AMATH 352 Summer 2012
Midterm

Wednesday, July 25

Name: ____

Problem	Points	Score
1	10	
2	20	
3	20	
4	20	
5	20	
6	10	

Total	

1. **(15)** The trace of an 3×3 matrix $A \in \mathcal{M}_{3\times3}$ is defined to be the sum of its diagonal elements: $\operatorname{tr} A = a_{11} + a_{22} + a_{33}$. Show that the set of trace zero matrices, $\operatorname{tr} A = 0$, is a subspace of $\mathcal{M}_{3\times3}$.

2. **(5 each)** Answer the following questions in one sentence or less giving brief justification.

(a) If $A = A^T$ then how are corng A and rng A related?

(b) If the range of A is a plane in $\mathbb{R}^3$, what is rank A?

(c) Is $S = \{\mathbf{x} \in \mathbb{R}^3 : (x-1) + y + z = 0\}$ a subspace of $\mathbb{R}^3$?

(d) If A is an $n \times n$ square matrix and $\ker A = \{0\}$ then are there any solutions of $A\mathbf{x} = \mathbf{b}$? If so, how many?

3. **(20)**

(a) *(15)* Find a basis for the kernel, cokernel, range and corange of the following matrix

$$B = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \end{bmatrix}.$$

(b) *(5)* Verify the fundamental theorem of linear algebra for the matrix B in 3(a).

4. **(25)**

(a) *(15)* Compute the the $PA = LU$ factorization of the following matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 0 \\ -1 & -3 & 4 \end{bmatrix}.$$

(b) *(5)* Does A have a Cholesky factorization? Why or why not?

(c) *(5)* Compute $\det A$.

5. **(20)**

(a) *(15)* Let T_n be the permutation matrix that interchanges rows of an $n \times n$ matrix in the following way:

- row j is moved to row $j+1$, $j = 1, \ldots, n-1$ and
- the last row is moved to the first.

Find

i. $\det T_3$

ii. $\det T_4$

iii. $\det T_k$ for $k > 1$

(b) *(5)* Let K be the $n \times n$ matrix that copies the first row and replaces the second row with this copied row. All other rows, including the first row, are left the same. What can you say about $\det K$?

(c) **(10)** Find a basis for the following subspace, S, of $\mathbb{R}^3$:

$$S = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \right\}.$$