### **DRAFT** QUANTUM MECHANIC'S MANUAL II Winter 2022

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# Contents

1	Introduction and Dedication	<b>5</b>			
<b>2</b>	From the Black Body and back	7			
3	The humble black body goes to Big Bang.         3.1       Introduction         3.1.1       From Black Body to Blackbody to the Big Bang         3.2       Baryon Acoustic Oscillation         3.2.1       Dark Matter         3.2.2       Cosmic Sound Wave         3.2.3       The story of BAO         3.2.4       Max Planck Conclusions, in his own words	9 9 11 14 14 14 14 14			
4	Epilogue				
<b>5</b>	Nature follows Gauge Quantum Field Theory				
6	The infinite complexity of Nature				
7 From the infinite to infinitesimal, and back 7.0.5 What is the World made of					
8	contd         8.0.6       blackbody spectrum summarized         8.0.7       photoelectric effect         8.0.8       Compton effect:         8.0.9       diffraction of electrons         8.0.10       quantization of physical quantities:         8.1       The Puzzle Illustrated by the Double Slit Experiment:         8.1.1       The Interferometer         8.2       Fundamentals of Quantum Physics         8.2.1       The Quantum Mechanical Solution of the Puzzle:         8.2.2       Particle in a potential "box"         8.2.3       Development of quantum physics continued	<ul> <li>23</li> <li>23</li> <li>23</li> <li>24</li> <li>24</li> <li>24</li> <li>25</li> <li>26</li> <li>26</li> <li>27</li> <li>27</li> <li>27</li> </ul>			
	8.3 Figures	28			

### **Introduction and Dedication**

This short work represent actually a fairly great amount of work. It deals with two great parts of contemporry physics: Gauge Field theory, and Cosmology. Busy as I was with all kinds of other endeaverosr, I negl;ected the goal/ambition

### From the Black Body and back

The amount of experimental evidence for quantum physics is overwhelming. As noted above, *all* available data agrees with quantum mechanics, and much of our civilization is actually based on it. In general, I am not a great advocate of a historical approach to science, but there are some foundational experiments of extreme significance and great beauty that simply *have* to be discussed in detail,

By the end of the 19th century physics has achieved amazing successes:

- Newton, building on contributions of Kepler and Galileo, provided the foundations. unifying everyday mechanics with the celestial motion.
- Maxwell, in a spectacular accomplishment, unified electricity, magnetism and optics. He predicted that a disturbance of the electromagnetic field should travel at a speed of  $1/\sqrt{\epsilon_0\mu_0} \sim 3 \times 10^8 m/s$ . In a truly historic moment, he learned, upon returning from his country house, that astronomers just measured speed of light to be  $\sim 3 \times 10^8 m/s$ .

However, some even more dramatic and far-reaching developments were awaiting around the corner of the 20th century.

# The humble black body goes to Big Bang.

### 3.1 Introduction

A black body is an object that does not reflect any light illuminating it – it absorbs everything. But by absorbing the EM energy it gains it and therefore it emits EM energy back, in general with a different spectrum. The classical realization has been a cavity with its inner walls maintained at a constant temperature T. After some time an equilibrium is established, where the energy received each second is equal to the energy radiated. The radiation in the cavity can studied by analyzing the radiation coming from a small hole (see Figure 8.1)

It was soon recognized that the equilibrium is characterized by the temperature T the body achieves, and – what is more interesting – the spectrum of that radiation depends only on T and on the frequency  $f^1$  (or equivalently  $\lambda^2$  on the wavelength

$$u(f,T) = \frac{c}{f^2} u(\lambda,T)$$

The independence of the energy spectrum on the blackbody details shows that this is an important, universal aspect of the thermodynamics, and therefore a significant effort centered on find the expression for the energy density u(f, T)

Using very solid classical physics arguments, Rayleigh derived the expression

$$u(f,T) = \frac{8\pi f^2}{c^3} kT$$
(3.1)

and it was a disaster – see Figure 8.2. It agreed with the experimental data for low frequencies, but it literally exploded, unphysically, at high f. Because no one was able to find fault with Rayleigh reasoning, it became "the ultraviolet catastrophe".

<sup>&</sup>lt;sup>1</sup>NOTE: I refuse the Greek  $\nu$  for frequency: just think of the equation  $\nu = v/\lambda$  (especially when handwritten ...)

<sup>&</sup>lt;sup>2</sup>Plotting the dependence on the wavelength seems more familiar; the dependence of frequency makes clear the physics contents of the final Planck solution (see below).

On the other hand, Wien proposed an expression he found (based on a model that is no longer of interest)

$$u(f,T) = af^{3}e^{-bf/T} (3.2)$$

and this, with the right choice of the adjustable parameters a and b worked quite well (see Figure 8.2). The agreement at high frequency was now perfect (no UV catastrophe!) but now the low frequencies were showing a small but definite and unacceptable discrepancy.

So at the very beginning of the twentieth century there were two formulas; one OK at high frequencies, the other one at low f. And in the year 1900, in a singular development in physics, Max Planck discovered the correct formula by purely mathematical trial and error:

$$u(f,T) = \frac{8\pi h}{c^3} \frac{f^3}{e^{hf/kT} - 1}$$
(3.3)

He *knew* it was correct because it agreed with the experimental data so excellently and over the full range of the variables. In addition, it beautifully made the transition between the Rayleigh and the Wien formulas (see Figure 8.2). So now Planck set out to find out what physics assumptions would yield his result.

After two months of work, he found and – as he was an old-school, careful and conservative<sup>3</sup> physicist – reluctantly accepted the result: it appeared that the cavity walls emit EM radiation in "quanta" of energy hf, 2hf, 3hf, ....nhf - and nothing in between. As a consequence, the Boltzman classical average of energy

$$\langle E \rangle = \int E.Prob(E)dE = \int E \frac{e^{-E/kT}}{\int e^{-E/kT}dE}dE$$
 (3.4)

becomes (with x = hf/kT where h is the Planck constant and k is the Boltzman constant)

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} nxe^{-nx}}{\sum_{n=0}^{\infty} e^{-nx}} = \frac{-x\frac{d}{dx}\sum_{n=0}^{\infty} e^{-nx}}{\sum_{n=0}^{\infty} e^{-nx}} = x\frac{e^{-x}}{1-e^{-x}} = \frac{x}{e^{x}-1}$$
(3.5)

which, when multiplied by the classical density of states  $\frac{8\pi f^2}{c^3}$ , becomes the Planck formula. And it is a nice exercise to show it is indeed a smooth transition between Rayleigh and Wien:

$$\lim_{x \to 0} eq.3.3 = eq.3.1$$
$$\lim_{x \to \infty} eq.3.3 = eq.3.2$$

It also is a simple and very nice exercise on integral calculus. Normally, the definite integral is defined as the surface beneath the curve y(x) calculated as a limit of a sum, i.e. the sum is an approximation. But sometimes Nature insists on using the sum, and it is the integral that is an approximation. Equations 3.4 and 3.5, illustrated on Figure 8.4, show how this comes about.

And finally, and remarkably, the Planck formula does not have any free parameter - not even for overall normalization (just two universal constants: the Planck constant h and

<sup>&</sup>lt;sup>3</sup>I say "conservative" in the best sense of the word; more about this in chapter XXX

the Boltzmann constant k). Both dependencies are quite dramatic (for the h on frequency, for the k on temperature, so Planck was able to determine both values with remarkable accuracy).

Note: physicists often use the value of the "hbar" defined as

$$\hbar = \frac{h}{2\pi}$$

#### Summary

This was the first crack in the safe where Nature kept the quantum from us, and Max Planck is immortal for making the breakthrough.

#### 3.1.1 From Black Body to Blackbody to the Big Bang

As fundamental as the blackbody was for the origin of Quantum Physics, there is more – much more.

• Relic from the Big Bang.

As early as 1948, based on not much more than the data on present-day abundances of the light elements (H, D, ...) George Gamow and collaborators predicted that a Big Bang should have left behind a relic of very low temperature radiation. In 1955, in an experiment aimed at reducing the noise interfering with the signals from the first active communication satellites, Penzias and Wilson observed a persistent excess of radiation around the temperature of about 3 K. Fortunately, they knew that Jim Peebles and collaborators were looking for the Big Bang relics in that temperature range. So they persisted, and published their results; Peebles and Co. published the interpretation in the same issue of the journal, and the age of experimental cosmology was born. Penzias and Wilson received a Nobel Prize in 1978; Jim Peebles continued a remarkable involvement in cosmology and received an overdue, much deserved Nobel Prize in 2019.

• The COBE spacecraft experiment

Experimental cosmology continued mostly with high altitude balloons, and came of age with the launch of the COBE satellite in 1989. The first great result was obtained very soon: their measurement on the spectrum of the "relic" - now officially called the Cosmic Microwave Background Radiation (CMBR) produced a fantastic agreement with the Planck formula – see Figure 8.5. By now it was very difficult to think of any other origin of the CMBR than a hot Big Bang. Therefore, this is a good time for a brief overview.

Figure 8.7 shows the (by now) standard chronology of the Big Bang showing how the part of the Universe currently observable by us started extremely small and extremely hot, and in some 13 billions years gradually cooled down to the present temperature of 3K and expanded to the present radius of some 42 light years (this is more than 13

billion due to the Universe expansion). It is a very busy diagram, and one can (and should) spend a lot of time inspecting and reflecting its many features. It is mind boggling that we can interpolate our knowledge over so many orders of magnitude, and it us only natural that one's degree of confidence decreases as we go closer and closer to the Big Bang itself. Here, we start very conservatively at the end of the period of nucleosynthesis that lasted from about  $10^{-10}$  to  $10^{-9}$  seconds (it sounds really weird to use the word "lasted" in that sentence ...). The Universe was about one second old and "only" 13 million[sic] light years across and contained a mix of protons, neutrons, electrons, neutrinos and photons violently agitated at temperature of 10 billion K. And then light nuclei were produced, and nothing much happened for about 350 000 years – Universe was gradually expanding and cooling down. And then, quite suddenly, something dramatic happened. As the temperature decreased to about 3,000 K, electrons became for the first time able to combine with the nuclei to produce neutral atoms, and the Universe became transparent. For some obscure reason, this is called "recombination" but those electron and nuclei were never combined previously ... Figures 8.11 and 8.8 illustrate the process.

The 3,000 K photons then traveled unimpeded and kept cooling by the continuing expansion until they reached their todays's temperature T=2.7 K. This is the Cosmic Microwave Background Radiation that Gamow (and others) predicted, Penzias and Wislon (and others) observed, and the COBE experiment could begin to investigate in detail. We already mentioned that their very first result (said to have been achieved in the first nine minutes!) was finding a spectacular agreement of the spectrum with the Planck black body formula. The whole Universe was a black body when CMBR was emitted because it originated in the whole Universe that is an isolated system by definition, and all its constituents were in thermal equilibrium because the expansion rate was much slower than the rate of the thermal motion. In fact, the CMBR is the best black body found in the Universe.

• The absolute speed of Earth. By 1992 COBE accumulated enough data for a detailed investigation of the CMBR, Figure 8.6 show that the results were almost too good to be true:

a) the temperature was uniformly 2.7 K in all directions. This was great news for the theory: "we" were originally "in the middle" of everything (because everything was in the middle of everything). Now we are at the center of a "Hubbel bubble" of the radius of the observable Universe, and CMBR comes uniformly in as it has been emitted from the inner surface of a sphere called the surface of last scattering (see Figure 8.9).

b) when restricting the data a narrow temperature range of  $\pm 3.5mK$  a dipole signal appeared, consistent with the Earth moving through the CMBR background with a speed about 600 km/s. Special relativity is still valid, but clearly there is one reference frame that is more equal than the others<sup>4</sup>:-)

<sup>&</sup>lt;sup>4</sup>Expanding on this and previous paragraph: the emission of CMBR happened everywhere at the same time, and it was a very brief (relarively speaking :-) flash emitted at about (13,000,000,000 + 300) years after

#### 3.1. INTRODUCTION

c) And finally, the great prize: when removing the dipole signal and restricting the data to  $\pm 20\mu K$  [sic !!!] there appeared a signal from the radiation of our own galaxy – which is not much more than a nuissance. But the data out of the galactic plane show a clear variation of randomly looking cold and hot spots. This was the Holy Grail, the hope of cosmologists which came to be fulfilled. Of course, the Nobel Prizes did not fail to come (to Mather and Smoot, the COBE leaders, in 2006).

• The (rare) definitive experiment.

The COBE experiment was followed up – with significantly more detailed results – by the WMAP project, and then by a definitive experiment called – appropriately – Planck. The cumulative progress can be seen on Figure 8.10. The pattern of cold and hot spots turned out to be anything but random. In the manner of decomposing a sound signal into contributions of various harmonics, one can similarly express the twodimensional pattern into different "spherical harmonics" that are called "multipoles". Naturally, the subsequent experiments show more and more details, so we show on Figure 8.12 the results from the Planck experiment. And this is the real Holy Grail. Nature has been kind to us: the peaks of the distribution contain priceless information on various cosmological models and their parameters.

The peaks seen on the Figure are called "acoustic peaks". It may sound strange that we speak of acoustics in the emptiness of space. But the space was not empty when CMBR was emitted – as you can see from Figure 8.7 the density was considerable, and it was sufficient to enable mechanical waves fully analogous to sound waves in air. But what is important is the amount of information that is provided by the exact locations and heights of the peaks. As a dramatic example, it was possible to determine from the spectrum that the metric of the observable Universe is not curved but compatible with flat. This means that the whole Universe is very much larger than our observable Universe, maybe infinite. It is awe inspiring that we can meaningfully discuss the whole Universe that we have never seen and will never see ...

I called the experiment definitive because its determination of the acoustic peaks cannot be improved. The resolution of the peaks is limited by the physical effects along the path of the CMBR photons from the surface of last scattering to us. When the resolution of the measurements exceeds this limit, any additional measurement will not change the results (unless, of course, the Planck team committed some grave errors, which is very unlikely). So the experiments is definitive and this is rare (and very valuable).

And finally: I find it remarkable to what degree the theory and the experiment cooperated in achieving the truly breathtaking results that we have discussed. Often it happens that the theory follows the experiment, and the work of the theorists may then have the nature of an after-the-fact postdiction. This significantly reduces the power of any claim that the successful experiments "confirmed" the theory. A dramatic example of the impressive lead of the theory in cosmology can be seen on Figure 8.13.

the Big Bang. We keep seeing it as if it kept happening, but this is an illusion due to the CMBR photons coming at us from larger and larger distances.

In fact, an even much more dramatic example concerns the cosmic inflation. So far we refrained from discussing it because limiting ourselves to the more reliable concepts seemed are-inspiring enough. Inflation refers to the possibility that the Universe went from initial size of e.g.  $10^{-10}$  to a size of e.g.  $10^{+15}$  meters, within the time of e.g.  $10^{-28}$  seconds; then the "regular" Big Bang followed (well, this would make the Big Bang look not so spectacular, after all -:) This has been motivated by the hope that it would remove some of the remaining difficulties with the theory. It so happens that some (many?) cosmologists now claim that the data from the Black Body research provide (or will provide, when polarization measurements improve) experimental evidence that inflation has in fact happened. As I remarked when discussing the metric of the "whole Universe": the mere fact that we can meaningfully discuss the cosmic inflation is mind boggling.

### 3.2 Baryon Acoustic Oscillation

Speaking of "mind boggling" we will end this chapter with a relatively new development. It will require us to first (re-)visit the CMBR.

#### 3.2.1 Dark Matter

First: DM is a misname. Everybody agrees it is there, interacts only gravitationally, even how much of it is there () — but no one has seen any of it yet. So it probably it is not really dark, an frankly: it is probably not very interesting.

### 3.2.2 Cosmic Sound Wave

The fluid composed of baryons, photons and electrons IS very interesting.

### 3.2.3 The story of BAO

#### 3.2.4 Max Planck Conclusions, in his own words

Life of Max Planck was very rewarding but also very difficult.

# Epilogue

Near the end of the Prelude I noted the hope in an eventual recovery of my youthful optimism in our young people, children and grandchildren. Stories like the one I told you in this chapter reinforce that hope. I will close with the words of the Great Man himself:

"Science enhances the moral value of life, because it furthers a love of truth and reverence; love of truth displaying itself in the constant endeavor to arrive at a more exact knowledge of the world of mind and matter around us, and reverence, because every advance in knowledge brings us face to face with the mystery of our own being.

# Nature follows Gauge Quantum Field Theory

### The infinite complexity of Nature

QM and its use vs our understanding exploration of the Big Bang relativities , timing of GPS and LIGO Quantum Gauge Theory LIFE JSB / KdF : origin/ evolution and human/animal brain and at the same time: Einstein/Darwin JSB vs Trump war vs philosophy the so various and powerful behavior of the MOB especially when combined with a powerful.charismatic individiuals (Christ, Mohammed, WWI+, Hitler, Lenin/Stalin/ Mao, Trump

# From the infinite to infinitesimal, and back

On a Gauge Theory of Elementary Interactions.

A. SALAM Imperial College -London J. C. WARD Carnegie Insitute of Technology -Pittsburgh (ricevuto il ] 5 Settembre 1960) ....

Summary. -A theory of strong as well as weak interactions is proposed using the idea of having only such interactions which arise from generalized gauge transformations.

One of the problems engaging current interest in field theory is the problem of determining which fields are elementary in some fundamental sense, and which are not. And equally, if not more, important, problem is that of finding a guiding principle for writing fundamental interactions of fields. The only such principle which exists at the present time seems to be the gauge-principle. Whenever a symmetry property exists, the associated gauge transformation leads in a fairly definite manner to the postulation of an interaction through the mediation of a number of intermediate particles. In this note we wish to reconsider the problem. Our basic postulate is that it should be possible to generate strong, weak and electro-magnetic interaction terms (with all their correct symmetry properties and also with clues regarding their relative strengths), by making local gauge transformations on the kinetic-energy terms in the free Lagrangian for all particles. This is the statement of an ideal which, in this paper at least, is only very partially realized. It may however be of interest to set down the procedure which has been followed.

#### 7.0.5 What is the World made of

When I was in high school in the late 1950s, we were taught that everything was made of the elementary electrons and equally elementary protons. Adding some equally elementary neutrons you got the nuclei, adding electrons produced atoms and so on. Everything was of course subject to Quantum Mechanics but otherwise made complete sense.

Then, in 1964 Gellmann and independently Zweig proposed that protons and neutrons are in fact triplets of the truly fundamental constituents (called quarks by G, aces by Z; quarks won ...). However, dedicated attempts to actually find the quarks all failed, and they became considered as useful but purely mathematical devices for classification of the rapidly increasing number of objects made out of "quarks".

However, the belief in the actual existence of quarks kept steadily increasing, and by xxxx there remained essentially no physicists with any doubts. Our current knowledge is summarized on Figure

Then

### contd

#### 8.0.6 blackbody spectrum summarized

The dependence of the "blackbody spectrum" on wavelength requires quantization of energy levels of electromagnetic radiation inside the black body cavity, with

$$E = hf$$
  $h = 6.6 \ 10^{-34} \ J.s$ 

The "Planck constant" is a fundamental property of Nature, and honors the scientist who discovered it (in addition to the Nobel prize he received).

Note the time sequence of Planck's discovery (correct formula first, its derivation two months later), and do not miss a useful lesson on integration and averaging.

A fascinating example of black-body radiation is the "cosmic background" of radiation emitted when, after the Big Bang, radiation "decoupled" from matter. This happened when Universe was about 400,000 years old, and the temperature was 3000 K. Since then, the expansion of the Universe by a factor of about 1,000 has cooled down the radiation by the same factor to about 3 K today. The precise measurements of the "ripples" in the cosmic background are starting to provide some incredibly detailed evidence on the early Universe (as well as information on our "absolute" speed).

#### 8.0.7 photoelectric effect

The second foundational quantum effect was discovered by Hertz in 1887, studied by Lenard<sup>1</sup> and interpreted by Einstein in 2005.

Electrons are ejected from an anode by a stream of photons, and collected by a cathode (see Figure 8.14). The main results are

1) The collected current depends on the light intensity, but the "stopping potential" (necessary to prevent any electrons from reaching the cathode ) does not

2) On the other hand, the stopping potential does depend on the frequency of the light used.

$$eV + W = hf$$

<sup>&</sup>lt;sup>1</sup>Lenard won the Physics Nobel Prize for his research. This did not prevent him from becoming a fervent Nazi supporter, together with Johannes Stark, another Nobelist.

where W is the "work function" of the particular metal the anode is made of – only voltage above this value can accelerate the electrons. And the constant h "happens to be' the same number as the h in a) as well as below. This might remind you of Maxwell's triumph showing that his  $1/\sqrt{\epsilon\mu}$  equaled the speed of light as measured directly.

Explanation of the photoeffect is what Einstein received his Nobel Prize for<sup>2</sup>: radiation of frequency f consists of quanta called photons, each of energy hf. You should see, and be able to show to anyone, the extraordinary beauty of the reasoning.

#### 8.0.8 Compton effect:

photon-electron elastic scattering, with the "recoil" photon showing lower frequency than the initial photon. This shows that a photon is a "particle" with energy and momentum

$$E = hf \quad p = E/c$$

Recall the general formula

$$E = \sqrt{m^2 c^4 + p^2 c^2}$$

with the special cases (HW!) of

a) rest

b) non-relativistic motion (yielding the familiar expression for the kinetic energy)

c) ultra-relativistic case (i.e. EM)

#### 8.0.9 diffraction of electrons

although they are generally considered to be particles, they exhibit diffraction as a wave with the wavelength

$$\lambda = h/p$$

Note the serendipitous role of broken equipment, and also the family symbolism:

J.J.Thomson (Nobel prize 1906 for showing quantum nature of electricity)

G.P.Thomson (Nobel prize 1937 for showing wave nature of electrons)

#### 8.0.10 quantization of physical quantities:

atomic spectra: the spectra of light emitted by excited atoms are quantized as

$$hf_{ab} = E_b - E_a$$

where, as discovered later,  $E_b$  and  $E_a$  are the energies of the initial and final state (i.e. there was a "quantum jump" from state a to state b). We will be able to calculate that soon enough.

 $<sup>^2\</sup>mathrm{I}$  believe he should have received at least another Physics Nobel for General Relativity, plus a Peace Nobel Prize  $\ldots$ 

**Stern-Gerlach experiment:** the deflection of particles going through inhomogeneous magnetic field are quantized as if the had the angular momentum projection quantized as:

$$L_z = m\hbar/2\pi = m\hbar$$

where m is an integer; see Figure 8.15

### 8.1 The Puzzle Illustrated by the Double Slit Experiment:

(Feynman Vol. III, 1.-3.) see our Figure Figure 8.16

To fully appreciate the Puzzle, you should try to 'comprehend' that if you **open** the slit B, electrons will **stop arriving** at point P !

HW: impress your friends with this, the Central Mystery of Quantum Physics, using a modified version of the double-slit experiment using the "Mach-Zehnder" interferometer. This not only illustrates the puzzle directly and quantitatively, but it also serves as an example of the central role that interferometry (looking at a tiny *differences*) plays in modern physics.

#### 8.1.1 The Interferometer

Light impinges on a half-silvered mirror BS (BS is for 'Beam Splitter" !). The rest of the spectrometer consists of two ordinary mirrors M1 and M2, detectors D1, D2 and D1' and a screen, as indicated on Figure 8.21. As discussed in the Figure caption, the beam splitter splits the beam in a very remarkable way.

In addition: When we do something to one path and not to the other one as indicated by "B=?" (the simplest is to introduce a gradual delay along path 1, or magnetic field with which we can gradually change the phase of the beam) the whole spectrum of fringes gradually shifts.

The results show a big puzzle: when only one detector is in place, we observe that each and every particle came through path 1 or path 2. But without the detectors we observe the interference, as if SOMETHING passed through BOTH paths. But what is that "something" ???

We obtain similar results using all particles (photons, neutrons, atoms, even molecules). And note that this is not some weird quantum theory: these experimental facts will have to be explain by the present or ANY future theory (recall the quotes on top of the first page).

Using particles traveling slower than light, we can have fun of thinking about the ontology when we place. The path one detector in the position  $D'_1$ . Whenever this detector does not detect the particle when predicted, we know that (and exactly when) will the detector  $D_2$  hit. What is the ontological status of the particles between the non-detection by D1' and detection by D2?

And last but not least: the particles are detected as single impacts, no matter how far from the source, and the interference effects subsist when then flux of particle is so low that they come literally "on by one" (see Figure Figure 8.22).

So what happens?

Some say: well, the two particles 'simply come through both paths at the same time. That seems very difficult. Each copy of c. the particle behaves like the whole particle, with its full energy, momentum etc. And it is easy to extend the setup so that the initial beam is split into many beams, even infinitely many, even uncountably infinitely many.

In an apparent act of desperation, believers in the Many Worlds interpretation double down: with every single quantum decision to be made anywhere, the whole Universe splits into as many Universes as there are the possible options for that quantum event.

We have something real simple, say the proponent of the Transactional Interpretation: any single quantum event simply sends faster-then-light to all possible quantum partners and the deceision is made depending on which transaction prevails ...

With all due respect to my learned colleagues, all these cures seem (much) worse than the disease.

As we shall see, the standard interpretation of our present theory (quantum mechanics) claims that this question is meaningless. The formalism allows us to correctly calculate the probabilities of the outcome of any experiment we can perform, the experiments discussed above are different experiments, and "what really happened" is, at best, a question for philosophers (while the "real physicists" say: "Shut up and calculate" :-).

On the other hand, some of us say: just wait a minute (or a century or two...)! More on this in Lectures ...

### 8.2 Fundamentals of Quantum Physics

Summary of experiments

1) wave-particle duality

, , , , , , , , , , , , , , , , , , ,		LOUS EN	waves
es no	yes	s YES	
10 Ve	es YES	5 yes	
5	əs no	es no yes no yes YES	es no yes YES yes

(the answers in upper case are the essence of the quantum-mechanical particle-wave duality)

2) Many physical quantities are "quantized".

### 8.2.1 The Quantum Mechanical Solution of the Puzzle:

Planck (1901) : E = hf to explain blackbody spectrum

Einstein (1905) :  $p = E/c = hf/c = h/\lambda$  to explain photoelectric effect

deBroglie (1925) : a (complex) wave with  $\lambda = h/p$  also for electrons etc.

(all of the above with the same value of the Planck constant h =)

Born (1927) : probabilistic interpretation of  $\psi$ 

momentum p, energy  $E \iff \text{complex } \psi(x,t)$  with wavelength  $\lambda = h/p$  and period T = h/E

We shall see how boundary conditions impose quantization. A bit later, we shall see how all this is embedded in the abstract, elegant theory of Quantum Physics in Hilbert Space.

#### 8.2.2 Particle in a potential "box"

Already at this stage we can do some quite fundamental physics. Consider a very simple but very useful "playground" of Quantum Mechanics (see Figure ??). Particle of mass m is confined in a potential V(x) = 0 for 0 < x < a and  $V(x) = \infty$  outside. Standing waves with

$$L = n\lambda_n/2$$

immediately yield quantization of energy

$$E_n = n^2 E_1$$

where  $E_1 = h^2/8mL^2 \neq 0$ 

This is our first encounter with quantization as consequence of the boundary conditions. The hydrogen atom will turn out to be just like this (only a little more complicated mathematically). The remarkable, highly non-classical result that the ground energy is not zero will turn out to be required by the Uncertainty Principle.

#### 8.2.3 Development of quantum physics continued

Born rule elaborated

It would be hard to overestimate the importance of the Born rule giving the correct interpretation of the wave function. It is interesting to note that Born, in his Nobel lecture, gave credit to Albert Einstein (so much for "Einstein was wrong" etc ...).

 $|\psi(x)|^2 dx \equiv$  probability of finding the particle in (x, x + dx)

$$\int_{a}^{b} |\psi(x)|^{2} dx \equiv \text{probability of finding the particle in } (a, b)$$

 $|\psi(x)|^2 \equiv$  probability **density** 

 $\psi(x) \equiv \text{probability amplitude or the "possibility wave"}$ 

for a discrete case,  $|\psi_i|^2$  is the probability of the i-th outcome

several distinguishable alternatives: add probabilities

for several indistinguishable alternatives: add probability amplitudes or possibilities or wave functions

NOTE: When you think about it, the alternatives to get a given (i.e. fully specified) quantum state from a given (fully specified) initial state are ALWAYS indistinguishable. Failure to appreciate this is at the origin of many published papers.

### 8.3 Figures



Figure 8.1: Commercially available blackbody. With a heater (or cooler) and temperature lock.



Figure 8.2: Horizontal is the wavelength in mm. The UV catastrophe is obvious. To appreciate the low frequency behavior see the LogLog plot next.



Figure 8.3: LogLog of previous plot. I am mesmerized by this graph :-)



Figure 8.4: An elementary integral/sum lesson.



Figure 8.5: The error bars are smaller than the thickness of the line. Magnified at the bottom; the curves are no longer relevant.



Figure 8.6: The Nobel for COBE for this



Figure 8.7: Chronology of the Big Bang

Fermilab Photogra



Figure 8.8: Planck spectra for three temperatures. Horizontal scale is CMBR wavelength in mm; note the absolute vertical scale! The legends give Temp of CMBR, time since BB and energy of the CMBR



Figure 8.9: Cartoon of the surface of last scattering



Figure 8.10: The three spacecraft experiments. COBE resolution is barely sufficient to see the first peak (see Figure 8.13); the Planck resolution cannot be improved (see Figure 8.12)



Figure 8.11: Many interesting facts about the first million years.



Figure 8.12: The definitive acoustic spectrum from Planck. The error bars of the residuals are multiplied by 10 on the right of the dashed like,



Figure 8.13: The preWMAP (i.e. also prePlanck) spectrum from 1999. The solid line is the prediction they call "the standard model". Compare with the previous plot. Those theorists in 1999 were clairvoyant(s)!!!(see text on the bottom of p.6)



Figure 8.14: Caption for PhotoeffectCombo



Figure 8.15: Caption for SternGerlach



Fig. 1–1. Interference experiment with bullets.

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Fig. 1–2. Interference experiment with water waves.

Fig. 1-3. Interference experiment

with electrons.





Figure 8.17: The double slit setup with the lower slit blocked; the detector is placed at the point P on Figure 8.16. You hear clicks as electron arrive at the detector.



Figure 8.18: You OPEN previously closed slit, and the electrons STOP arriving



Figure 8.19: Now you close the top slit, and electrons are arriving again - click, click clikclik  $\ldots$ 



Figure 8.20: and the stop again when they have TWO paths open instead of just one ...



Figure 8.21: When any one of the three detectors  $(D_1, D_2 \text{ and } D'_1)$  is placed in the beam, the screen shows a broad, featureless signal as illustrated on a). When all three detectors are moved out of the beam, the screen shows a pattern of fringes as shown on Figure 8.22. For explanation of the element marked "B=?" see the Text.



Figure 8.22: Fringes created by impacts of single particles when the two beams on Figure 8.21 are allowed to interfere. The effect persists even if the flux of particles is so low that there never is more that one particles in the spectrometer at any given time.