Comparing Model I and Model II Formulations of Spatially-Explicit Harvest Scheduling Models with Adjacency Constraints

Abstract: This paper investigates whether Johnson and Scheurman's (1977) Model II formulation, which can dramatically reduce the size and difficulty of linear programming harvest scheduling models, offers similar potential for efficiency gains in solving spatially-explicit harvest scheduling models with area-based adjacency constraints. One hundred and fifty hypothetical problems and four real problems were formulated using Model I and using two Model II formulations. The hypothetical problems were distributed (30 each) in five categories: regulated forest problems with four, six, and eight planning periods, and over-mature forest problems with four and six periods. The length of the planning horizon was a key factor determining whether Model II formulations outperform Model I formulations in spatially-explicit forest management planning problems. Results from the hypothetical problems suggest that Model I formulations outperform Model II formulations for four-period problems. However, Model II formulations perform significantly better than Model I formulations for problems with planning horizons of six and eight planning periods. The alternative Model II formulation (Model IIa) performs slightly better than the basic Model II formulation, but not significantly. Real forest results are not as conclusive as with the hypothetical forests, reflecting the limitations of drawing conclusions from a single case study.

Key Words: Forest planning models, mixed-integer programming, area-based adjacency constraints, area-restriction models.

1. Introduction

Linear programming (LP) formulations of forest-wide management planning problems were first introduced in the 1960s (Curtis 1962, Loucks 1964, Kidd et al. 1966, Nautiyal and Pearse 1967, Ware and Clutter 1971). When these models were proposed, solving even small LP models was a challenge due to the limitations of computers at the time. Some addressed these computational challenges by developing alternative algorithms for solving the problems. For example, Walker (1976) developed the binary search method to schedule harvests over time to maximize the net present value of the harvest subject to a downward-sloping demand curve. Hoganson and Rose (1984) developed a Lagrangian decomposition approach that breaks large problems with forest-wide harvest targets into many smaller dual problems that optimize individual management unit (stand) decisions. The smaller problems are tied together by the global problem of finding a set of shadow prices that result in the approximate satisfaction of forest-wide constraints when the individual management unit solutions are aggregated. A significant breakthrough in both model size and solution time came when Johnson and Scheurman (1977) proposed an alternative formulation of the linear programming (LP) harvest scheduling problem, which they called Model II, that reduces the number of variables needed to formulate large LP forest management problems, potentially by several orders of magnitude – from 265,665 to 506 in one example (p. 9).

While dramatic advances in computing technology have made even the largest LP problems of past decades relatively easy to solve today, new, more complex problems continue to challenge the limits of today's computers and software. In the past two decades, considerable forest management planning research has focused on developing spatially-explicit planning models (e.g., Meneghin et al. 1988, Nelson and Brodie 1990, Lockwood and Moore 1993, Weintraub et al. 1994, Murray and Church 1995a,1995b, 1996, Snyder and ReVelle 1996, 1997, Hoganson and Borges 1998, Borges et al. 1999, Boston and Bettinger 1999, McDill and Braze 2000, McDill et al. 2002, Falcão and Borges. 2001, 2002, Caro et al. 2003, Richards and Gunn 2003, Crowe et al. 2003, Rebain and McDill 2003a, 2003b, Goycoolea et al. 2005, 2009, Tóth and McDill 2008, and Constantino et al. 2008). Much of this research has focused on addressing adjacency constraints, which limit the size of openings created by harvesting operations and require a minimal green-up period before adjacent areas can be harvested. Adjacency is an important problem, as many forestry organizations face such constraints (e.g., Barrett et al. 1998, AF&PA 2000, Boston and Bettinger 2002). However, other important forest management issues – including mature patch size (Rebain and McDill 2003a, 2003b) and shape (Barrett 1997, Tóth and McDill 2008), patch size distribution, core area (Öhman and Eriksson 1998, Öhman 2000, Wei and Hoganson 2007, Zhang et al. 2011), connectivity and habitat fragmentation (Franklin and Forman 1987, Gustafson and Crow 1998), and landscape susceptibility to catastrophic fire (Acuna et al. 2010) – also benefit from the application of spatially-explicit models.

Many spatially-explicit forest management planning problems that include a wide variety of spatial management objectives and constraints have been formulated as mixed-integer linear programming (MIP) models (e.g., Meneghin et al. 1988, Murray and Church 1995a, 1996, Snyder and ReVelle 1996, 1997, McDill and Braze 2000, McDill et al. 2002, Crowe et al. 2003, Rebain and McDill 2003a, 2003b, Goycoolea et al. 2005, 2009, Tóth and McDill 2008, and Constantino et al. 2008). Typically, binary variables are used to indicate whether a particular management regime (including the timing of various management actions) will be applied to a particular management unit. These models can be quite large, due to a large number of management units and/or management regimes. Since their solution space is non-continuous – consisting of feasible points rather than a convex, continuous feasible region as in the case of LP – and because the number of feasible solutions is a combinatorially large function of the number of management units and management regimes, these models can be extraordinarily hard to solve.

Some have addressed these challenges by developing heuristic algorithms for solving spatially-explicit forest planning problems (e.g., Nelson and Brodie 1990, Lockwood and Moore 1993, Weintraub et al. 1994, Murray and Church 1995b, Hoganson and Borges 1998, Borges et al. 1999, Boston and Bettinger 1999, Falcão and Borges. 2001, 2002, Caro et al. 2003, Richards and Gunn 2003). Others have developed and evaluated alternative MIP formulations (e.g., Meneghin et al. 1988, Murray and Church 1995a, 1996, Snyder and ReVelle 1996, 1997, McDill and Braze 2000, McDill et al. 2002, Goycoolea et al. 2005, 2009, and Constantino et al. 2008). Nearly all of the MIP formulations of spatially-explicit forest management planning models (e.g., Meneghin et al. 1988, Murray and Church 1995a, McDill and Braze 2000, McDill et al. 2002, Goycoolea et al. 2005, 2009, and Constantino et al. 2008) have used a Model I formulation where each management unit variable represents a single management regime for that unit for the entire planning horizon (Johnson and Scheurman 1977). This is not surprising, for a couple of reasons. First, one of the key advantages of the Model II formulation of an LP harvest scheduling model is that areas harvested in the same planning period that otherwise differ only in terms of their initial age class can be "merged" and represented by a single common variable after the first harvest. This is not an option in a spatially-explicit model if the modeler wishes to maintain the spatial uniqueness of areas represented by each variable. Furthermore, at least in LP formulations, the advantages of Model II compared with Model I increase as the planning horizon is increased, and most of the spatially-explicit forest planning models in the literature do not have very long planning horizons. Most assume that a management unit will be harvested at most once during the planning horizon, and the planning horizons are generally less than one rotation (e.g., Meneghin et al. 1988, Murray and Church 1995a, McDill and Braze 2000, McDill et al. 2002, Goycoolea et al. 2005, 2009, and Constantino et al. 2008).

Since few papers have explicitly discussed the reason for choosing a particular planning horizon, we can only speculate that in at least some cases relatively short planning horizons were used because problems with longer planning horizons were simply too hard to solve. (The authors of this paper have certainly experienced this.) Some authors might also argue that in many cases it does not make sense to formulate and solve spatially-explicit forest planning models with long planning horizons since unpredictable events will almost certainly require a change of plans in later periods, so there is no point in planning in so much detail very far into the future. Nevertheless, good reasons can also be given for using longer planning horizons, even for spatially-explicit forest planning. The most important is that forest sustainability issues generally require one to consider more than one rotation. For example, how else can planners ensure that a given harvest level can be sustained for more than one rotation? Furthermore, target forest conditions, such as a target age-class distribution, often take longer than one rotation to achieve (Hoganson and McDill 1993), and it may take longer than one rotation to project the cumulative effect of harvesting decisions on the spatial structure of the future forest, e.g., on its patch size distribution. As with non-spatial plans, even if it is unlikely that solutions for future periods will be followed precisely, it is useful to show that if all works out as planned the plan can actually produce the desired future conditions. Finally, if a model does not consider a complete rotation in forest planning, it is possible – even likely, especially with profit-maximizing objective functions – that the model will harvest the best sites during the planning horizon, leaving poorer quality or inaccessible sites to be harvested after the end of the planning horizon (McQuillan 1986, 1991). Finally, it is possible – even likely, especially if management units are significantly smaller than treatment units (harvest blocks) – that the model will leave unharvested areas in pieces that are too small or too oddly shaped to be manageable.

This paper investigates whether Johnson and Scheurman's (1977) Model II formulation – which can dramatically reduce the size and solution difficulty of LP harvest scheduling models – offers similar potential for efficiency gains in solving spatially-explicit harvest scheduling models. As mentioned above, in a Model I formulation of a harvest scheduling problem a variable represents a single management regime for an area for the entire planning horizon (Johnson and Scheurman 1977). In contrast, a Model II formulation uses four types of variables to track the management of various areas (Fig. 1). One type of variable represents the decision not to cut a management unit during the planning horizon. The second and third types represent the decisions to cut a unit in a given period for the first time or for the last time, respectively. Finally, the fourth type of variable represents the decision to harvest a management unit in a particular period after it has been cut in another period. In aspatial, LP harvest scheduling models, management units are aggregated into “analysis areas” that combine areas that are similar in terms of forest type, site class, stocking class, initial age class and possibly other characteristics, and continuous variables are defined representing the number of hectares from these analysis areas assigned to specific management regimes. In spatial MIP models, binary variables are used to represent whether or not a given management regime will be applied to a given management unit. The models used in this research are basic spatially-explicit forest planning models (MIPs) with harvest flow constraints, ending condition constraints, and area-based adjacency constraints (McDill et al. 2002). Nevertheless, we expect that the results presented here will also apply to spatially-explicit models formulated to address a broader range of spatially-explicit constraints and objectives.

While most spatially-explicit harvest scheduling models in the literature have used a Model I formulation, this paper is not the first to apply the Model II formulation to spatially-explicit harvest scheduling models. Snyder and ReVelle (1996, 1997) present a network formulation of the spatially-explicit harvest scheduling problem that is very similar to the Model II LP formulation. Snyder and ReVelle’s (1996) results suggest that the formulation is quite efficient. For example, they report solving in seconds a problem with 50 cutting units and 15 ten-year planning periods having 13,350 variables and 1,871 constraints (p. 1086), which was a very large MIP model at the time and one of the longer planning horizons reported ever in the literature for spatially-explicit forest management models. However, Snyder and ReVelle (1996, 1997) never directly compared the Model I and Model II formulations, and the number of problems that they report is relatively small. Since then, to our knowledge, no spatially-explicit harvest scheduling models using a Model II formulation have appeared in the literature.

2. Methods

To compare the performance of Model I and Model II formulations of spatially-explicit forest management planning problems, four real and 150 hypothetical problems were formulated and solved in Model I format and in two Model II formats. The hypothetical problems are based on data from 60 forests created with MakeLand (McDill and Braze 2000). The 60 forests are based on 30 maps, each having 50 irregularly-shaped polygons, representing management units averaging 20 ha in size. Each map was randomly populated with a regulated forest and an over-mature forest. The target regulated and over-mature age-class distributions are shown in Table 1. The actual age-class distributions of specific forests vary slightly from the target age-class distributions because management units must be assigned in their entirety to only one age-class.

The four real problems are based on a loblolly pine plantation forest in the southeastern U.S. Although the problems are based on actual data, both the spatial data and the growth and yield information were intentionally perturbed to prevent the disclosure of private information. The plantation has 280 management units (4,884 ha in total) that range in age from 0 to 60 years. Three different timber products are produced in this plantation: pulpwood, sawtimber and chip-and-saw. For the flow constraints, the harvest variables h_t were defined as the harvest revenue generated at time t, rather than the volume harvested, and flow constraints with 10% bounds were used. Flow constraints for the hypothetical forest problems allowed the volume harvested to increase by up to 15% per period, but only allowed decreases of 3%.

We hypothesized that the Model II formulation would be especially efficient for problems with longer planning horizons. Thus, problem instances with four, six, and eight 20-

year planning periods were formulated for each hypothetical regulated forest, and instances with four and six 20-year planning periods were formulated for each hypothetical over-mature forest. Eight-period models were not formulated for the over-mature forests because they would have been too difficult to solve to optimality within the time limit used in this study. The optimal rotation for the hypothetical forests is 80 years, and the minimum rotation is 60 years, so it is possible to harvest a management unit up to two times within the planning horizon for problems with four or six planning periods, and up to three times with eight planning periods. Four problem instances were formulated for the real loblolly pine forest: one with five 5-yr periods, one with ten 5-yr periods, one with fifteen 5-yr periods, and one with twenty 5-yr periods. These planning horizons correspond roughly to one, two, three and four rotations.

Each model maximizes the discounted net revenue, plus a discounted residual forest value based on the state of the forest at the end of the planning horizon, subject to flow constraints, an ending age constraint, and adjacency constraints. In all cases, the minimum average ending age was set at one half of the optimal rotation. A variety of formulations that impose adjacency restrictions have been proposed in the literature (McDill and Braze 2000, McDill et al. 2002, Goycoolea et al. 2005, and Constantino et al. 2008). The problem instances in this study were formulated with McDill et al.'s (2002) Path constraints. These constraints allow contiguous groups of management units to be harvested concurrently as long as their combined area does not exceed a given maximum harvest opening size. The mathematical formulation of these constraints is described below. Alternative adjacency constraint formulations were not considered, but we believe it is unlikely that the general conclusions of the study would change if a different adjacency formulation was used.

Programs written in MS Visual Basic 2008 were used to formulate the problem instances. Problem instances were then solved with IBM ILOG CPLEX Version 12.1.0 (64-bit) on 4 threads on a 4-core Intel Xeon X5160 (2.99 GHz) machine with 4 GB of RAM and a 64-bit Windows 7 Operating System. The solver was set to terminate if one of the following two stopping criteria was met: 1) an optimal solution was found, or 2) twelve hours of solution time had passed. A formulation of a given problem was considered superior to another formulation of the same problem if either 1) an optimal solution was found faster with that formulation, or 2) if the optimality gap at the end of 12 hours was smaller for that formulation than for the other formulation. The "optimality gap" refers to the difference between the dual bound on the objective function value of the problem and the primal bound at the end of 12 hours of solution time, divided by the primal bound and expressed as a percent (McDill and Braze 2001). One of the advantages of the branch-and-cut algorithm implemented by CPLEX, compared with most purely heuristic solution approaches, is that it provides a dual bound on the solution, which is an important measure of solution quality.

The following sections describe the mathematical formulations of Johnson and Scheurman's (1977) Model I and Model II in the context of spatially-explicit forest planning models.

2.1 Model I

To specify the spatial version of Johnson and Scheurman's (1977) Model I used in this paper, let S denote the set of management units, $t=1, 2, \ldots, T$, the time periods in the planning horizon, and k be the minimum rotation age. In Model I, a prescription corresponds to a complete sequence of all management actions to be applied to a given area over the entire planning horizon. In our simplified models, the only management activities that are applied are regeneration harvests. Since harvests are assumed to occur only at the midpoints of the planning periods, the number of times a unit can potentially be harvested over the planning horizon is $\lceil T/k \rceil$. The set of all possible prescriptions that can be assigned to a management unit can be represented by a set of vectors of length *T*: $P = \{(0,...,0),(1,0,...,0),...\}$. The first element of the vector corresponds to period 1, the second to period 2, and so on. The elements in each $p \in P$ are either zeros or ones, where a one represents the decision to harvest the management unit in the corresponding planning period and a zero indicates no harvest should occur in that period. Let the 0-1 variable x_{sp} represent the decision whether unit s should follow prescription p. If it should, $x_{sp} = 1$, otherwise it is 0. Decision variables are created only for those prescriptions that would not lead to premature harvests. In other words, all prescription variables with first harvests that occur before the unit reaches its minimum rotation age are excluded from the model during pre-processing.

As previously mentioned, the cover/path formulation of McDill et al. (2002) was used to address adjacency constraints. This adjacency formulation requires the identification of each group of contiguous management units whose combined area just exceeds the maximum harvest area; that is, if any one unit is removed from the set, either the combined areas of the remaining units in the set will no longer exceed the maximum harvest area or the remaining units will no longer be contiguous. Let Λ^+ denote all, and $C \in \Lambda^+$ denote one such group of units.

For management unit $s \in S$, let a_s denote the area, e_s the regeneration cost, and v_{sp}^{t} the volume per unit area in period t given prescription p. Let E_{sp} represent the ending value of management unit *s* if prescription *p* is followed. We use h_t as an accounting variable for the volume harvested from the entire forest in period t, we use c for unit volume timber price, i for the real interest rate, and r_{sp} for the revenues associated with harvesting unit s according to prescription p. Bounds $f_{\min}$ and $f_{\max}$ denote the allowable percent decrease and increase in harvest volume in consecutive planning periods. Finally, let $\overline{ET}$ denote the minimum average age for the forest at the end of the planning horizon, and let δ_{sp} be the age of unit *s* at the end of the planning horizon if prescription *p* is followed. Then, the Model I formulation of the basic spatially-explicit harvest scheduling problems formulated and solved for this study is:

$$\max \sum_{s,p} r_{sp} x_{sp} \tag{1}$$

s.t.:

$$\sum_{p} x_{sp} = 1 \qquad \forall s \in S \tag{2}$$

$$\sum_{s,p} a_s \cdot v_{sp}^{t} \cdot x_{sp} = h_t \qquad \forall \mathrm{t} \le \mathrm{T} \tag{3}$$

$$\left(1 - f_{\min}\right) \cdot h_t \le h_{t+1} \qquad \forall t \le T - 1 \tag{4}$$

$$\left(1 + f_{\max}\right) \cdot h_t \le h_{t+1} \qquad \forall t \le T - 1 \tag{5}$$

$$\sum_{s \in C} x_{st} \le \left|C\right| \text{-} 1 \qquad \forall C \in \Lambda^{+}, t \le \left|T\right| \tag{6}$$

$$\sum_{s,p} \left(\delta_{sp} - \overline{ET}\right) a_s x_{sp} \ge 0 \tag{7}$$

$$x_{sp} \in \left\{0,1\right\} \qquad \forall s \in S, p \in P \tag{8}$$

The objective function (1) maximizes the net discounted timber revenues associated with the entire forest across the planning horizon of *T* periods, plus the discounted ending value of the forest, with the objective function coefficients being:

$$r_{sp} = \sum_{t} \left(c \cdot a_s \cdot v_{sp}^t - e_s \right) \cdot \left(1+i\right)^{-t} + E_{sp} \left(1+i\right)^{-T} \qquad \forall s \in S, p \in P \tag{9}$$

The ending value of a management unit is calculated under the assumption that unit will be harvested at the financially optimal rotation in all periods beyond the planning horizon. Constraint (2) ensures that each unit is assigned exactly one prescription. Constraint (3) calculates the harvest volume in each time period, and Constraints (4) and (5) ensure that the harvest volumes in adjacent planning periods do not fluctuate by more than the lower or upper bounds, $f_{\min}$ and $f_{\max}$, respectively. Constraint (6) ensures that no contiguous group of units whose combined area exceeds the maximum harvest area is harvested concurrently. Constraint (7) ensures that the average age of the forest at the end of the planning horizon is greater than or equal to the minimum average ending age. Finally, Constraint (8) defines the decision variables as binary.

2.2 Model II

To specify Model II for the purposes of this research, let the variable sets $b_{st},\ l_{st} \in \{0,1\}$ $\forall s,t$ denote the decision whether unit *s* should be cut in period *t* the first time and whether it should be cut in period *t* the last time, respectively. If unit *s* is to be cut the first time in period *t*, then $b_{st} = 1$, 0 otherwise. If unit *s* is to be cut the last time in period *t*, then $l_{st} = 1$, 0 otherwise. Further, let variable set $g_{s,t',t} \in \{0,1\}$ $\forall s,t$ denote the decision whether unit *s* should be harvested in period t after it had previously been cut in period t'. If it is, then $g_{s,t',t} = 1$, 0 otherwise. Finally, let $n_s \in \{0,1\} \ \forall s,t$ represent the "do-nothing" prescription. If unit *s* is not to be cut during the planning horizon, then $n_s = 1$, 0 otherwise. Then, the following objective function and inequality constraints define Model II:

$$\max \sum_{s,t} \left[r_{st}^b \cdot b_{st} + \sum_{t' \le t-k} r_{s,t',t}^g \cdot g_{s,t',t} + r_{st}^l \cdot l_{st} \right] \tag{10}$$

st.

$$n_s + \sum_t b_{st} = 1 \qquad \forall s \in S \tag{11}$$

$$b_{st} + \sum_{t' \le t-k} g_{s,t',t} = \sum_{t^* \ge t+k} g_{s,t,t^*} + \ell_{st} \qquad \forall s \in S, t \le T \tag{12}$$

$$\sum_s \left[a_s \cdot v_{st}^b \cdot b_{st} + \sum_{t' \le t-k} a_s \cdot v_{s,t',t}^g \cdot g_{s,t',t} \right] = h_t \qquad \forall t \le T \tag{13}$$

$$(1 - f_{min}) \cdot h_t \le h_{t+1} \qquad \forall t \le T-1 \tag{14}$$

$$(1 + f_{\max}) \cdot h_t \ge h_{t+1} \qquad \forall t \le T-1 \tag{15}$$

$$\sum_{s \in C, t' \le t-k} \left(b_{st} + g_{s,t',t} \right) \le |C| \text{-}1 \qquad \forall C \in \Lambda^+, t \le |T| \tag{16}$$

$$\sum_{s,p} \left[a_s \left(\delta_{st} - \overline{ET} \right) \left(n_s + l_{st} \right) \right] \ge 0 \tag{17}$$

$$b_{st}, \ell_{st} \in \{0,1\} \qquad \forall s \in S, t \le T \tag{18}$$

$$g_{s,t',t} \in \{0,1\} \qquad \forall s \in S, t \le T, t' \le T-k \tag{19}$$

$$n_s \in \{0,1\} \qquad \forall s \in S \tag{20}$$

As in Model I, the objective (Equation 10) is to maximize net timber revenues over the planning horizon, plus an ending forest value, subject to 1) logical constraints (11) that allow exactly one first harvest or no harvest of a unit, 2) harvest volume accounting and flow constraints (13-15) that are analogous to inequalities (3-5) in Model I, 3) maximum harvest opening size constraints (16), and 4) a minimum ending average age constraint (17), where δ_{st} is the age of unit *s* at the end of the planning horizon if it was last cut in period *t*. At $t=0$, δ_{s0} corresponds to the initial age of unit *s* plus the length of the planning horizon. Revenue coefficients associated with the first harvest of unit *s* in period *t* were calculated using relation $r_{st}^{b}=\left(c\cdot a_{s}\cdot v_{st}^{b}-e_{s}\right)\cdot\left(1+i\right)^{-t}$, where v_{st}^{b} is the harvest volume of unit *s* in period *t* given that this is the first time unit *s* is cut. Similarly, the formula $r_{s,t',t}^{g}=\left(c\cdot a_{s}\cdot v_{s,t',t}^{g}-e_{s}\right)\cdot\left(1+i\right)^{-t}$ was used to calculate the net discounted revenues associated with harvesting cutting unit *s* in period *t* after having cut it previously in period *t′*. Parameter $r_{s,t}^{l}$ gives the present value of management unit s at the end of the planning horizon if it is harvested for the last time in period *t*. Parameter $v_{s,t',t}^{g}$ represents the unit area volume associated with the harvest of unit s in period t when the previous harvest occurred in period t'.

Identity set (12) is a "network flow" constraint that ensures that whenever a management unit is cut in a particular period, whether it is the first or an intermediate harvest, there must be another variable that either declares this harvest to be the last for that unit or that forces it to be cut again during the planning horizon. In other words, this constraint forces each management unit to have a complete prescription plan for the entire planning horizon. Along with the logical constraints (11), these flow constraints create a network representation of the harvest scheduling problem with nodes representing the starting (source), the ending (sink) and the intermediate states of the units (Fig. 1). Whatever harvest action takes the unit to a given harvest-period state, there has to be another decision, such as a declaration that this harvest was the unit's last or a decision to cut the unit in a subsequent period, that takes it out of that state. Finally, constraints (18-20) define the four sets of decision variables as binary.

The following additional set of constraints can be added to Model II to potentially tighten the formulation, possibly leading to shorter average solution times.

$$n_s + \sum_t l_{st} = 1 \qquad \forall s \in S \qquad (21)$$

These constraints are logical constraints similar to Constraints (11) that require the unit to be cut last in exactly one period or to not be cut at all. Snyder and ReVelle (1996) also include a similar constraint set in their model (their Constraint set [4]) which "forces a final harvest to occur during the final supernode year, or sink node" (p. 1083). All problem instances were formulated with and without this constraint set to test whether adding them improves model performance. We refer to the Model II formulation without these constraints as Model II and the formulation with the constraints as Model IIa. Thus, each problem was formulated using Model I, Model II, and Model IIa.

Model formulations were initially compared on the basis of model size: i.e., numbers of variables and constraints, as model size is one factor that can influence model performance. Model performance was evaluated based on solution time when different formulations of a given problem instance were both solved to optimality in less than 12 hours of run-time. If one formulation of a problem instance was solved to optimality within 12 hours, it is obviously superior (for that instance) to another formulation of that instance that was not solved in 12 hrs. For instances where both formulations being compared were not solved within 12 hours, performance was evaluated based on the optimality gap achieved within 12 hours. With the potential of having to use mixed criteria for evaluating model performance, the only measure of model performance that can always be applied to all instances is simply the number of "wins," "ties" and "losses" for each formulation.

However, since there were only a few cases where problems were not solved to optimality in 12 hours, average solution times for the different formulations were also compared. As each problem instance was formulated each way, a paired comparison of the solution times for the different formulations could be made. Due to the large variability in solution times for different problem instances, a big difference in solution times for a single instance could potentially dominate the mean solution time difference for any given comparison. To minimize this effect, paired comparisons were normalized by dividing the difference in the solution times for two formulations of the same problem instance by the maximum solution time for the two formulations being compared. The formula for this normalization is:

$$\text{Normalized Difference}_{ijk} = \frac{time_{ij} - time_{ik}}{\max\left(time_{ij}, time_{ik}\right)}$$

Where $time_{ij}$ is the solution time for formulation *j* of problem instance *i*. This normalized difference is always in the interval (-1, 1). When the normalized difference is greater than zero, then formulation *k* can be considered superior to formulation *j* for instance *i*; conversely, when the normalized difference is less than zero, formulation *j* can be considered superior to formulation *k* for instance *i*. The normalized difference can be interpreted as a percentage improvement in solution time for the superior formulation relative to the inferior formulation.

Paired t-tests were used to test the two-tailed hypotheses that the mean normalized (percent) differences between solution times for different formulations are not equal to zero.

3. Results and Discussion

One measure of model efficiency is model size. As a general rule, smaller models tend to be easier to solve than larger models. Table 2 compares the number of variables and number of constraints needed to formulate the hypothetical forest planning problems with four, six and eight planning periods using the Model I and Model II formulations. The number of variables and constraints for the Model IIa formulation are not shown because that formulation always has the same number of variables as Model II plus one additional constraint for each management unit. Model I produces smaller models than Model II in all cases. On average, the Model II formulation has approximately 30 percent more variables and 50 percent more constraints than the Model I formulation. There is no obvious trend suggesting that the ratio of the number of Model II variables to the number of Model I variables is getting bigger or smaller as the number of periods increases. However, the ratio of the number of Model II constraints to the number of Model I constraints does appear to increase as the number of periods is increased.

The results in Table 2 show that adding two periods to the model generally results in an approximate doubling in the number of variables. For Model I, this result is quite consistent, whether it is from four periods to six or from six periods to eight. However, for Model II, the relative increase in the number of variables between four and six periods – which increases the number of variables by roughly 150%, on average – is greater than between six and eight periods – where the number of variables increases by only 80%. Increasing the number of periods has a proportionately larger effect on the number of constraints for the Model II formulation than for the Model I formulation. For example, increasing the number of planning periods from four to six increases the number of Model I constraints by about 50% (roughly in proportion to the increase in the number of periods), whereas, by comparison, the number of Model II constraints is approximately doubled.

Clearly, Model II MIP formulations result in larger models for the problem instances considered here. Model II results in smaller LP models because analysis areas that would have been tracked by separate variables can be "merged" in later periods. In a LP model that aggregates similar areas into "analysis areas," when areas that were otherwise similar but represented as separate analysis areas because they start out in different age classes are harvested in the same period their areas can be re-aggregated into a single analysis area and assigned to a single variable. However, in a spatially explicit model, this doesn't happen because areas always retain their unique representations because they are spatially unique.

Of course, model size is only a secondary concern. In most cases, the critical limiting factor in solving MIP formulations of spatially-explicit forest planning problems is solution time. Tables 3 and 4 compare the performance of Model I with Models II and IIa, respectively, for the hypothetical problems by identifying the number of problem instances where one formulation outperforms the other. Both tables show a very clear trend where Model I clearly outperforms both versions of Model II in problems with only 4 planning periods, and where both versions of Model II outperform Model I for problems with either 6 or 8 periods. Model IIa outperforms Model I slightly more often than Model II.

There were only eight instances where the hypothetical problems were not solved to optimality within the 12-hour solution-time limit. All of them were over-mature forest problems formulated with 6 planning periods. Of the eight, seven were Model I formulations and one was a Model II formulation; none were Model IIa formulations. Since in the majority of these cases Model I had the poorest performance, excluding these cases from a pairwise analysis of the solution time differences biases our results in favor of Model I, making this a conservative analysis. Tables 5 and 6 compare the solution times for the hypothetical forest problems formulated with Model I versus those formulated with Models II and IIa, respectively. The tables show the average solution time for each formulation and for each initial age-class structure and planning horizon. Average solution times for the over-mature forests tend to increase by roughly three orders of magnitude with the addition of two time periods, while average solution times for the regulated forests tend to increase by one to two orders of magnitude with the addition of two time periods. Average Model I solution times are superior for over-mature forest planning problems with four-period planning horizons. In all other cases, there is either little difference between the two formulations or the Model II formulations are superior. Furthermore, the advantage of the Model II formulations clearly grows as the length of the planning horizon increases.

The normalized difference in solution time is shown in Tables 5 and 6 as a percent difference. Positive values can be interpreted as indicating that Model I outperformed the respective version of Model II. The p-value gives the probability of obtaining the observed average percent improvement in solution time if the true mean difference in solution times is equal to zero. Because of the normalization, observations can be combined across different planning horizon lengths, as the normalized differences can be seen as coming from the same population, regardless of the planning horizon length. The results in both tables show quite clearly that both versions of Model II outperform Model I for planning horizons of 6 or 8 periods, and in all these cases we can confidently reject the null hypothesis that there is no difference in average solution time. While Model I outperforms both versions of Model II for problems with 4 planning periods, the difference is only statistically significant at the 0.05 level when comparing Model I and Model II on over-mature forest problems with four planning periods.

With the hypothetical forests, the comparisons between the two Model II formulations and Model I are generally stronger for Model IIa than for Model II. However, in a similar comparison between Model II and Model IIa, none of the differences in solution times were statistically significant. Nevertheless, the results do suggest that Model IIa is somewhat superior to Model II.

The results for the real loblolly pine forests are shown in Table 7. None of the problem instances were solved to optimality in 12 hours, although the 5-period models were solved to very small gaps. Conclusions as to which formulation is superior are less clear than for the hypothetical problems. The basic Model II formulation performed better than the other two formulations with 5- and 20-period planning horizons; Model I performed best with a 10-period planning horizon; and Model IIa performed best with a 15-period planning horizon. In fact, with Model IIa, no integer feasible solution was found within 12 hours for the problem with a 20-period planning horizon, and this formulation performed the worst of the three formulations for three out of the four planning horizons. Most likely, these results merely highlight the shortcomings of trying to draw conclusions from a single example. There is a lot of variability in solution times for different problems and with different formulations.

To explore this issue further, we calculated correlations between solution times for different formulations within a given class of hypothetical problems, e.g., regulated forest problems with 60-year planning horizons. These correlations are reported in Table 8. For regulated forest problems with shorter (4 and 6-period) planning horizons, solution times were fairly consistent (correlations between 0.59 and 0.94) between formulation methods. That is, if a problem instance took a relatively long time to solve with one formulation, it also took a long time with the other formulations and vice versa. However, for the regulated forest problems with 8 planning periods and for the overmature forest problems, the correlations between solution times for different formulations were much weaker (correlations generally less than 0.5 and as low as 0.09). This indicates that for these problems, when a problem took a long time to solve with one formulation it did not necessarily take a long time to solve with another. In general, we concluded from this that as problems become more difficult to solve, solution times become less predictable.

4. Conclusions

The results from this study show that the length of the planning horizon is a key factor in determining whether Model II formulations are superior to Model I formulations in spatially-explicit forest management planning problems. The results from the hypothetical problems suggest that Model I tends to be superior to Model II for problems with relatively short planning horizons (less than or equal to one rotation). However, at some point, as the length of the planning horizon is increased, Model II eventually becomes the superior formulation. This occurs even though Model II formulations generally require substantially more variables and more constraints than equivalent Model I formulations. While current spatially-explicit model formulations tend to use planning horizons that are shorter than one rotation, there are good reasons to consider using longer rotations. These include consideration of long-term sustainability issues and the cumulative impact of harvesting decisions on the spatial structure of the forest, e.g., the patch size distribution. As computing power increases and solution algorithms improve, our ability to solve models with longer planning horizons will also improve. Using a Model II formulation also appears to help make problems with longer planning horizons more tractable.

Results from our real test case are not entirely consistent with our results from our hypothetical problems. In particular, Model IIa does not perform particularly well for this test case; although it produces the lowest solution time with a 15-period planning horizon, it gives the worst solution times with the remaining planning horizons and does not even produce a feasible solution within 12 hours for the problem instance with the longest planning horizon. We interpret this lack of consistency with the hypothetical problems as a reflection of the variability and unpredictability of results for any single case, especially with harder problems.

An important question that often arises with spatially-explicit forest planning problems is “what makes some problems so much harder to solve than other problems?” The results presented here reaffirm the result reported by McDill and Braze (2000) that initial age-class distribution is a key factor. For a given planning horizon, our problems based on forests with over-mature age-class distributions took two or more orders of magnitude longer to solve than problems based on forests with regulated age-class distributions. Furthermore, the results presented here show that problems with longer planning horizons are harder to solve than problems with shorter planning horizons, with increases in solution time typically of two or three orders of magnitude for six versus four planning periods and for eight versus six planning periods. One key advantage of the Model II formulations, however, is solution time does not increase as fast with increases in the planning horizon with this formulation as it does with the Model I formulation. Model II solution times tended to be longer than Model I solution times with short rotations, but Model II eventually becomes the superior formulation as the length of the planning horizon is increased.

We note at the beginning of this paper that spatial forest planning models do not benefit as much from using a Model II formulation compared with aspatial LP models because with aspatial planning models, areas that differ only in terms of their initial age class cannot be merged after they are harvested in the same period. However, the advantages of Model II over Model I will also increase if more intermediate treatment options, such as thinning, are included, and this advantage applies to spatial as well as aspatial models. This is because increasing the number of treatment options between regeneration harvests only increases the Model II model size additively for each rotation, whereas it increases Model I model size multiplicatively. To see this, consider the simple case where the rotation is fixed and the planning horizon is two rotations long. If there are three intermediate treatment (e.g., thinning) options for each rotation, this can be modeled with six variables in Model II, but it will require nine variables with Model I.

An interesting potential extension of the Model II format would be to use spatially-explicit variables to represent management areas up until the first or second time they are harvested and then to allow them to be merged into aspatial analysis area variables after the first or second cut. This would allow modelers to maintain a high degree of spatial specificity for the initial periods of the model when it is probably more critical while still modeling certain aspatial sustainability considerations for longer planning horizons. For problems where planners are interested in projecting the long-term spatial configuration of the forest under different management policies, however, this would not be a suitable approach.

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Table 1. Target distribution of area by age class for hypothetical forests for regulated and over-mature initial age-class distributions.

Age Classes	Percent of Total Area	
	Regulated	Over-mature
1 to 20	25	10
21 to 40	25	15
41 to 60	25	20
61 to 80	25	25
81 to 100	-	30

Table 2. Average number of variables and constraints for Model I and II formulations of the hypothetical forest problems.

Initial Age-Class Distribution	Number of Periods	Avg. Number of Variables		Avg. Number of Constraints	
		Model I	Model II	Model I	Model II
Over-Mature	4	273	347	254	328
	6	588	851	371	635
Regulated	4	240	289	224	272
	6	515	738	340	563
	8	1,100	1,351	457	819

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Table 3. Number of wins and ties by forest initial age-class distribution and planning horizon when comparing Model I with Model II.

Initial Age-class Distribution	Planning Periods	No. of Obs.	Model I Wins	Ties	Model II Wins
Over-mature	4	30	22	1	7
Over-mature	6	30	8	0	22
Over-mat. Total		60	30	1	29
Regulated	4	30	13	8	9
Regulated	6	30	7	1	22
Regulated	8	30	4	1	25
Regulated Total		90	24	10	56

Table 4. Number of wins and ties by forest initial age-class distribution and planning horizon when comparing Model I with Model IIa.

Initial Age-class Distribution	Planning Periods	No. of Obs.	Model I Wins	Ties	Model II Wins
Over-mature	4	30	15	0	15
Over-mature	6	30	7	0	23
Over-mat. Total		60	22	0	38
Regulated	4	30	13	7	10
Regulated	6	30	5	1	24
Regulated	8	30	5	1	24
Regulated Total		90	23	9	58

Table 5. Comparison of average solution times and average percent difference in solution time for Model I and Model II by forest initial age-class distribution and planning horizon. (P-values test the hypothesis that the average percent difference equals zero.)

Initial Age-class Distribution	Planning Periods	No. of Obs.	Average Solution Time (sec)		Avg. % Difference	P-value
			Model I	Model II		
Over-mature	4	30	6.9	10.0	21.1%	0.027
Over-mature	6	22	4687.6	2981.2	-34.2%	0.039
Over-mat. Total		52	1987.2	1267.0	-2.3%	0.802
Regulated	4	30	0.1	0.1	3.5%	0.532
Regulated	6	30	1.7	0.7	-23.1%	0.003
Regulated	8	30	326.4	26.4	-56.6%	0.000
Regulated Total		90	109.4	9.1	-25.4%	0.000

Table 6. Comparison of average solution times and average percent difference in solution time for Model I and Model IIa by forest initial age-class distribution and planning horizon. (P-values test the hypothesis that the average percent difference equals zero.)

Initial Age-class Distribution	Planning Periods	No. of Obs.	Average Solution Time (sec)		Avg. % Difference	P-value
			Model I	Model II		
Over-mature	4	30	6.9	33.5	11.8%	0.263
Over-mature	6	23	5,753.5	2,823.3	-42.7%	0.008
Over-mat. Total		53	2,500.7	1,244.1	-11.9%	0.209
Regulated	4	30	0.1	0.1	4.6%	0.378
Regulated	6	30	1.7	0.6	-32.9%	0.000
Regulated	8	30	326.4	7.6	-52.6%	0.000
Regulated Total		90	109.4	2.7	-27.0%	0.000

Table 7. Comparison of optimality gaps after 12 hours of solution time for the real forest problems formulated with four planning horizons, using Model I, Model II and Model IIa.

Planning horizon	Model I Opt. gap (%)	Model II Opt. gap (%)	Model IIa Opt. gap (%)
5 periods	0.00000268	0.00000228	0.00023
10 periods	0.07	0.09	0.10
15 periods	5.44	0.78	0.46
20 periods	1.40	0.96	Infinite*

*No feasible solution found.

Table 8. Correlation between solution times for different formulations within different groups of problem instances.

Age-Class Distribution	Planning Periods	MI with MII	MI with MIIa	MII with MIIa
Regulated	4	0.59	0.70	0.81
	6	0.94	0.80	0.92
	8	0.77	0.36	0.34
Over Mature	4	0.34	0.39	0.09
	6	0.10	0.22	0.48

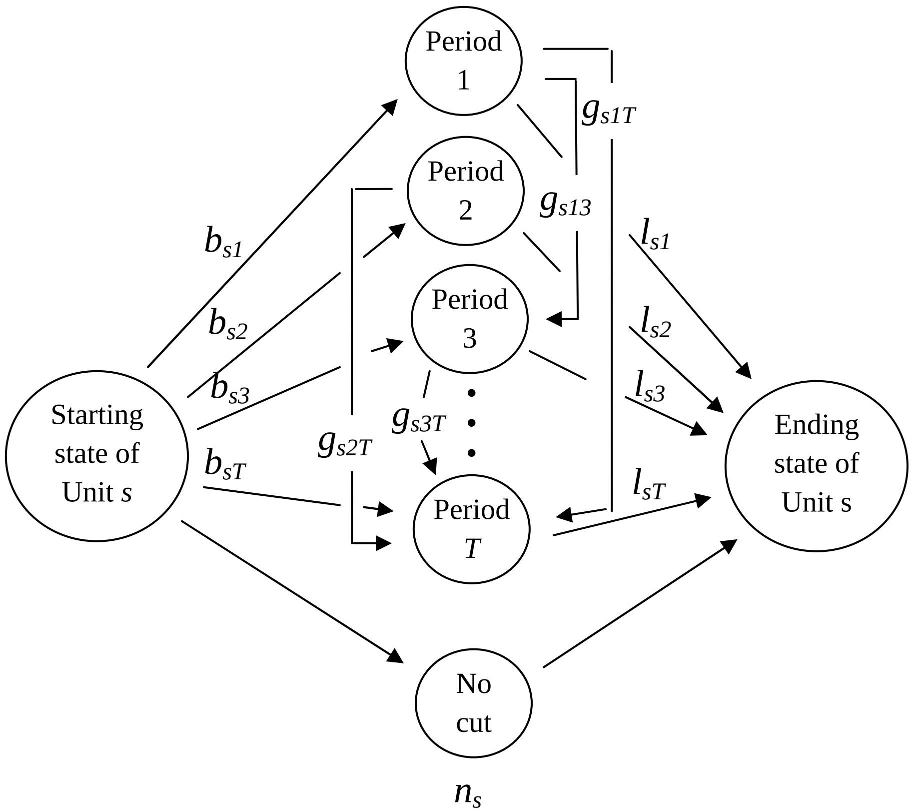

Figure 1. Network flow representation of spatial Model II. The arrows correspond to the variables that "move" unit *s* across the planning horizon.

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