1 SOLVING PROBLEMS BY DIAGRAM

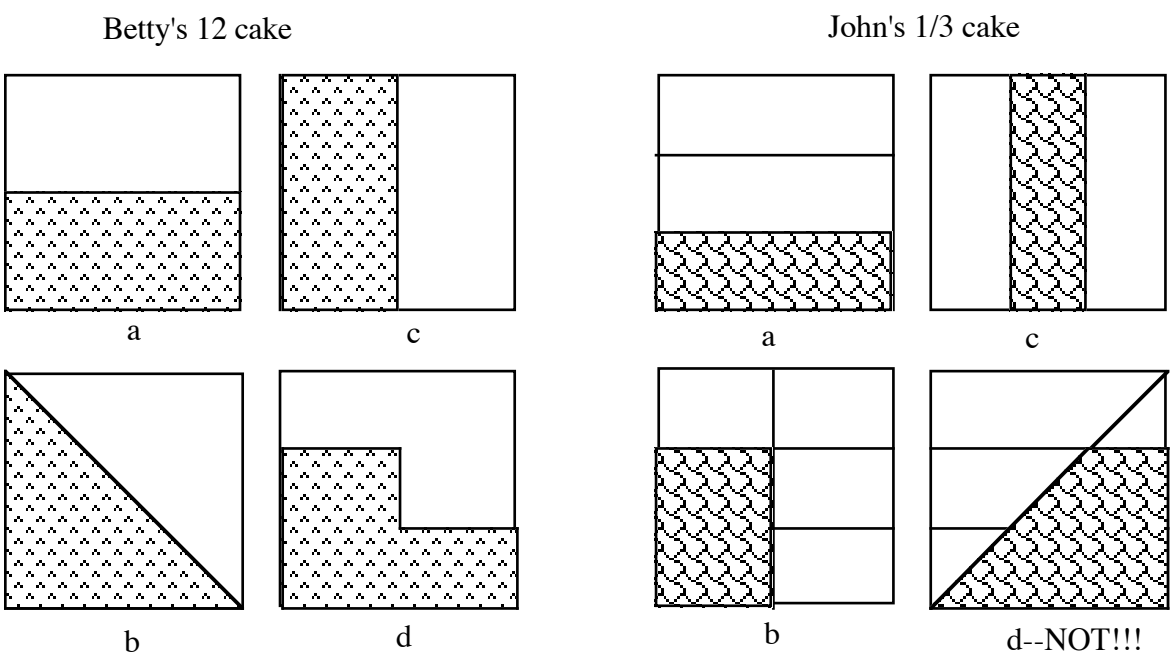
This section involves problems in "real" situations in which fractions must be added, subtracted, multiplied, divided or compared. In solving these problems, however, you will not use the usual arithmetic rules for working with fractions. Instead, you will use diagrams. To solve such a problem, you must produce a diagram which clearly shows all of the fractions involved in the situation, and also clearly shows the relationships between the fractions, as well as how you arrived at your solution. For instance, let's work on the following problem:

Betty and John made a rectangular cake. [Note that for ease in sketching accurate subdivisions, not only will the cakes in this course be rectangular—so will the pizzas!] Betty ate 1/2 of the cake and John ate 1/3 or the cake. How much is left?

To say that Betty ate 1/2 of the cake is to say that the cake was divided into two equal pieces and Betty ate one. To say that John ate 1/3 of the cake is to say that the cake was divided into three equal pieces and John ate one. There fractions can be represented in a variety of ways—as can be seen in the diagrams below:

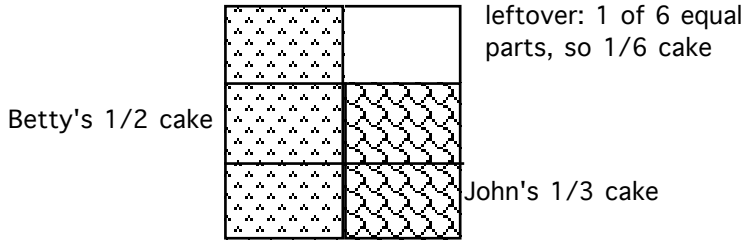
Betty's 1/2 cake

John's 1/3 cake



Notice that as the ways of dividing up the cake get more complicated, saying WHY the shaded area represents the fractional part ("1/2 of the cake" or "1/3 of the cake") becomes more and more difficult. In particular, to use (b) to represent John's 1/3 cake, we must state that the shaded area can be broken up into two pieces, and that suitable shuffling of those pieces can produce a horizontal stripe equal to two other horizontal stripes.

If, however, you agree that these diagrams do represent Betty and John's portions of cake, then solving the problem is not very difficult. You draw the rectangle and divide it up into six equal pieces, as shown below. [*Where did the number SIX come from?*--It works, and it is what we need in order to combine the two diagrams.] Then you shade in Betty's part of the cake and John's part of the cake. Since we have divided the cake into six equal pieces, and the friends between them have eaten five of the pieces, they have eaten 5/6 of the cake. We also see that one of the pieces is left. It represents 1/6 of the cake.



GROUND RULES FOR SOLUTIONS BY DIAGRAM:

a) The diagram must visually represent the problem. Thus, if the problem is about a rectangular cake, you should show rectangles; if the problem is about distance, you should show a line with distance marked off; if the problem is about cups of lemonade, you should show cups.

b) If the problem involves a fraction M/N of some thing (where M and N are counting numbers with M<N), then you must show the thing, show it clearly divided into N equal parts, and then show M of those parts shaded in.

c) Your diagram must be clear, accurate, and convincing. When you divide things into equal parts, the parts should look equal.

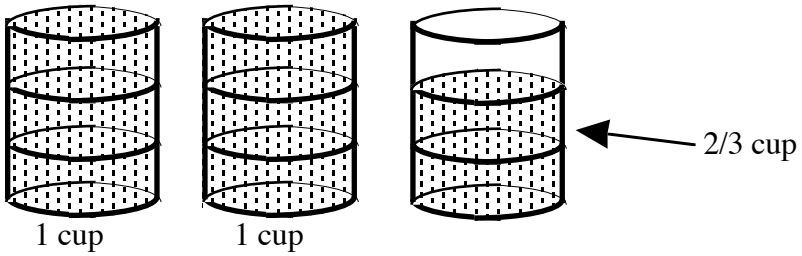
d) All of the information in the problem must be included in the diagram.

a) Your final diagram must illustrate to someone else (another student, for instance) **how** you arrived at your answer and/or **why** it is correct.

SOME WORKED EXAMPLES OF SOLVING FRACTION PROBLEMS BY DIAGRAM

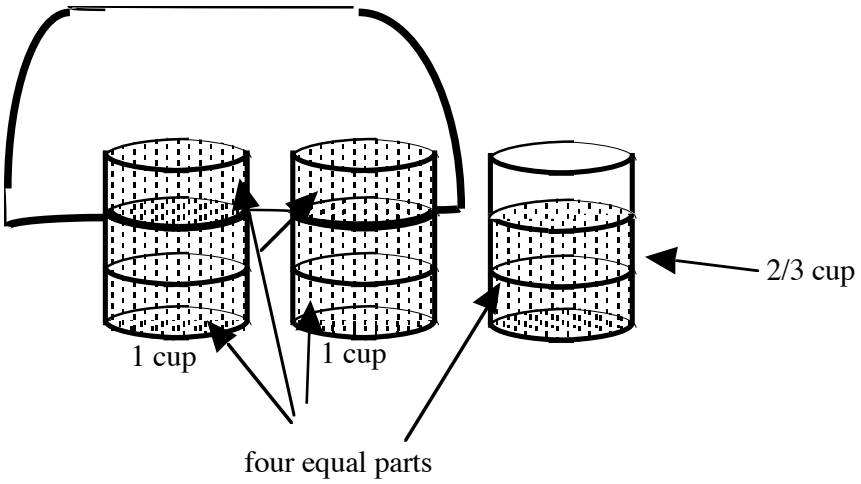
A) Mrs. Jones's class used 2 2/3 cups of sugar in making 4 batches of cookies. Assuming that each batch took the same amount of sugar, how much sugar is in each batch of cookies?

Step 1: The diagram below shows the 2 2/3 cups of sugar. We know that this is 2 2/3 cups because the last cup is divided into 3 equal parts, and 2 of these are used.

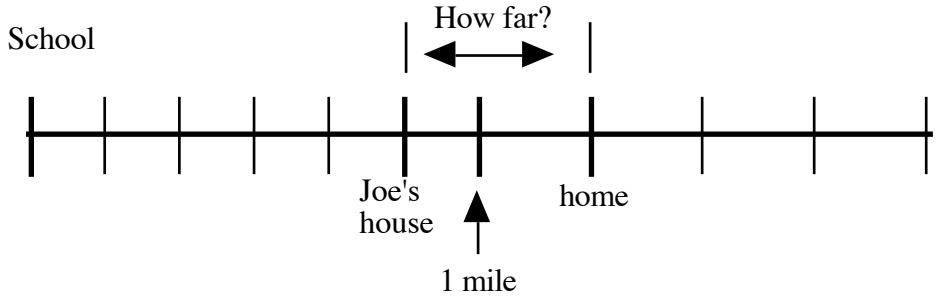


Step 2: In order to get the answer we must divide the cups of sugar shown into 4 equal parts. We *could* do this by subdividing the already divided cups. If there were 5 batches, that's what we would have to do. However, if we look at the

part which has been "circled" below, we see that if it were removed, the amount in each of the two full cups would be equal to the amount in the partially filled cup, and furthermore, the amount removed would also be equal to it. Hence we have divided the sugar into four equal parts. Each of these sugar is enough for one batch. Each is 2/3 of a cup of sugar, because each results from dividing the cup of sugar into 3 equal parts and selecting 2 of these parts. The answer is 2/3 cup of sugar.



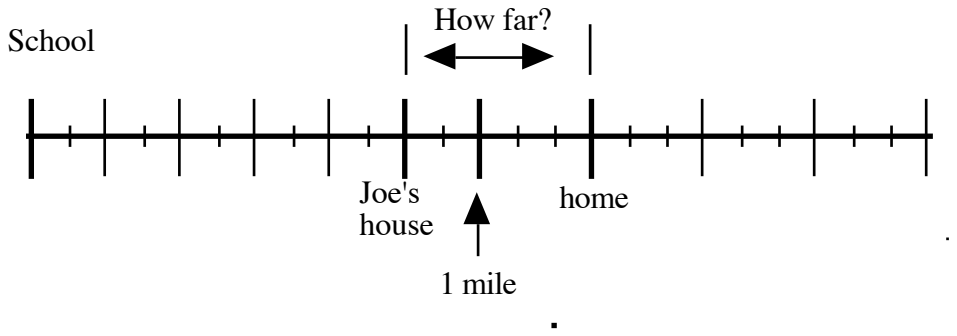
B) Ralph walks 1 1/4 miles home from school every day. He stops at Joe's house, which is 5/6 of a mile from school. How far is it from Joe's house to Ralph's house?



In this diagram, the first mile is divided into 6 equal pieces in order to show 5/6 of a mile. The second mile is divided into 4 equal pieces in order to show the 1/4 mile part of Ralph's walk home.

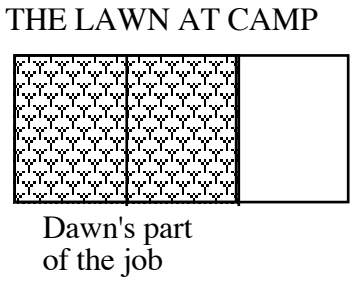
To solve the problem, we divide each of the 6 divisions of the first mile into 2 subdivisions. This means that the first mile is divided into 12 equal pieces—each therefore 1/12 of a mile long. Then we divide each of the four pieces of the second mile into 3 subdivisions. This means that the second mile is divided into 12 equal pieces—each therefore 1/12 of a mile long. The diagram

below then shows that the distance between Joe's house and Ralph's is 5/12 of a mile.



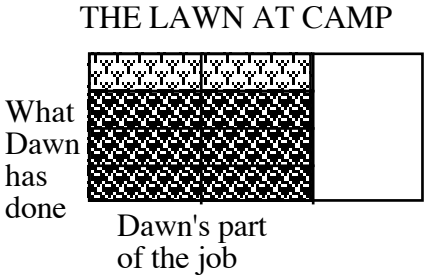
C) Dawn has agreed to mow 2/3 of the lawn at camp. She has done 3/4 of her job. What portion of the **lawn** has Dawn done?

Step 1:

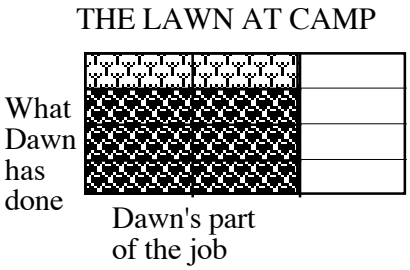


In this diagram the lawn has been divided into three equal parts. Each represents 1/3 of the lawn. Two of the parts represent Dawn's part of the job—which is 2/3 of the lawn.

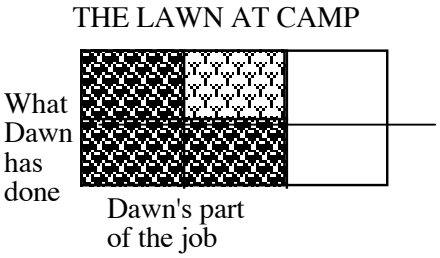
Step 2: Next we divide Dawn's part of the job into four equal parts, because she did 3/4 of the job.



Step 3: If we continue the horizontal lines that divide Dawn's part of the line, we can see that the lawn is divided into 12 equal parts. We see from the diagram that Dawn has done 6 of these, so she has done 6/12 of the lawn.

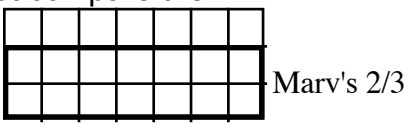


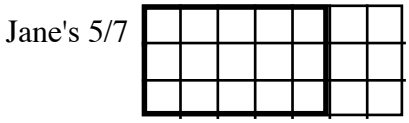
Step 4: But we know that 6/12 = 1/2, so we need to improve the diagram. We need a diagram which has all of the information of the problem, and on which the lawn is divided in half, one of the halves being what Dawn has done. One of **many** such possibilities is shown below:



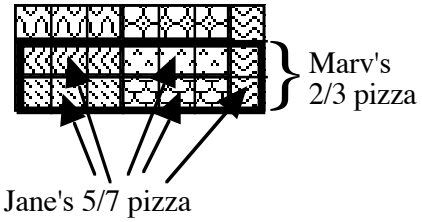
D) Marv ate 2/3 of a (rectangular) pizza. Jane ate 5/7 of a pizza the same size. Who ate more?

In the diagram below, we see the pizza cut up in two ways. One shows Marv's portion and one shows Jane's. The two portions are so different in shape that we cannot compare them:





By combining the two ways of dividing the pizza we have divided it into 21 equal parts. We can see that Marv ate 14 of these while Jane ate 15. So Jane ate more. But that's an answer that uses counting and not the diagram. The diagram below gives an answer that does not involve counting:



There are seven equal portions, each with its own funny shading. If Jane ate the ones which have arrows pointing to them, she clearly ate more than Marv's two horizontal stripes' worth.

SOME TERMINOLOGY FOR FRACTION PROBLEMS

In solving mathematics problems, it can be very useful to find some other (solved) problem which is "the same" as the one we are looking at One way to see how fraction problems are the same as or different from each other is to notice that in each of the problems there are three possible components:

There is the **whole**, which is given as some amount of stuff—like lemonade or acreage, or distance. If you solve the problem by diagram, it will be what you first draw. In the examples above, it would be Mrs. Jones's sugar supply, or the camp lawn, or the distance from home to school.

There is a **part** of a whole, which is the smaller piece that each problem has. This is the sugar for one batch of cookies, or the distance to Joe's house, or the part of the lawn that Dawn has mowed.

There is a **portion**, which is the ratio between the **part of the whole** and the **whole.** For instance, we are told that each batch of cookies used 1/4 of the available sugar (that is the **portion** of the sugar used to make a batch), and that Dawn has mowed 3/4 of what she needs to (that is, the **portion** of her job).

One of the things that makes these problems tricky is that any of the quantities—the whole, the part or the portion—can be expressed as a fraction, so that you cannot tell which is the portion by looking for the fraction. However, you might notice that in general the whole and the part have units attached, like quarts or miles (or at least could have—for instance, the lawn could just as well be 1 acre of lawn.) The portion, on the other hand, doesn't. This is because the portion is a *relationship* between the part and the whole and is not an actual quantity.

11 FRACTION PROBLEMS TO BE SOLVED BY DIAGRAM

Directions: Solve the problems below by diagram **USING THE GROUND RULES ABOVE**. Look over the example solutions above, but remember *there are many ways to solve any particular problem by diagram.* Be creative—don't just follow.

1) Ms. Jones had 6 pints of lemonade. She gave 1/4 of it to her class. How many pints did she keep?

2) Ms. Alvarez has 2 1/2 bars of candy. She wants to divide it evenly among her 4 tap-dance students. How many candy bars does each student get?

3) Nan's go-cart requires 2/3 of a gallon of gas to fill it up. She has 2 2/3 gallons. How many times can she fill it up?

4) In the January White Sale, Grant bought towels for which he paid $48. All prices in the sale were "1/3 off the regular price." What would he have paid for the towels at the regular price?

5) Pietro has 3/4 of a quart of milk. He uses 2/3 of it for a milk shake. What part of a quart of milk did he use?

6) Miguel picked 3 3/4 quarts of blueberries. He gave his friend Sally 3 quarts of blueberries. What portion of the blueberries he picked does he have left?

7) Ming ran 1/4 of a kilometer and then walked 5/6 of a kilometer. What was the total distance she covered?

8) Jose has 2 1/3 yards of material. He uses 1 1/2 yards to make a vest. How much does he have left?

9) At a pizza party Frank ate 1/3 of each of four pizzas. Sarah ate 1/2 of each of three different pizzas. Who ate more?

10) Pablo has 1 1/3 gallons of paint. If he uses 1/4 of what he has, how much paint has he used?

11) It takes Farouk 12 hours to paint a room. He has been working for 8 hours. What portion of the room has he done?

12) Hubert's job is to shovel 3/4 of the driveway whenever it snows. If he gets 1/3 of the driveway shoveled right after the snow stops, and does another 1/4 of the driveway after he has taken a chocolate break, what portion of **his job** has he done?

13) Sally, Joe and Chris own a Tofu Bar. If Sally owns 2/5 and Joe owns 1/4, what part of the Bar does Chris own?

14) Gretchen walks 1 1/2 miles home from school. When she is 2/3 of the way home, she stops in at the 7-11 for her daily candy bar. How far is it from the 7-11 to her home?

15) Vassily runs 3 miles every day. He has run 1 3/4 miles today. What portion of his run does he have left?

16) Which is more, 2/3 of a box of cookies or 5/7 of the same box?

III YET MORE FRACTION PROBLEMS

For this set you are to use a two-pronged approach.

A) First solve the problem using a diagram.

B) Then solve the problem using ordinary fraction arithmetic, **explaining what you are doing.**

1) Nadim has 1/2 cup of flour. He knows that this is 1/3 of what he needs for his pancake recipe. How much flour does he need to make pancakes?

2) Louise had 1 2/3 gallons of Kool-Aid and gave away 1 1/3 gallons of it. What portion of her Kool-Aid did she give away?

3) Paul had 4/5 of a quart of milk. He used 3/4 of it for a milk shake. What part of a quart of milk did he use?

4) Carmen picked 1 3/4 bushels of coffee beans on Monday and 7/8 of a bushel on Tuesday. How much did she pick altogether on the two days?

5) Each of Bob's four friends gave him 3/8 of a large pizza. How much pizza did he receive?

6) Rosalyn ate 1 1/2 cups of trail mix on her hike. This is 3/4 of what she had. How much trail mix did she bring?

7) Cheng found 3/4 of a gallon of ice cream in her freezer. She gave Galen 2/3 of what she had. What part of a gallon did Cheng give away?

8) Lance intends to run 5 1/2 miles today. He has just finished running 3 3/4 miles. What portion of his run does he have left?

9) Aviva ran 5/6 of a mile the first day of summer when she started running regularly. On the last day of summer, she was able to run 1 1/4 miles. How much further did she run at the end of the summer than when she started?

10) Ned bought a pack of audio tapes that were marked down by 25%. The usual price for these tapes is $6.00 a pack. How much did Ned pay?

11) Charlene ate 3/4 of a cup of ice cream. This was 3/5 of the ice cream that she had. How much ice cream did she have to begin with?

IV A RANDOM COLLECTION OF FRACTION PROBLEMS

These are to be done as homework, with a general no-holds-barred approach. Help from your group, your friends, your parents and/or your kids are all fine. The only restriction is that what you actually write you should understand.

1) An oil tanker holds 25,000 gallons of crude oil. In an oil spill, 3/5 of the load is lost. How many gallons remain in the tanker?

2) The furniture in a house is valued at $12,500. The house is worth 4 and a half times as much as the furniture. How much are both together worth?

3) 5 and a half pounds of chocolate chip cookies are to be packed into bags each of which holds a third of a pound. How many bags will be filled and how much will be left for the packer to munch on?

4) A moped gets 42 2/3 miles per gallon of gas. How far can it go on 3 1/2 gallons?

5) A Cadillac gets 6 1/4 miles per gallon. How many gallons are required for a trip of 50 miles?

6) My front lawn measures 2 1/2 yards by 5 1/3 yards. To fertilize it, I need 1/2 ounce of fertilizer per square foot. How much fertilizer must I buy?

7) Dick and his sisters Jane and Sally each have their own marijuana plot. Dick raises 1 2/3 pounds of it, Jane raises 1 1/2 pounds and Sally tops them both with 2 1/3 pounds. If they pool their resources and divide them with equal shares for each of them and each of their parents, how much marijuana does each one get?

8) Willie left home with a bunch of flowers for Suzabelle. Halfway to her house, he sneezed very hard and knocked the petals off 2/3 of them. He threw away the petal-less ones and hastily went home and added 4 more to the remaining bunch. Opening the gate to her front yard he dropped 4/5 of the flowers he was carrying into a mud puddle. So he picked three flowers from Suzabelle's mother's prize gladiola patch and rang the doorbell. "How lovely", said Suzabelle. "Now I'll have a dozen flowers--I only had 5 before." How many did Willie set out with the first time?

9) A batch of cookies requires 2 1/4 cup of flour and 1 egg. You have in your kitchen 8 cups of flour and a half dozen eggs.

a) How many batches of cookies can you make?

b) After you have made the cookies, how much flour and how many eggs will you have left? What portion of the original amount of flour and the original number of eggs will you have left?