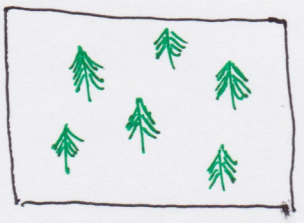
Bio-Example #6: Nature Reserve Design

In conservation, it is common to try to design nature reserves that are the most cost-effective given a set budget. For example, suppose I want to create a rectangular reserve that is x miles long and y miles wide:



I have determined the cost of maintaining this reserve is $2x^2 + y^2 + xy$. Moreover, I have a budget of 100.

If we use the entire budget, then $2x^2 + y^2 + xy = 100$.

But should we make x really big at the expense of y, or should we do the opposite? One way to quantify the tradeoff between x and y is to find $\frac{dy}{dx}$. This will tell us how much we have to decrease y if we increase x and vice-versa. Using implicit differentiation,

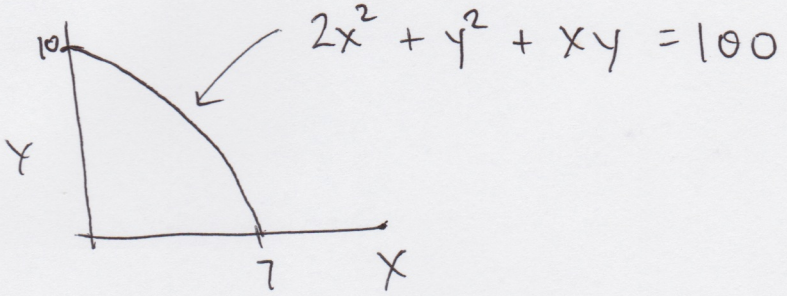
$$\frac{d}{dx}\left[2x^2 + y^2 + xy\right] = \frac{d}{dx}\left[100\right]$$

$$4x + 2y \cdot y' + y + xy' = 0$$

$$y'(2y+x) + 4x + y = 0$$

$$\frac{dy}{dx} = \boxed{y' = \frac{-4x-y}{x+2y}}$$

If $x \geq 0$ and $y \geq 0$ (which they should be), then $\frac{dy}{dx}$ is always negative, meaning an increase in x leads to a decrease in y.



Later on, we'll learn how to find the best choice of (x,y) that will provide the most conservation given the budget.