A Stress and Relative Density-Dependent Dynamic Compliance Spectra Model of the Creep Response of Microcellular Polycarbonate

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The present paper reports a new discrete complex compliance spectra method in which each frequency component is a direct function of stress and relative density. Comparisons between model calculations and experimental measurements show that the model exhibits excellent quantitative agreement at all experimental stress-relative density states and significantly smoothes the experimental input data. It is anticipated that design engineers will use the present method to accurately predict the creep strain histories resulting from a broad range of specific stress-relative density combinations. [S0021-8936(00)01804-3]

1 Introduction

Microcellular foams are generally closed-cell foams with average cell size of order 10 micrometers ([1]). Such foams are characterized by a high density of cells, usually exceeding 100 million cells per cubic centimeter. The high cell density makes it possible to achieve significant reductions in density of the polymer while keeping the average cell size small. Such foams have been created in polycarbonate using carbon dioxide as the gas for bubble nucleation ([2]). The polycarbonate-carbon dioxide system has been a model microcellular system, and a number of investigations have focused on the properties of this system. For example, tensile, fatigue, and fracture toughness studies have uncovered some unique properties of these novel foams ([3–6]). A key advantage offered by microcellular foams is the potential to reduce the amount of material needed by replacing the solid polymer by a microcellular polymer of reduced density. As polycarbonate is used in many load-bearing applications, a knowledge of the creep behavior of microcellular polycarbonate is of special interest. An experimental investigation of the time dependent response of microcellular polycarbonate foams was conducted by Wing [7] who found that creep strains of the foams contain a significantly higher viscoplastic component compared to the unfoamed material. Wing obtained creep data on foams of relative densities (density of foam devised by the density of polycarbonate) of 1.00, 0.95 and 0.81. The specimens were subjected to various levels of constant tension for eight hours.

Wing et al. [8] have presented a model for the creep response of microcellular polycarbonate based on the Schapery's theory of nonlinear viscoelasticity. The present paper reexamines the experimental measurements of Wing [7] with a new stress and relative density-dependent discrete dynamic compliance spectra method. Comparison between model calculations and experimental measurements show that the new spectra model exhibits excellent quantitative agreement at all experimental stress-relative density states. A materials scientist may use the present model interactively to design experiments which will allow the accurate modeling of nonlinear creep properties of the microcellular material over a broad range of stress and relative density. A designer may use the present model to identify the creep strain as a function of time resulting from a given stress-relative density combination, or to identify relative density range over which the material provides relatively stable creep properties.

The present model requires a set of creep curves measured under variable stress and material relative density as input, and then predicts the creep strain under general conditions of time, stress, and relative density as output. The method in essence uses many thousands of creep parameters, a process made mathematically tractable by the use of fast Fourier transforms. This method was selected in order to construct a single mathematical entity which may be accurately applied to the prediction of the response of a highly nonlinear viscoelastic media over very broad stress and relative density conditions. The method is quite general, and may be applied to many other multivariable time-dependent problems in engineering.

2 A Stress and Relative Density-Dependent Discrete Dynamic Compliance

The present method will make use of fast Fourier transforms ([9]). The first analysis task is, therefore, to fit time-sampled experimental creep data with an appropriate set of continuous functions which may be evaluated at a regular interval. The present calculation uses an $n=0$ to 2 series of Chebyshev polynomials of the first kind for this purpose, the Chebyshev polynomials of the first kind, T_n, being defined by

$$T_n(\mathrm{Cos}\ \theta)=\mathrm{Cos}(n\theta) \tag{1}$$

and obeying the orthogonality condition.

$$\int_{-1}^{1} T_m(x)T_n(x)(1-x^2)^{-1/2}dx=0, \quad \{m\neq n\} \tag{2}$$

A discrete creep strain frequency spectra for each of the individual creep tests may now be constructed by fast Fourier transforming a strain list with an initial element equal to zero and subsequent elements obtained for the creep strain time function ([10–16]). The present calculations set the first strain time sample equal to zero and then evaluate the fitted creep strain functions every 10 seconds from 1 to 651 seconds to produce a 67-element

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time-sample list. The experimental discrete strain frequency components may then be obtained by fast Fourier transformation ([15])

$$\tilde{E}\left(\frac{n}{NT}\right)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}\varepsilon(kT)\exp\left[2\pi i\frac{nk}{N}\right] \tag{3}$$

where N is the number of elements in the time sample list and T is the time-sampling interval. The same numerical procedure is used for the stress, assuming an ideal creep stress step time sequence.

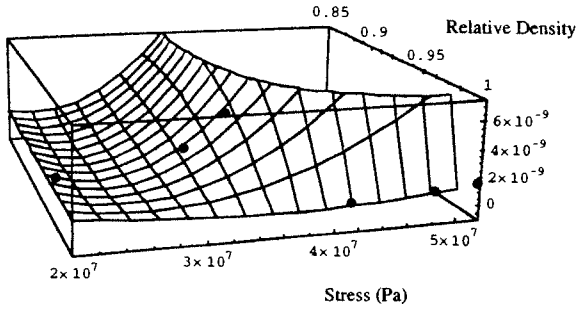
$$\sigma(kT)=\{0,\sigma_0,\sigma_0,\ldots,\sigma_0\} \tag{4}$$

A discrete dynamic compliance may then be obtained by complex division of stress frequency components with corresponding strain frequency components.

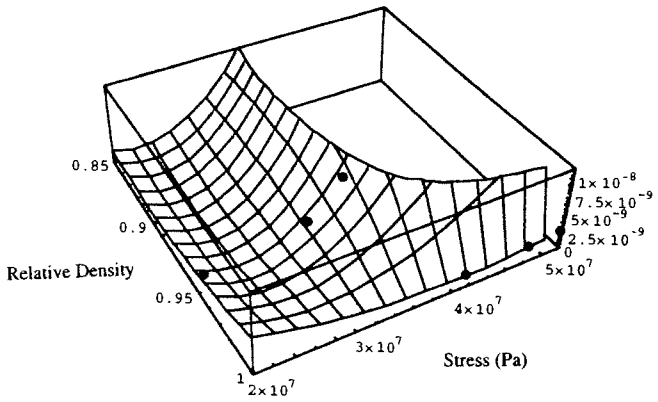
$$\tilde{J}=\left\{\frac{\tilde{E}\left(\frac{0}{NT}\right)}{\tilde{S}\left(\frac{0}{NT}\right)},\frac{\tilde{E}\left(\frac{1}{NT}\right)}{\tilde{S}\left(\frac{1}{NT}\right)},\frac{\tilde{E}\left(\frac{2}{NT}\right)}{\tilde{S}\left(\frac{2}{NT}\right)},\ldots,\frac{\tilde{E}\left(\frac{N-1}{NT}\right)}{\tilde{S}\left(\frac{N-1}{NT}\right)}\right\} \tag{5}$$

We now assume that the real and imaginary parts of each discrete frequency component smoothly vary with applied stress and relative density. We may then assemble a stress and relative density-dependent discrete dynamic compliance spectra by regression for the real and imaginary parts of each experimental discrete dynamic compliance component. The regression basis used for the present calculations is given by

$$\sum_{i=0}^{2}\sum_{k=0}^{2}T_i(\rho_{\mathrm{rel}})T_k\left(\frac{\sigma}{60\,10^6}\right). \tag{6}$$

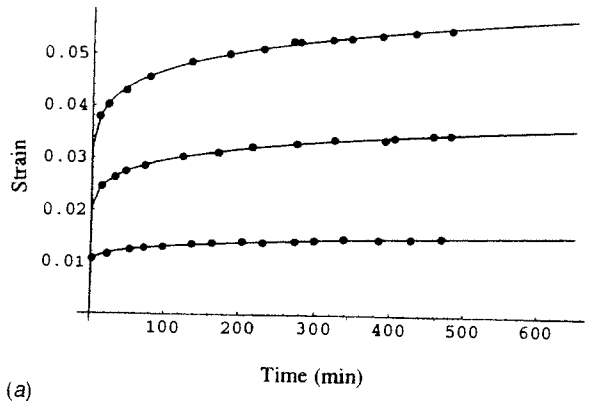


(a)

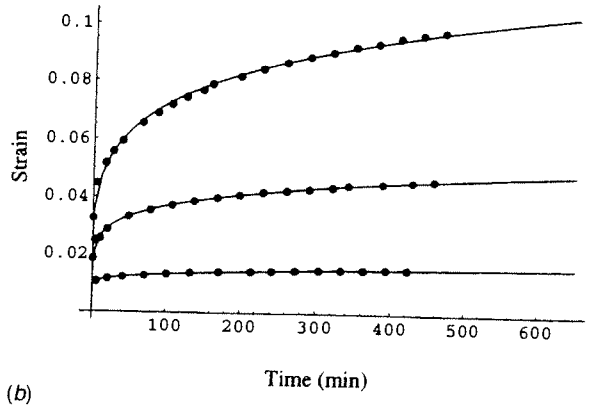


(b)

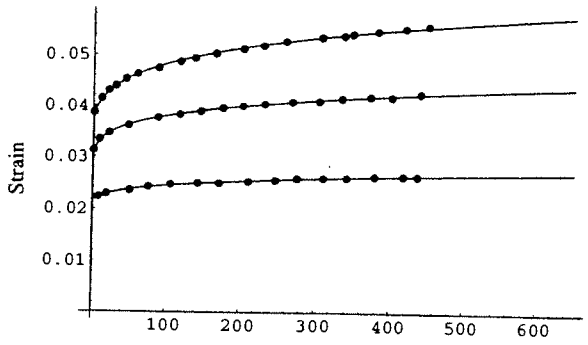
Fig. 1 The stress-relative density dependencies of the real, (*a*), and imaginary, (*b*), parts of the third discrete dynamic compliance component. Dots indicate experimental measurements.



(a)



(b)



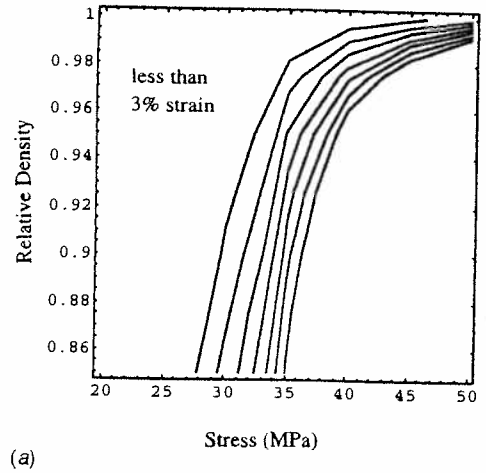
(c)

Fig. 2 Comparison between experimental creep measurements and model calculations; (*a*) relative density=0.81, s =27.6, 24.1, and 13.8 Mpa, (*b*) relative density=0.95, s=34.5, 31.0, and 20.7 Mpa, and (*c*) relative density=1.00, s=51.7, 48.2, and 41.3 Mpa.

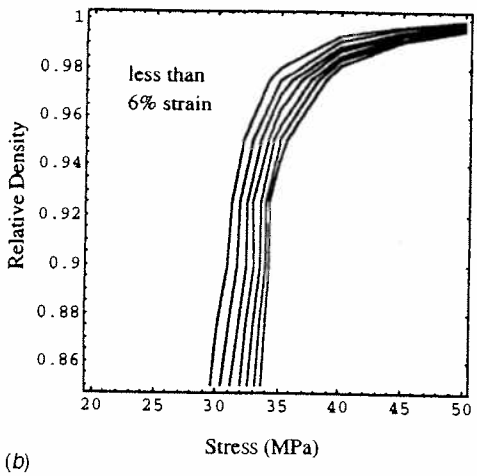
Figure 1 shows the real, (*a*), and imaginary, (*b*), response surface of the third discrete dynamic compliance frequency component as functions of relative density and applied stress. Experimental stress-relative density states are identified with black dots. The figure shows that the real and complex parts of the dynamic compliance appear to be smooth, easily fit functions of stress and relative density within the experimental range. The present creep behavior is clearly under sampled at low relative density and high stress. An additional measurement at a relative density of approximately 0.81 and a stress of approximately 34 MPa would very likely have improved model predictions in this stress-relative density region.

3 Evaluation of the Creep Model

The calculation of creep processes begins by the evaluation of the real and complex parts of the stress-dependent dynamic com-



(a)



(b)

Fig. 3 Predicted creep strain as a function of applied stress and relative density; (*a*) creep strain after 31 minutes with one percent creep contours ranging evenly from 3 percent strain to 9 percent strain, and (*b*) creep strain after 501 minutes with one percent creep contours ranging evenly from six percent strain to 12 percent strain.

pliance spectra for a particular level of applied stress and relative density. The discrete creep strain frequency spectrum may then be obtained by multiplying dynamic compliance spectra components by corresponding stress frequency components. The creep strain as a function of time is then obtained by inverse discrete Fourier transformation.

$$\varepsilon(kT) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \tilde{E}\left(\frac{n}{NT}\right) \exp\left[-2\pi i \frac{nk}{N}\right] \tag{7}$$

Figure 2 compares experimental creep measurements with model calculations at each experimental applied stress-relative density condition. The figure shows that the model exhibits excellent quantitative agreement at all experimental stress-relative density states. The model also provides further benefit by significantly smoothing the experimental data. The ability of the model to precisely match all experimental measurements along with the smoothness of the dynamic compliance response surfaces indicate that the model should provide accurate interpolation and some range of accurate extrapolation to nonexperimental stress-relative density states when the dynamic compliance surfaces are well sampled. A materials scientist may therefore use the present model interactively to design experiments which will allow the accurate modeling of the nonlinear creep of the microcellular materials over a broad range of stress and relative density.

The model may be further used to calculate the creep strain corresponding to any time sample coordinate as a generalized function of stress and relative density. Figure 3 shows two examples of how the resulting isochronal creep prediction may be displayed as a contour plot. A designer may use such a plot to identify stress-relative density combinations which will not result in an excessive accumulation of creep strain, or to identify relative density ranges over which the microcellular material provides stable creep properties. For example, Fig. 3(*a*) shows that a microcellular polycarbonate sample with a relative density of 0.86 under a 30 Mpa stress will strain nearly the same amount after 501 seconds as a 0.99 relative density sample under a 40 Mpa stress. The plot further shows that the creep of microcellular polycarbonate is very sensitive to changes in relative density when the relative density is between 1 and 0.96, and much less sensitive to changes in relative density when the relative density is between 0.95 and 0.85. Therefore, it would appear to be very difficult to obtain consistent creep measurements from materials with relative densities above 0.95.

4 Summary and Conclusions

The present paper reports a new stress and relative density-dependent discrete dynamic compliance spectra method which may be used to model the creep behavior of nonlinear viscoelastic microcellular materials. Comparisons between model calculations and experimental measurements show that the model exhibits excellent quantitative agreement at all experimental stress-relative density states. The model also significantly smoothes the experimental data. A materials scientist may use the present model interactively to design experiments which will allow the accurate modeling of the nonlinear creep properties of the microcellular material over a broad range of stress and relative density. A designer may use the present model to identify the creep strain resulting from a given stress-relative density combination after a given amount of time, or to identify relative density ranges over which the material provides relatively stable creep properties.

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