as follows:

$$\Pi_2(\beta_1, \beta_2) = \begin{cases} 0, & \beta_2 < \lambda - \Delta P \\ \frac{1}{2} - \frac{\beta_2}{4}, & \beta_2 = \lambda - \Delta P \\ 0, & \beta_2 > \lambda - \Delta P \end{cases} \quad (29)$$

We can find that service provider 2 has a positive profits only when $\beta_2 = \lambda - \Delta P$. If $\lambda - \Delta P \leq 1$, we can get $\beta_2^* = \lambda - \Delta P$. Otherwise, service provider 2's profits is always 0 and we let the optimal strategy of service provider 2 to be 0.

6 SIMULATION RESULTS

In this section, we perform simulations to verify the results we derived.

6.1 Nash Equilibrium

As shown in Fig.4, the simulation results demonstrate the equilibrium results under different conditions of ΔP and λ. The horizontal coordinate represents the difference in cost, i.e., ΔP, while the vertical coordinate represents the users' stickiness, i.e., λ. For the green region, there exists no equilibrium. As for the orange region, there exists two co-existing equilibriums $(\beta_1^*, \beta_2^*) = (\widehat{\beta}_1, 0)$ and $(\beta_1^*, \beta_2^*) = (1, \widehat{\beta}_2)$. For the remaining grey region, there exists a unique equilibrium and we demonstrate the equilibrium results (β_1^*, β_2^*) in the figure. The simulation results verify the results of our previous analysis in Lemma 5.5 and Lemma 5.6.

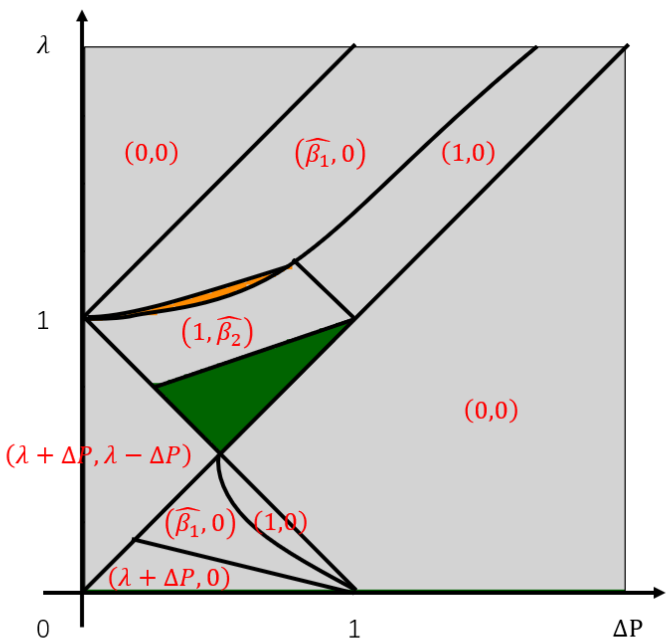


Figure 4: Simulation Results

6.2 Service Provider's Profits

In this subsection, we perform simulations on service providers' utilities with different parameter settings. With the simulation results, we try to find out the reason why there are different equilibrium results in different parameter settings.

6.2.1 When $\lambda + \Delta P < 1$ and $\lambda - \Delta P < 0$. In Fig.5(a) and Fig.5(b), we let $\lambda + \Delta P = 0.9$ and modify $\lambda - \Delta P$ from -0.1 to -0.9, with a step size of 0.2.

Firstly, we consider the case when the optimal strategy of service provider 2 is 0 in Fig.5(a). We can find that the optimal strategy of service provider 1 is $\widehat{\beta}_1$ when $\beta_1 > \lambda + \Delta P$. In order to better demonstrate the results, we demonstrate the simulation results with the value of β_1 range from 0 to 2.5. However, since the incentive level needs to be smaller than 1 and larger than $\lambda + \Delta P$, the optimal strategies of service provider 1 have three possible values: $\lambda + \Delta P, \widehat{\beta}_1, 1$.

While in Fig.5(b), we perform simulations to show the optimal strategies of service provider 2 when service provider 1's optimal strategy is 1. We can find that the optimal strategies is 0 even when service provider 1 choose his maximum incentive level. We can conclude that service provider 2's optimal strategy is always 0 due to the cost advantage. Hence, there exists three equilibriums $(\beta_1^*, \beta_2^*) = (\widehat{\beta}_1, 0)$, $(\beta_1^*, \beta_2^*) = (\lambda + \Delta P, 0)$ and $(\beta_1^*, \beta_2^*) = (1, 0)$ in this case.

6.2.2 When $\lambda + \Delta P > 1$ and $\lambda - \Delta P > 0$.
In Fig.5(c) we set the parameter $\lambda + \Delta P = 2.7$ and modify the parameter $\lambda - \Delta P$ from 0.3 to 1.1, with a step size of 0.2. We consider the case when the optimal strategy of service provider 2 is 0. In order to better demonstrate the results, we perform simulations with different value of β_1 range from -2 to 2. From the simulation results, we can find that the optimal strategy of service provider 1 is $\widehat{\beta}_1$. However, due to the limit of the incentive level's domain, the optimal strategies of service provider 1 have three possible values: $0, \widehat{\beta}_1, 1$.

In Fig.5(d), we set the parameter $\lambda + \Delta P = 1.7$ and modify the parameter $\lambda - \Delta P$ from 0.1 to 0.9, with a step size of 0.2. We demonstrate the optimal strategy of service provider 2 when service provider 1's strategy is 1. The optimal strategies of service provider 2 is $\widehat{\beta}_2$ when $\beta_2 < \lambda - \Delta P$. We can find that $\beta_2^* = \widehat{\beta}_2$ when $\widehat{\beta}_2 < \lambda - \Delta P$ and $\beta_2^* = \lambda - \Delta P$ otherwise.

In Fig.5(e), we set the parameter $\lambda - \Delta P = 0.4$ and modify the parameter $\lambda + \Delta P$ from 1.4 to 2.2, with a step size of 0.2. Similar to Fig.5(d), we also demonstrate the optimal strategies of service provider 2 when service provider 1 set up the maximum incentive level. The optimal strategies of service provider 2 is $\widehat{\beta}_2$ when $\beta_2 < \lambda - \Delta P$. We can find that $\beta_2^* = \widehat{\beta}_2$ when $\widehat{\beta}_2 > 0$ and $\beta_2^* = 0$ otherwise.

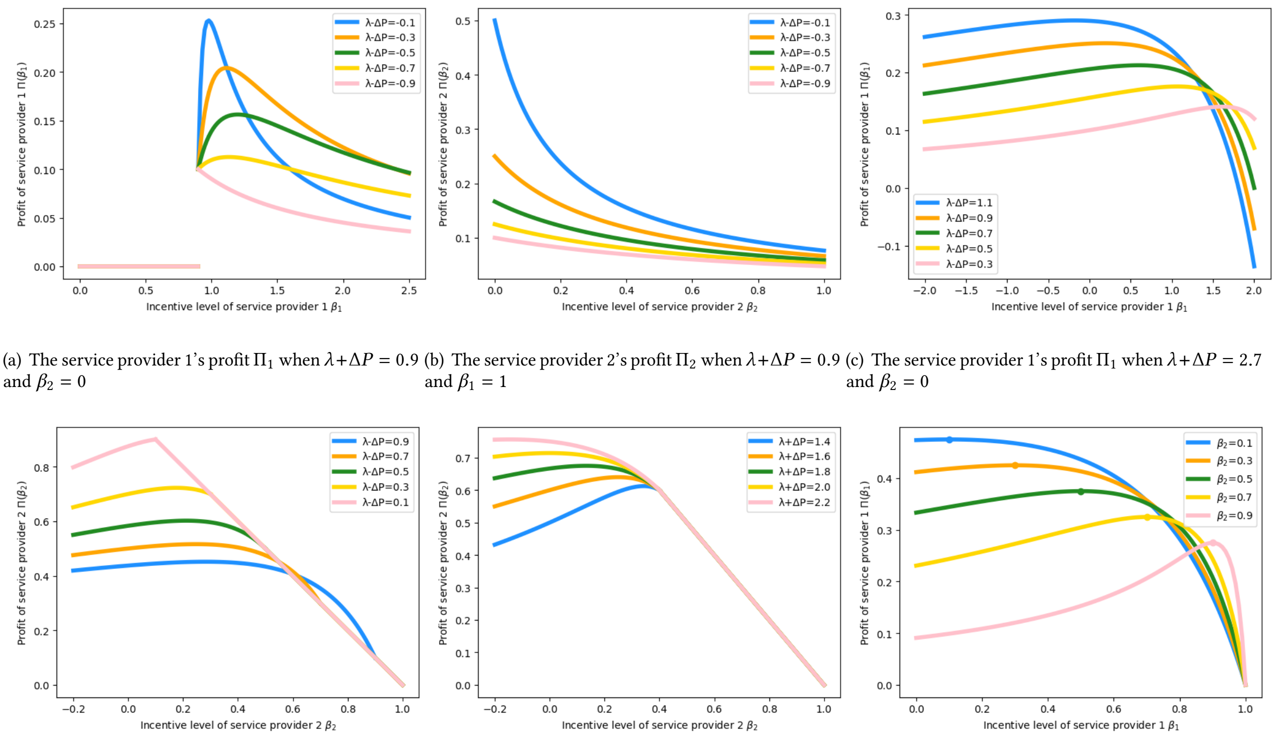
In conclusion, we can explain the reason why the equilibrium results in this case is so complex. Due to the cost advantage, service provider 2's optimal strategy is 0 under most conditions. Meanwhile, service provider 1's optimal strategy is $\widehat{\beta}_1$, correspondingly. Due to the domain of incentive level is $\beta_1 \in [0, 1]$, the optimal strategy of service provider becomes 0 when $\widehat{\beta}_1 < 0$ and becomes 1 when $\widehat{\beta}_1 > 1$. Hence, there exists three equilibrium $(\beta_1^*, \beta_2^*) = (0, 0)$, $(\beta_1^*, \beta_2^*) = (\widehat{\beta}_1, 0)$ and $(\beta_1^*, \beta_2^*) = (1, 0)$.

However, when the incentive level of service provider is 1, the optimal strategy of service provider 2 is not always 0 as shown in Fig.5(e). The optimal strategy of service provider 2 becomes $\widehat{\beta}_2$ when $\widehat{\beta}_2 > 0$. Hence, there exists an equilibrium $(\beta_1^*, \beta_2^*) = (1, \widehat{\beta}_2)$.

Meanwhile, when $\widehat{\beta}_2 > \lambda - \Delta P$, the optimal strategy of service provider 2 becomes $\lambda - \Delta P$. However, the optimal strategy of service provider 1 is not 1. Hence, there's no equilibrium in this case.

6.3 When $\lambda - \Delta P = 1$ and $\lambda + \Delta P = 1$

From the condition we can know that $\lambda = 1$ and $\Delta P = 0$. In Fig.5(f), we demonstrate the utility of service provider 1 and scatter his optimal strategy when the strategy of service provider 2 varies. We



(a) The service provider 1's profit Π_1 when $\lambda + \Delta P = 0.9$ and $\beta_2 = 0$

(b) The service provider 2's profit Π_2 when $\lambda + \Delta P = 0.9$ and $\beta_1 = 1$

(c) The service provider 1's profit Π_1 when $\lambda + \Delta P = 2.7$ and $\beta_2 = 0$

(d) The service provider 2's profit Π_2 when $\lambda + \Delta P = 1.7$ and $\beta_1 = 1$

(e) The service provider 2's profit Π_2 when $\lambda - \Delta P = 0.4$ and $\beta_1 = 1$

(f) The service provider 1's profit Π_1 when $\lambda = 1, \Delta P = 0$

Figure 5: Service Providers' Profits

can find that the optimal strategy is always same as the strategy of service provider 2.

6.4 Other cases

When $\lambda + \Delta P < 1$ and $\lambda - \Delta P > 0$: When $\beta_2 = \lambda - \Delta P$, the utility of service provider 1 is positive only when $\beta_1 = \lambda + \Delta P$, vice versa. *When $\lambda + \Delta P > 1$ and $\lambda - \Delta P < 0$:* Even when $\beta_2 = 0$, the utility of service provider 1 is always 0. These two cases are too simple so that we would not demonstrate the results of service providers.

7 CONCLUSION

This paper study a novel and interesting question on the DEX market competition with transaction fee mining mechanism. We propose a two-stage game to formulate the interactions between DEXs and users. Different from the price competition discussed in state-of-the-art platform competition work, we consider service providers' incentive levels in our model and derive their optimal incentive levels in this paper. Besides, we also derive the equilibriums under different conditions of the transaction cost difference and users' stickiness and analyze these equilibrium results. We show that though the service provider with a lower transaction cost can win the market, users' stickiness can offset the market advantage. In the most intense competition cases, service providers compete with each other, and both of them incentivize users with transaction fee mining.

ACKNOWLEDGEMENTS

This work was supported by Shenzhen Science and Technology Program (Grant No. JCYJ20210324124205016), in part by the Project AC01202101038 from Shenzhen Institute of Artificial Intelligence and Robotics for Society.

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APPENDIX

A USERS' DECISIONS

When $\beta_1 > \lambda + \Delta P$ and $\beta_2 < \lambda - \Delta P$, we can derive:

$$U_m^1 - U_m^2 = (\beta_1 - \lambda - \Delta P)x_m - (\beta_2 - \lambda + \Delta P)(1 - x_m) > 0 \quad (30)$$

Hence, the utility of choosing service provider 1 is always greater than that of service provider 2. Similarly, we can get $U_m^2 > U_m^1$ when $\beta_1 < \lambda + \Delta P$ and $\beta_2 > \lambda - \Delta P$.

In other cases, we can consider there's a marginal user n in the DEX market, where the payoffs of choosing service providers 1 and 2 are same, i.e., $U_n^1 = U_n^2$. Hence, we can derive the location of marginal user is:

$$x_n = \frac{1}{2} + \frac{\beta_1 N_1 - \beta_2 N_2 - \Delta P}{2\lambda} \quad (31)$$

Since $N_1 = x_n, N_2 = 1 - x_n$, we can derive the number of user N_1 and N_2. When $2\lambda - \beta_1 - \beta_2 \neq 0$, the number of users choose service provider 1 is:

$$N_1 = \frac{\lambda - \beta_2 - \Delta P}{2\lambda - \beta_1 - \beta_2} \quad (32)$$

Meanwhile, when $\beta_1 = \lambda + \Delta P$ and $\beta_2 = \lambda - \Delta P$, we can derive $N_1 = N_2 = \frac{1}{2}$. The proof is now completed.

B NASH EQUILIBRIUM

In order to achieve the Nash Equilibrium, we conduct the analysis in the following cases:

- $\lambda - \Delta P < 0, \lambda + \Delta P < 1$
- $\lambda - \Delta P < 0, \lambda + \Delta P = 1$
- $\lambda - \Delta P < 0, \lambda + \Delta P > 1$
- $0 \le \lambda - \Delta P < 1, \lambda + \Delta P < 1$
- $0 \le \lambda - \Delta P < 1, \lambda + \Delta P = 1$
- $0 \le \lambda - \Delta P < 1, \lambda + \Delta P > 1$
- $\lambda - \Delta P = 1, \lambda + \Delta P = 1$, i.e., $\lambda = 1$ and $\Delta P = 0$
- $\lambda - \Delta P \ge 1, \lambda + \Delta P > 1$

(1) When $\lambda - \Delta P < 0$ and $\lambda + \Delta P < 1$,
The optimal strategy of service provider 1:

$$\beta_1^* = \begin{cases} \min\{\max\{\lambda + \Delta P, \widehat{\beta_1}\}, 1\} & ,0 \le \beta_2 < 2\lambda \\ \lambda + \Delta P & ,2\lambda \le \beta_2 \le \lambda - \Delta P + 1 \\ \underset{\beta_1 = \{\lambda + \Delta P, 1\}}{\arg\max} \{\Pi_1(\beta_1, \beta_2)\} & ,\lambda - \Delta P + 1 < \beta_2 \le 1 \end{cases} \quad (33)$$

The optimal strategy of service provider 2:

$$\beta_2^* = \begin{cases} 0, & 0 \le \beta_1 < \lambda + \Delta P \\ 0, & \beta_1 = \lambda + \Delta P \\ 0, & \lambda + \Delta P < \beta_1 \le 1 \end{cases} \quad (34)$$

When $\beta_2 = 0$, $\beta_1^*(\beta_2)$ has three possible values: $\lambda + \Delta P, \widehat{\beta_1}, 1$, which depends on the value of $\widehat{\beta_1}$. When $\lambda + \Delta P < \widehat{\beta_1} < 1$, $\beta_1^*(\beta_2) = \widehat{\beta_1}$ when $\beta_2 = 0$. When $\widehat{\beta_1} \ge 1$, $\beta_1^*(\beta_2) = 1$ when $\beta_2 = 0$. When $\widehat{\beta_1} \le \lambda + \Delta P$, $\beta_1^*(\beta_2) = \lambda + \Delta P$ when $\beta_2 = 0$. Meanwhile, we can get $\beta_2^*(\beta_1) = 0$ for any $\beta_1 \in [0, 1]$. Hence, there exists equilibriums $(\beta_1^*, \beta_2^*) = (\min\{\max\{\lambda + \Delta P, \widehat{\beta_1}\}, 1\}, 0)$.

(2) When $\lambda - \Delta P < 0$ and $\lambda + \Delta P = 1$,
The optimal strategy of service provider 1:

$$\beta_1^* = 0, \forall \beta_2 \in [0, 1] \quad (35)$$

The optimal strategy of service provider 2:

$$\beta_2^* = 0, \forall \beta_1 \in [0, 1] \quad (36)$$

For any $\beta_1 \in [0, 1]$, we can achieve $\beta_2^*(\beta_1) = 0$. Meanwhile, we can achieve $\beta_1^*(\beta_2) = 0$ for any $\beta_2 \in [0, 1]$. Hence, there exists equilibrium $(\beta_1^*, \beta_2^*) = (0, 0)$.

(3) When $\lambda - \Delta P < 0$ and $\lambda + \Delta P > 1$,
The optimal strategy of service provider 1:

$$\beta_1^* = 0, \forall \beta_2 \in [0, 1] \quad (37)$$

The optimal strategy of service provider 2:

$$\beta_2^* = 0, \forall \beta_1 \in [0, 1] \quad (38)$$

For any $\beta_1 \in [0, 1]$, we can achieve $\beta_2^*(\beta_1) = 0$. Meanwhile, we can achieve $\beta_1^*(\beta_2) = 0$ for any $\beta_2 \in [0, 1]$. Hence, there exists equilibrium $(\beta_1^*, \beta_2^*) = (0, 0)$.

(4) When $0 \le \lambda - \Delta P < 1$ and $\lambda + \Delta P < 1$,
The optimal strategy of service provider 1:

$$\beta_1^* = \begin{cases} \min\{\widehat{\beta_1}, \lambda + \Delta P\} & ,0 \le \beta_2 < \lambda - \Delta P \\ \lambda + \Delta P & ,\beta_2 = \lambda - \Delta P \\ \min\{\max\{\lambda + \Delta P, \widehat{\beta_1}\}, 1\} & ,\lambda - \Delta P < \beta_2 < 2\lambda \\ \lambda + \Delta P & ,2\lambda \le \beta_2 \le 1 \end{cases} \quad (39)$$

The optimal strategy of service provider 2:

$$\beta_2^* = \begin{cases} \min\{\widehat{\beta_2}, \lambda - \Delta P\} & ,0 \le \beta_1 < \lambda + \Delta P \\ \lambda - \Delta P & ,\beta_1 = \lambda + \Delta P \\ \max\{\widehat{\beta_2}, \lambda - \Delta P\} & ,\lambda + \Delta P < \beta_1 < 2\lambda \\ \lambda - \Delta P & ,2\lambda \le \beta_1 \le 1 \end{cases} \quad (40)$$

When $\beta_1 = \lambda + \Delta P$, $\beta_2^*(\beta_1) = \lambda - \Delta P$. When $\beta_2 = \lambda - \Delta P$, $\beta_1^*(\beta_2) = \lambda + \Delta P$. Hence, we can easily find the equilibrium $(\beta_1^*, \beta_2^*) = (\lambda + \Delta P, \lambda - \Delta P)$.
When $\beta_1 = 1$, $\beta_2^*(\beta_1) = \lambda - \Delta P$. When $\beta_2 = \lambda - \Delta P$, $\beta_1^*(\beta_2) \neq 1$. Hence, there's no equilibrium when $\beta_1^* = 1$.
When $\beta_1 = \widehat{\beta_1}$, $\beta_2^*(\beta_1)$ has two possible value: $\lambda - \Delta P, \widehat{\beta_2}$. If $\beta_2 = \lambda - \Delta P$, $\beta_1^*(\beta_2) \neq \widehat{\beta_1}$. If $\beta_2 = \widehat{\beta_2}$ and $\beta_1 = \widehat{\beta_1}$, we can derive $\beta_1^* = \lambda + \Delta P$ and $\beta_2^* = \lambda - \Delta P$ by solving the equations. Hence, we can find the equilibrium $(\beta_1^*, \beta_2^*) = (\lambda + \Delta P, \lambda - \Delta P)$.

(5) When $0 \le \lambda - \Delta P < 1$ and $\lambda + \Delta P = 1$,
The optimal strategy of service provider 1:

$$\beta_1^* = \begin{cases} \min\{\widehat{\beta_1}, 1\} & ,0 \le \beta_2 < \lambda - \Delta P \\ 1 & ,\beta_2 = \lambda - \Delta P \\ 0 & ,\lambda - \Delta P < \beta_2 \le 1 \end{cases} \tag{41}$$

The optimal strategy of service provider 2:

$$\beta_2^* = \begin{cases} 0 & ,\beta_1 = 0 \\ \min\{\widehat{\beta_2}, \lambda - \Delta P\} & ,0 < \beta_1 < 1 \\ \lambda - \Delta P & ,\beta_1 = 1 \end{cases} \tag{42}$$

When $\beta_2 = \lambda - \Delta P$, $\beta_1^*(\beta_2) = 1$. When $\beta_1 = 1$, $\beta_2^*(\beta_1) = \lambda - \Delta P$. Hence, we can easily find the equilibrium $(\beta_1^*, \beta_2^*) = (1, \lambda - \Delta P)$.
When $\beta_2 = 0$, $\beta_1^*(\beta_2)$ has two possible values: $\widehat{\beta_1}, 1$. However, $\beta_2^*(\beta_1) \neq 0$ when $\beta_1 = \widehat{\beta_1}$ or 1. Hence, there's no equilibrium when $\beta_2 = 0$.
When $\beta_2 = \widehat{\beta_2}$, $\beta_1^*(\beta_2)$ has two possible values: $\widehat{\beta_1}, 1$. If $\beta_1 = 1$, $\beta_2^*(\beta_1) \neq \widehat{\beta_2}$. If $\beta_1 = \widehat{\beta_1}$, $\beta_2^*(\beta_1)$ has two possible values: $\widehat{\beta_2}, \lambda - \Delta P$. From previous discussion we know that when $\beta_1^* = \widehat{\beta_1}$ and $\beta_2^* = \widehat{\beta_2}$, we can get $\beta_1^* = \lambda + \Delta P$, $\beta_2^* = \lambda - \Delta P$. Hence, there's no equilibrium when $\beta_2^* = \widehat{\beta_2}$.

(6) When $0 \le \lambda - \Delta P < 1$ and $\lambda + \Delta P > 1$,
The optimal strategy of service provider 1:

$$\beta_1^* = \begin{cases} \min\{\widehat{\beta_1}, 1\}, & 0 \le \beta_2 < \lambda - \Delta P \\ 0, & \beta_2 = \lambda - \Delta P \\ 0, & \lambda - \Delta P < \beta_2 \le 1 \end{cases} \tag{43}$$

The optimal strategy of service provider 2:

$$\beta_2^* = \begin{cases} 0, & 0 \le \beta_1 \le \lambda + \Delta P - 1 \\ \min\{\widehat{\beta_2}, \lambda - \Delta P\}, & \lambda + \Delta P - 1 < \beta_1 \end{cases} \tag{44}$$

- If $\lambda + \Delta P < 2$,
When $\beta_2 = 0$, $\beta_1^*(\beta_2)$ has two possible values: $\widehat{\beta_1}, 1$. If $\beta_1 = 1$, $\beta_2^*(\beta_1) \neq 0$. If $\beta_1 = \widehat{\beta_1}$, $\beta_2^*(\beta_1) = 0$ if $\widehat{\beta_1} \le \lambda + \Delta P - 1$. Hence, there exists equilibrium $(\widehat{\beta_1}, 0)$ if $\widehat{\beta_1} < 1$ and $\widehat{\beta_1} \le \lambda + \Delta P - 1$.
When $\beta_2 = \lambda - \Delta P$, $\beta_1^*(\beta_2) = 0$. When $\beta_1^* = 0$, $\beta_2^*(\beta_1) \neq \lambda - \Delta P$. Hence, there's no equilibrium when $\beta_2 = \lambda - \Delta P$.
When $\beta_2 = \widehat{\beta_2}$, $\beta_1^*(\beta_2)$ has two possible values: $\widehat{\beta_1}, 1$. From previous discussion we know that $\beta_1 = \lambda + \Delta P$ and $\beta_2 = \lambda - \Delta P$ when $\beta_2 = \widehat{\beta_2}$ and $\beta_1 = \widehat{\beta_1}$, which does not satisfy the domain of incentive level. Hence, there's no equilibrium if $\beta_1 = \widehat{\beta_1}$. If $\beta_1 = 1$, $\beta_2^*(\beta_1) = \min\{\widehat{\beta_2}, \lambda - \Delta P\}$. Hence, there exists equilibrium $(\beta_1^*, \beta_2^*) = (1, \widehat{\beta_2})$ if $\widehat{\beta_1} > 1$ and $\widehat{\beta_2} < \lambda - \Delta P$.
- If $\lambda + \Delta P \ge 2$,
When $\beta_2 = 0$, $\beta_1^*(\beta_2)$ has two possible values: $\widehat{\beta_1}, 1$. When $\beta_1 = \min\{\widehat{\beta_1}, 1\}$, $\beta_2^*(\beta_1) = 0$ since $\min\{\widehat{\beta_1}, 1\} \le \lambda + \Delta P - 1$. Hence, there exists equilibrium $(\beta_1^*, \beta_2^*) = (\min\{\widehat{\beta_1}, 1\}, 0)$.

(7) When $\lambda - \Delta P = 1$ and $\lambda + \Delta P = 1$, i.e., $\lambda = 1$ and $\Delta P = 0$,
The optimal strategy of service provider 1:

$$\beta_1^* = \begin{cases} 0, & \beta_2 = 0 \\ \min\{\widehat{\beta_1}, 1\}, & 0 < \beta_2 < 1 \\ 1, & \beta_2 = 1 \end{cases} \tag{45}$$

The optimal strategy of service provider 2:

$$\beta_2^* = \begin{cases} 0, & \beta_1 = 0 \\ \widehat{\beta_2}, & 0 < \beta_1 < 1 \\ 1, & \beta_1 = 1 \end{cases} \tag{46}$$

From the condition, we know that $\Delta P = 0$ and $\lambda = 1$. By substituting them into equation, we can get $\min\{\widehat{\beta_1}, 1\} = \widehat{\beta_1} = \beta_2$ and $\widehat{\beta_2} = \beta_1$. When $\beta_2 = \beta_1 \in [0, 1]$, $\beta_1^*(\beta_2) = \beta_2$. When $\beta_1 = \beta_2 \in [0, 1]$, $\beta_2^*(\beta_1) = \beta_2$. Hence, there exists equilibrium $(\beta_1^*, \beta_2^*) = (\beta_2, \beta_1)$.

(8) When $\lambda - \Delta P \ge 1$ and $\lambda + \Delta P > 1$,
The optimal strategy of service provider 1:

$$\beta_1^* = \begin{cases} 0 & ,0 \le \beta_2 \le \lambda - \Delta P - 1 \\ \min\{\widehat{\beta_1}, 1\} & ,\lambda - \Delta P - 1 < \beta_2 < \lambda - \Delta P \\ 0 & ,\beta_2 = \lambda - \Delta P \end{cases} \tag{47}$$

The optimal strategy of service provider 2:

$$\beta_2^* = \begin{cases} 0 & ,0 \le \beta_1 \le \lambda + \Delta P - 1 \\ \widehat{\beta_2} & ,\lambda + \Delta P - 1 < \beta_1 \end{cases} \tag{48}$$

- If $\lambda + \Delta P \ge 2$,
When $\beta_2 = 0$, $\beta_1^*(\beta_2) = 0$. When $\beta_1 = 0$, $\beta_2^*(\beta_1) = 0$. Hence, we can easily find the equilibrium $(\beta_1^*, \beta_2^*) = (0, 0)$.
- If $\lambda + \Delta P < 2$,
When $\beta_2 = 0$, $\beta_1^*(\beta_2) = 0$. When $\beta_1 = 0$, $\beta_2^*(\beta_1) = 0$. Hence, we can easily find the equilibrium $(\beta_1^*, \beta_2^*) = (0, 0)$.
When $\beta_2 = \widehat{\beta_2}$, $\beta_1^*(\beta_2)$ has three possible values: $0, \widehat{\beta_1}, 1$. If $\beta_1 = 0$, $\beta_2^*(\beta_1) \neq \widehat{\beta_2}$. If $\beta_1 = \widehat{\beta_1}$, $\beta_2^*(\beta_1) = \widehat{\beta_2}$ if $\widehat{\beta_1} > \lambda + P - 1$. From previous discussion we know that $\beta_1 = \lambda + \Delta P$ and $\beta_2 = \lambda - \Delta P$ when $\beta_2 = \widehat{\beta_2}$ and $\beta_1 = \widehat{\beta_1}$, which does not satisfy the domain of incentive level. If $\beta_1 = 1$, $\beta_2^*(\beta_1) = \widehat{\beta_2}$. The equilibrium exists if $\min\{\widehat{\beta_1}, 1\} = 1$ and $\lambda - \Delta P - 1 < \widehat{\beta_2} < \lambda - \Delta P$. However, there's no solution. Hence, there's no equilibrium when $\beta_2^* = \widehat{\beta_2}$.