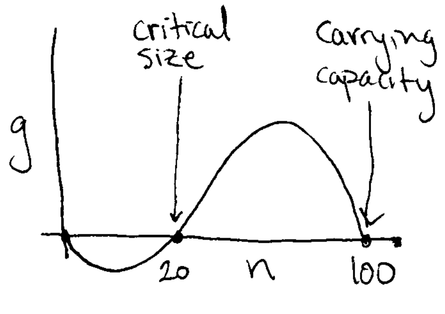
Bio-Application # 5 : Allee Effect

In many biological populations, growth is density-dependent. That means growth depends on how many individuals are already in the population. Most organisms exhibit a carrying capacity, so if the population gets too big, the growth rate becomes negative. In a population exhibiting the Allee effect, there is also a critical threshold of individuals the population has to reach before there is a positive growth rate. For example, if there are enough meerkats in a population of meerkats (think Timon), they will be able to warn each other about predators, thus having a positive growth rate. One way to model the Allee effect mathematically is with the equation for growth rate g as a function of population size n

$$g(n) = 1.3n\left(1-\frac{n}{100}\right)\left(\frac{n}{20}-1\right).$$



The population is in equilibrium if $n=0$, $n=20$, or $n=100$, because $g(n)=0$ at those values of n. One way to

Find out if a certain equilibrium is stable or unstable over time is to look at the derivative of g at each equilibrium point, $n = 0, 20$, or 100. If the derivative is negative, then the population is likely to stay at that size, but if the derivative is positive, it is likely to find a new equilibrium over time. To find the derivative of $g(n)$, you can either use the product rule twice, or use the "triple product rule," which says

$$(u \cdot v \cdot w)' = u' \cdot vw + uv' \cdot w + uvw'.$$

Then

$$g'(n) = 1.3\left(1-\frac{n}{100}\right)\left(\frac{n}{20}-1\right) + 1.3n\left(-\frac{1}{100}\right)\left(\frac{n}{20}-1\right) + 1.3n\left(1-\frac{n}{100}\right)\left(\frac{1}{20}\right)$$

If we plug in each equilibrium value $n = 0, 20, 100$ into this,

$$\star \quad g'(0) = 1.3(1)(-1) = \boxed{-1.3 < 0}$$

$$g'(20) = 1.3(20)\left(1-\frac{20}{100}\right)\left(\frac{1}{20}\right) \boxed{> 0}$$

$$\star \quad g'(100) = 1.3\left(-\frac{1}{100}\right)\left(\frac{100}{20}-1\right) \boxed{< 0}$$

So the population is likely to be at its carrying capacity ($n=100$) or extinct ($n=0$), depending on if there are initially more or less than 20 meerkats.