AMATH 351 Homework Five

Tom Trogdon

Due July 29, 2011

Exercise 1

Consider the oscillator given by

$$x'' + \gamma x' + 2x = 0.$$

(a) Find the value of γ so that the system is critically damped.

(b) For this value of γ, solve the initial value problem with $x(0) = 0, \;\; x'(0) = 1$.

Exercise 2

Consider the differential equation

$$x'' + \omega^2 x = C\cos(\Omega t), \quad \omega, \Omega, C \text{ are constants.}$$

Further assume that $\omega, \Omega > 0$ and $\omega \neq \Omega$.

(a) Find the general solution.

(b) Impose the initial conditions $x(0) = x'(0) = 0$.

(c) Write the solution in the form

$$x(t) = c_1 \sin(\omega_1 t)\sin(\omega_2 t),$$

determining all constants.

(d) Using the notation from page 75 of the notes: Which frequency governs the modulation of the amplitude? Which frequency governs the underlying carrier wave?

Exercise 3

If

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 0 \\ 3 & 2 & -1 \\ -2 & 1 & 3 \end{bmatrix}, \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 4 & -2 & 3 \\ -1 & 5 & 0 \\ 6 & 1 & 2 \end{bmatrix},$$

find

(a) **AB**

(b) **BA**

(c) Are they (**AB** and **BA**) equal to each other?

Exercise 4

Determine if the columns of $\mathbf{A}$ are linearly independent. Then find all solutions x of $\mathbf{A}x = 0$.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tag{1}$$

Note: These solutions form what is called the nullspace of $\mathbf{A}$.