AMATH 351 Homework 2

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Due Wednesday, July 6

1 Method Of Integrating Factors (B& D 2.1)

Exercise 1

For the following differential equations, find the general solution.

1. $y' + 3y = t + e^{-2t}$.

2. $y' + y/t = 3\cos 2t, \quad t > 0$.

Exercise 2

Solve the initial value problem

$$ty' + 2y = t^2 - t + 1, \quad y(1) = \frac{1}{2}, \quad t > 0. \tag{1}$$

Exercise 3

Consider the initial value problem

$$y' - \frac{3}{2}y = 3t + 2e^t, \quad y(0) = y_0. \tag{1}$$

Solve this equation and obtain an explicit solution $y = y(t)$. You can see that, for different y_0 values, $y(t)$ approaches different values when $t \to +\infty$. Then, find the value of y_0 that separates solutions that grow positively as $t \to +\infty$ from those that grow negatively. How does the solution that corresponds to this critical value of y_0 behave as $t \to +\infty$?

Exercise 4

A tank with a capacity of 500 gal originally contains 200 gal of water with 100lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the mixture is allowed to flow out of the tank at a rate of 2 gal/min. Find the amount of salt in the tak at any time prior to the instant when the solution begins to overflow. Find the concentration (in pounds per gallon) of salt in the tank when it is on the point of overflowing. Compare this concentration with the theoretical limiting concentration if the tank had infinite capacity.

2 Stability and Phase Plane Analysis

Exercise 5

Consider the following equation

$$y' = (1 - y^2)(y - 2)e^y. \tag{1}$$

Find all equilibrium solutions and classify their stability. Use this information to draw some solution curves.

3 Exact Equations and Integrating Factors (B&D 2.5)

Exercise 6

Determine whether each of the equations is exact. If it is exact, find the general solution.

(a) $(e^x \sin y + 3y)dx - (3x - e^x \sin y)dy = 0.$

(b) $(y/x + 6x)dx + (\ln x - 2)dy = 0.$