AMATH 352 Homework 7

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Due Friday, August 10

Exercise 1

Consider the following least-squares approach to approximately integrating a function $f(x) = \exp(x)$ over $[-1, 1]$.

- For a given n, use the code `x = cos((0:n)*pi/n)'` to generate a non-uniform grid on $[-1, 1]$.
- For each point x_i in this grid we get a data point $(x_i, f(x_i))$.
- Using the ideas from Exercise 1 of Homework 6 set up a least squares system to fit this data with a polynomial $p(x) = ax^2 + bx + c$.
- Solve the least-squares system for a, b and c.
- Integrate this polynomial exactly to find

$$\text{approx} = \int_{-1}^{1} (ax^2 + bx + c)dx.$$

- Define actual $= \int_{-1}^{1} \exp(x)dx$ and compute the error: `abs(actual-approx)`.

Answer the following questions:

(a) Give the error with $n = 10$.

(b) As you increase n does the error limit to zero?

Please upload your code, as usual.

Exercise 0 *Extra Credit, 5 pts*

Modify the code above to allow for the degree of the polynomial to increase with the number of grid points, *i.e.* $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$. Note that you now have a square matrix and no longer need to use a least-squares solve. Upload your code. How large does n need to be so that machine precision is reached (when MATLAB returns 0)?

The following exercises should be done by hand, showing all steps.

Exercise 2

Olver & Shakiban— 5.3.27bdf

Exercise 3

Find the $A = QR$ factorization of

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -2 \\ -1 & -3 \\ 0 & 5 \end{bmatrix},$$

and use this to solve the least-squares problem

$$\min_{\mathbf{x}\in\mathbb{R}^2} \|A\mathbf{x} - \mathbf{b}\|, \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

Exercise 4

Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation. Let $\mathbf{e}_j$, $j = 1, 2, 3$ be the standard basis vectors. Use the following information to write out the matrix representation for L:

$$L[\mathbf{e}_1 + \mathbf{e}_2] = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad L[\mathbf{e}_3 + \mathbf{e}_2] = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad L[\mathbf{e}_1] = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

What is the rank of L? Can you determine this before you construct the matrix representation?

Exercise 5

Olver & Shakiban 8.2.14

Exercise 6

Olver & Shakiban 8.3.16