Midterm - QSci 291 - Winter 2014

February 14, 2014

Each question is worth 5 pts. Put your name on the other side of each page.

(1) Solve $|7x - 16| \geq 24$ for x (illustrate answer on graph):

Case 1: $7x-16 \geq 0$

$7x - 16 \geq 24$

$7x \geq 40$

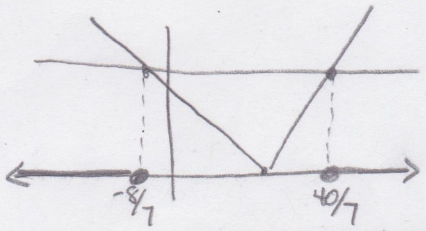
$\boxed{x \geq 40/7}$

Case 2: $7x-16 < 0$

$7x - 16 \leq -24$

$7x \leq -8$

$\boxed{x \leq -8/7}$



$\boxed{x \in (-\infty, -8/7] \cup [40/7, \infty)}$

(2) Find the equation of the line (in slope-intercept form) containing $(-3, 3/5)$:

$(2,2)$

$$m = \frac{2 - 3/5}{2+3} = \frac{7/5}{5} = \frac{7}{25}$$

$$y - 2 = \frac{7}{25}(x-2) \longrightarrow y = \frac{7}{25}x - \frac{14}{25} + \frac{50}{25} = \boxed{\frac{7}{25}x + \frac{36}{25}}$$

(3)Find:

$$\lim_{x\to -4} \frac{x^2 - x - 20}{x+4} = \lim_{x \to -4} \frac{(x-5)(x+4)}{x+4}$$

$$= \lim_{x\to -4} x - 5 = \boxed{-9}$$

(4) Find a recursion for a population that quadruples in size every 25 minutes and has 1024 individuals at 50 minutes:

t	0 min	25 min	50 min
N_t	64	256	1024

$\div 4$ $\quad$ $\div 4$

$$\boxed{N_{t+1} = 4 \cdot N_t \ , \quad N_0 = 64}$$

where t represents units of 25 mins.

(5)Use the limit laws to determine:

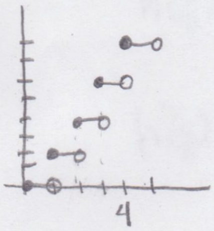
$$\lim_{x\to\infty}\left(\frac{2}{n} - \frac{1}{n^2+1}\right)$$

$\lim\limits_{n\to\infty} \frac{2}{n} = 2 \cdot \lim\limits_{n\to\infty} \frac{1}{n} = 2 \cdot 0 = 0;$

$\lim\limits_{n\to\infty} \frac{1}{n^2+1} = 0;$ so

$\lim\limits_{n\to\infty} \frac{2}{n} - \frac{1}{n^2+1} = \boxed{0}.$

(6)Investigate if the floor function $g(x) = 2\lfloor x \rfloor$ is continuous at $x = 4$:



$g(4) = 8$, so $g(4)$ is defined, but

$\lim\limits_{x\to 4^-} g(x) = \boxed{6}$ and $\lim\limits_{x\to 4^+} g(x) = \boxed{8}$ so

$\boxed{g(x) \text{ is not continuous at } x=4.}$

(7) Find $f'(x)$ for $f(x) = (3x - 2)^2$:

$f(x) = (3x-2)(3x-2) = 9x^2 - 12x + 4$

Using the power rule,

$$\boxed{f'(x) = 18x - 12.}$$

(8) Find the equation of the tangent line (in slope-intercept form) to the curve $y = 7x^4$ at the point $(7, 16807)$:

$y'(x) = 28x^3$, so $y'(7) = 28 \cdot 7^3 = 9604 =$ slope.

$16{,}807 = 9604 \cdot 7 + b$

$b = -50{,}421 \longrightarrow \boxed{y = 9604x - 50421}$

(9) Evaluate $f'(x)$ for $f(x) = \frac{a+2x^3}{ab^2} - abx + (a+2b)x - ab$ with respect to x. Assume that a and b are constants:

$f(x) = \frac{a}{ab^2} + \left(\frac{2}{ab^2}\right)x^3 - abx + (a+2b)x - ab$

$f'(x) = 0 + 3 \cdot \left(\frac{2}{ab^2}\right)x^2 - ab + (a+2b)$

$= \boxed{\frac{6x^2}{ab^2} - ab + a + 2b}$.

(10) Differentiate $h(q) = 3q^6 \cos \pi/3 + cos\pi/6$:

$h(q) = [3 \cdot \cos(\pi/3)]\, q^6 + \cos(\pi/6)$

$h'(q) = 6 \cdot [3 \cdot \frac{1}{2}]\, q^5 + 0$

$= \boxed{9q^5}$.