Temporal Connectivity of Mature Forest Patches In Spatially-Explicit Harvest Scheduling Models

Abstract. We present a forest harvest scheduling model that can ensure the temporal connectivity of mature forest habitat over time in a landscape managed for timber production. Past models addressed the spatial aspects of habitat connectivity by requiring that a certain amount of mature forest habitat is retained throughout a planning horizon in contiguous patches of minimum size and age. These models failed to recognize, however, that the dynamic patches of a managed forest ecosystem might not provide escape routes for certain wildlife unless there is temporal overlap among the patches. According to biologists, the lifespan of the patches is often more important than their size and contiguity for species survival. We propose a mixed integer programming formulation that guarantees escape routes among patches of mature forest habitat that arise and diappear over time as the forest ages and gets harvested. Using four real forests as examples, we illustrate the mechanics of the approach and show that the new model is not only tractable computationally, but it can also make harvest scheduling models with minimum patch size constraints easier to solve.

Keywords: forest fragmentation, temporal connectivity, mature forest patches, spatial harvest scheduling, mixed integer programming

1 Introduction

Anthropogenic land use such as agriculture, forestry or urban sprawl can transform natural landscapes and fragment wildlife habitat. Sensitive species might not be able to persist in an ecosystem if dispersing individuals cannot utilize remaining patches of suitable habitat (Fischer and Lindenmayer, 2007; Kindlmann and Burel, 2008). Individual patches might be too small in size, too elongated in shape, or too far from each other to provide sufficient protection. In response to these concerns, connectivity modeling has become an important research area in the context of reserve design (e.g., Conrad et al., 2012; Önal and Briers, 2006; Tóth et al., 2009; Rebain and McDill, 2003). While both Conrad et al. (2012) and Önal and Briers (2006) focus on the optimal selection of a spatially connected network, the Tóth et al. (2009) and Rebain and McDill (2003) models target problems where, as opposed to full connectivity, only minimum contiguity thresholds need to be met by the reserves.

In managed ecosystems, such as timberlands, or in dynamic systems that are subject to frequent and catastrophic disturbances such as fire, habitat connectivity has both spatial and temporal dimensions. A forest stand that provides suitable habitat for a certain species today might be gone tomorrow due to a timber harvest or wildfire. Similarly, a forested site that is in an early successional stage of its development today, might become mature habitat in a future time period. The lifespan of different habitat patches might have important implications on just how well certain wildlife populations persist in the landscape over time. In fact, Fahrig (1992) has shown that the temporal dimension of habitat is often far more important than the spatial dimension for persistence. This is expecially true for species that are limited in mobility such as insects or amphibians. A forest planner might ask if it was possible to schedule timber harvests across the landscape and over time in such a way so that contiguous forest patches of a minimum size and age would always be overlapping with or adjacent to patches that develop in subsequent periods. If it was possible to ensure such dynamic ”escape routes” in the landsacpe, how much would the effort cost to the landowner in forgone timber revenues? None of the connectivity models that are currently available in the forest planning or reserve selection literatures can answer these questions. This is the knowledge gap that we would like to fill by introducing the temporal connectivity models. Before describing the formal, mathematical details of these model, we give an overview of the work that had been documented in the literature about habitat connectivity planning both in the context of forest harvest scheduling and reserve design.

The term connectivity can be defined based in the landscape structure (structural connectivity) and based on the needs and dispersal characteristics of a species (functional connectivity) (Kindlmann and Burel, 2008). Kindlmann and Burel (2008) define connectivity as *’the ease with which individuals of a species can move about within the landscape’*, which highlights the structural aspect of connectivity in concert with the functional or species-specific aspect of it.

Forest management is one of the land use forms that is often associated with habitat loss and fragmentation. The loss of mature – likewise late seral or old growth – forest patches is one of the reasons for the criticism. Mature forest patches provide irreplaceable habitat for numerous species (e.g. Franklin, 1997; Franklin and Forman, 1987; Lindenmayer and Franklin, 2002), and ecological guidelines suggest the protection of large mature forest patches in the landscape (e.g. Lindenmayer et al., 2006). As a result, models that aim to maintain mature forest habitat emerged from both the conservation reserve design and the harvest scheduling literature.

The reserve design problem (e.g. Haight and Travis, 2008; Williams et al., 2005; Önal and Briers, 2006) can be stated as (1) what is the minimal area that provides adequate habitat for the population of a group of species, or (2) what is the largest group of species that can be protected given a budget constraint (Önal and Briers, 2006). The spatial aspect of habitat connectivity – i.e. the design of wildlife corridors – has received attention in the reserve design literature including studies on the mathematical background of the problem (Cerdeira and Pinto, 2005), modeling approaches (Cerdeira et al., 2010; Conrad et al., 2012; Önal and Briers, 2006; Sessions, 1992; Williams, 1998), case studies on implementation (Fuller et al., 2006) as well as survey papers (Galpern et al., 2011; Williams et al., 2005). Notably, Önal and Briers (2006) have devised an MIP model in which they added spatial contiguity as an additional criterion for reserve site selection using graph theory. The study of Conrad et al. (2012) is also worth mentioning. Formulated as a so-called 'connected subgraph problem', their model not only ensures the connectivity of reserve sites at minimal cost, but it can also maximize the suitable habitat area along the corridor subject to a budget constraint. The temporal aspect of habitat connectivity has not been addressed in reserve design models as these models designate the chosen mature forest stands to be reserves. Therefore, reserve design models are static models, guaranteeing persistence of habitat patches over time.

The harvest scheduling problem that requires sufficiently large patches of mature forest habitat in a managed forest is called the Minimum Patch Size (MPS) problem (Rebain and McDill, 2003). Minimum size of mature patches, minimum area of mature habitat, and the minimum age requirement for maturity are predefined parameters of the forest management problem. The MPS problem allows the composition of mature forest habitat to change over time and space without restrictions. Thus, the problem is dynamic, a feature that may be crucial for ecologically and economically sustainable forest management. However, it does not guarantee persistence of habitat patches over time. While some species (such as birds or larger mammals) can easily relocate and find distant mature forest patches to be functionally connected, proximity of mature forest patches over time is fundamental for less mobile species. Such species are for example some amphibian (e.g. Baldwin et al., 2006) and arthropod species (e.g. Schowalter, 1995). This study addresses the above shortcoming of the MPS problem and proposes a temporal connectivity model that guarantees that mature forest patches provide persistent habitat even for species with limited capabilities for dispersal.

Therefore, this study considers habitat patches to be connected over time if patches of consecutive time periods overlap.

Many studies have modeled minimum patch size requirements (e.g. Bettinger et al., 2002; Caro et al., 2003; Falcão and Borges, 2002; Martins et al., 2005), but have used various heuristic methods to solve the resulting combinatorial problem. Bettinger et al. (2002) have compared eight heuristic algorithms applied to spatial forest planning problems (74 stands) with wildlife habitat objectives. They have reported that although most heuristic techniques find very good solutions of spatially unrestricted planning problems, the more complex the spatial requirements the less likely that these techniques find a solution within 1% of the global optimum. Falcão and Borges (2002) have proposed and tested a new heuristic algorithm (sequential quenching and tempering) that was able to provide solutions within 1.5% of the global optimum 90% of the time for forest planning problems (300-900 stands) with volume flow, minimum clearcut size and minimum mature patch size constraints. Caro et al. (2003) have developed and tested another heuristic technique (2-Opt tabu search) for forest planning problems (574-27,000 stands) with volume flow, minimum clearcut size, green-up period, and minimum mature patch size constraints. Their technique has found the optimal solution of 20-stand test problems and a solution within 8% of the global optimum of a real-size problem. Martins et al. (2005) have proposed an integer programming model and a heuristic technique based on column generation for solving forest planning problems with maximum clearcut size and mature patch size constraints. The technique is suited for problems with 100-225 stands, but it was unable to handle larger ones.

Rebain and McDill (2003) have proposed the first exact, mixed integer programming (MIP) formulation for the MPS problem along with volume flow, average ending age, and maximum clearcut size restrictions. Computational capacity at the time of the publication allowed for solving only a small, hypothetical problem consisting of 50 stands and 3 planning periods. The model has the following limitations. First, it does not consider the shape of, and therefore the ratio of interior space and edge habitat within mature patches. Although compact shape and high interior space - edge habitat ratio is desirable, managed forests tend to have less interior space. Therefore, it is important to address the issue of compactness. Second, the model does not guarantee temporal connectivity of the mature forest patches, a consideration that is the primary focus of this paper. Third, an enumeration algorithm is necessary to formulate the model. The algorithm used in the study was impractical for cases where the MPS was large relative to the average stand size. An improvement on the cluster enumeration algorithm is the secondary contribution of this paper.

Building on the above study, Tóth and McDill (2008) have addressed the compactness of mature forest patches using the total perimeter of the patches as shape indicator. They have tested the model on a hypothetical problem with 50 stands and 3 planning periods. They have found that enforcing low or minimal perimeter of mature patches results in not only fewer, larger and more compact patches, but it is also more likely that the same patches would form mature habitat in consecutive periods. This finding suggests that the lack of temporal connectivity among mature forest patches, what was a limitation of the MPS model Rebain and McDill (2003) proposed, can be addressed indirectly. However, the model may promote static instead of dynamic mature patches.

Hof et al. (1994) have considered connectivity of wildlife habitat over time indirectly in an MIP framework. They devised a model that can optimize the spatial layout of harvesting activity so that it maximizes wildlife viability over time. Temporal connectivity of habitat patches increases the viability of a wildlife population, it is therefore incorporated indirectly into the model. However, the devised MIP is not a harvest scheduling model. It considers no timber objective or constraints on harvesting.

According to our knowledge, there has been no peer-reviewed publication that addressed temporal connectivity of mature forest patches in harvest scheduling models. The temporal dimension of habitat connectivity has been addressed in other fields of natural sciences, for example in agriculture (Baudry et al., 2003), in marine conservation (Treml et al., 2008), and in wetland science (Leibowitz and Vining, 2003). Most of those studies, however, report analytical investigations or simulation efforts rather than optimization approaches.

This paper builds on the MPS model of Rebain and McDill (2003). As the primary contribution, we introduce a new MIP model ensuring that mature forest patches spatially overlap in consecutive planning periods while they may change place over time. We refer to this model as Simple Temporal Connectivity model (TC.0). We also propose two modeling improvements on TC.0. The first improvement (called TC.1) reduces the number of constraints that are necessary to describe the temporal connectivity relationships. The second improvement (called TC.2) is a preprocessing procedure that one can apply if the MPS problem satisfies a special condition, i.e. the minimum mature habitat requirement is not increasing in consecutive planning periods. The procedure eliminates those potential mature patches from the MPS model that cannot satisfy the temporal connectivity requirement in the special setting. The two improvements can be utilized simultaneously, and the resulting model is referred to as TC.1+2. As a secondary contribution of this paper, we introduce the Age Discriminative Cluster Enumeration algorithm (ADCE) that can significantly reduce the formulation time of the MPS model, and therefore, the formulation time of the temporal connectivity model.

The rest of the paper is organized as follows. Section 2.1 describes the existing exact model of the MPS problem extending the harvest scheduling problem. In Section 2.2, the ADCE is introduced. Section 2.3 introduces the temporal connectivity concept and models TC.0, TC.1, TC.2, and TC.1+2. Section 3 presents the design of a computational experiment with four real forest planning problems in which we tested TC.0, TC.1, TC.2, and TC.1+2, using the existing exact MPS model (Rebain and McDill, 2003) as a benchmark. Finally, in Sections 4-6, we present, analyze, and discuss the results of the experiment, and conclude.

2 Methods

2.1 The Minimum Patch Size (MPS) Problem

2.1.1 Terminology

Harvest scheduling models maximize timber revenues from a forest over a planning horizon P, subject to a set of ecological and other constraints. Let $p = 1, 2, ...P$ represent the periods of the planning horizon. Let N represent the set of management units or stands $i \in N$ in the forest. Each stand i has the following attributes: area a_i, initial age t_i (in terms of planning periods), forest type k_i, volume in period p v_{ip}, the set of adjacent stands D_i, and the expected revenue coefficient in period p r_{ip}.

For the purpose of this study, we considered two stands adjacent if they shared a common boundary. In any period p, a stand may either be harvested completely, or left unharvested. Therefore, a binary decision variable x_{ip} is assigned to each stand for each period, indicating that stand i is harvested in period p. Furthermore, we declare a binary decision variable x_{i0} for each stand, which represents the case when stand i is not harvested over the planning horizon. For simplicity, let $p = 0$ represent this 'no action' management alternative. A stand may not be harvested until it has reached the minimum rotation age of forest type k R_k. We assume that $R_k \geq P$, therefore, each stand may be harvested only once over the planning horizon. Consequently, volume and revenue coefficients are assumed to remain zero after a stand is harvested.

We consider four constraints: logical, harvest fluctuation, average ending age, and minimum clearcut size constraints. The *logical constraint* ensures that each stand may be harvested only once over the planning horizon. This constraint is the corollary of the assumption that $R_k \geq P$. The *harvest fluctuation constraints* ensure that harvested volume of one period is not less or more than some portion (L lower and U upper bound) of that in the preceding period. The *average ending age constraint* guarantees that the area-weighted average age of the forest by the end of the planning horizon is at least a certain target age $\overline{ET}$ (in terms of planning periods). Finally, the *maximum clearcut size constraint* ensures that the total area of any contiguous group of stands that are harvested in any period is less than a predefined limit A_{max}. We modeled the last restriction with the Path formulation (McDill et al., 2002) that uses minimal cover constraints to prohibit maximum clearcut size violations from occurring. A cover in this context represents a group of stands that are connected in the landscape and the sum of their areas just exceeds the maximum clearcut size. A cover is minimal if its area drops below the maximum clearcut size if we remove any stand from the group. Let C denote a cover and Λ^+ denote the set of minimal covers. Using this notation, $C \in \Lambda^+$ if and only if $\sum_{i \in C} a_i \geq A_{max}$, and $\sum_{i \in C \setminus \{j\}} a_i \leq A_{max} \quad \forall j \in C$.

2.1.2 Harvest scheduling model

The spatially-explicit harvest scheduling models is:

$$\max \sum_{i \in N} \sum_{p=1}^{P} r_{ip} x_{ip} \tag{1}$$

Subject to

$$\sum_{p=0}^{P} x_{ip} = 1 \quad \forall i \in N \tag{2}$$

$$\sum_{i \in N} v_{i,p+1} x_{i,p+1} \leq U \sum_{i \in N} v_{ip} x_{ip} \quad \forall p = 1, 2, ...P-1 \tag{3}$$

$$\sum_{i \in N} v_{i,p+1} x_{i,p+1} \geq L \sum_{i \in N} v_{ip} x_{ip} \quad \forall p = 1, 2, ...P-1 \tag{4}$$

$$\sum_{i \in N} \left[(t_i + P - \overline{ET}) x_{i0} + \sum_{p=1}^{P} (P - p - \overline{ET}) x_{ip} \right] a_i \geq 0 \tag{5}$$

$$\sum_{i \in C} x_{ip} \leq |C| - 1 \quad \forall C \in \Lambda^+, \quad p = 1, 2, ...P \tag{6}$$

$$x_{ip} \in \{0, 1\} \quad \forall i \in N, \quad p = 0, 1, ...P \tag{7}$$

Equation (1) is the objective function that maximizes net present value (NPV) of timber revenues over the planning horizon. Constraints (2)-(5) are logical, harvest fluctuation, and average ending age constraints, respectively. Constraint (6) describes the maximum clearcut size restriction. Constraint (7) is a binary restriction on the decision variables.

2.1.3 Minimum Patch Size Constraints

Rebain and McDill (Rebain and McDill, 2003) used the following set of constraints to ensure that (1) the total area of mature forest habitat is not smaller than the minimum mature habitat K_p in each period; (2) the size of the patches constituting the mature habitat is not smaller the minimum patch size A_{min}; and (3) each stand in a mature patch is at least of age T (in terms of planning periods).

$$\sum_{i \in M} \sum_{j \in J_{ip}} x_{i,j} - |M| O_{M,p} \geq 0 \quad \forall M \in \Omega, \quad p = j_M, ...P \tag{8}$$

$$\sum_{M \in \Omega} O_{M,p} - BO_{ip} \geq 0 \quad \forall i \in N, \quad p = j_i, ...P \tag{9}$$

$$\sum_{i \in N_{j,p}} a_i BO_{ip} \geq K_p \quad \forall p \in P \tag{10}$$

$$O_{M,p} \in \{0,1\} \quad M \in \Omega, \quad p = j_m, ...P \tag{11}$$

$$BO_{ip} \in \{0,1\} \quad i \in N, \quad p = j_i, ...P \tag{12}$$

where

M is a cluster i.e. a group of contiguous stands with a combined area just exceeding the minimum patch size A_{min} ($\sum_{i \in M} a_i \geq A_{min}$ and $\sum_{i \in M \setminus \{j\}} a_i \leq A_{min}$ for some $j \in M$),

Ω is the set of all clusters,

J_{ip} is the set of all prescriptions under which stand i can be mature in period p,

$O_{M,p}$ is a binary variable indicating if cluster M is old enough to be a mature patch in period p ($O_{Mp} = 1$ if and only if $M \in \Omega : t_i + p \geq T \quad \forall i \in M$)

j_M is the first period in which cluster M meets the age requirement for maturity ($j_M = \max_{i \in M}(T - t_i)$),

j_i is the first period in which stand i meets the age requirement for maturity ($j_i = T - t_i$),

BO_{ip} is a binary variable indicating if stand i in period p is part of at least one cluster that meets the age requirement for maturity ($BO_{ip} = 1$ if and only if $i \in N : i \in M,\ j_M \leq p$),

$N_{j,p}$ is the set of stands that can be mature in period p ($N_{j,p} := \{i \in N : t_i + p \geq T\}$).

Constraint set (8) determines if a cluster can reach the mature age in a given period. Constraint set (9) indicates if a stand is part of at least one mature patch in a given period. Moreover, this set ensures that only one of the possibly overlapping clusters is chosen as a mature patch (Rebain and McDill, 2003). Constraint set (10) ensures that the total area of mature forest habitat meets the predefined requirement in each period. Constraint sets (11) and (12) are binary restrictions on the indicator variables. We refer to the MPS model of Rebain and McDill (Rebain and McDill, 2003) as the Benchmark model from here on.

2.2 Formulating the cluster set – Age Discriminative Cluster Enumeration (ADCE)

Enumerating the clusters in set Ω is not trivial. The Benchmark model used a modification of the Path Algorithm (McDill et al., 2002) for cluster enumeration, that takes the area a_i and the adjacency relationships D_i of each stand $i \in N$, and the minimum patch size A_{min} as inputs. It gives the set Ω as output.

Let the function $f(E, A_{min}) \mapsto \Omega$ represent the modified Path Algorithm, where matrix E is defined on $N \times N$ by $e(i,j) = \begin{cases} a_i & \text{if } i = j \\ 1 & \text{if } j \in D_i \\ 0 & \text{otherwise} \end{cases}$. It contains the adjacency relationships for the entire forest and the area of each stand. This algorithm generates all possible stand combinations just exceeding the MPS regardless of the maturity of the stands building up the cluster. Maturity of the clusters is only considered during model formulation. However, excluding all the stands from the forest that cannot meet the minimum age requirement in a given period may reduce the complexity of the enumeration problem by reducing both the number of relevant stands in the forest and the vertex degree (i.e. the number of stands adjacent to each stand).

The ADCE (Figure 1) modifies the input and output of function f such that $f(E_P, A_{min}) \mapsto \Omega_P$, where matrix E_P is defined on $N_P \times N_P$. N_P is the set of stands that can reach the age of maturity by the last planning period, and Ω_P contains only clusters that can form a mature patch in some planning period.

Note that $\Omega_P \subseteq \Omega$. Hence, the ADCE performs at least as well as the modified Path algorithm. The difference in size between the two sets and the efficiency gain of ADCE depends on the age class distribution, the mature age requirement, and the length of planning horizon in a particular forest.

2.3 Temporal connectivity

This study considers mature patches in a harvest scheduling model to be connected over time if patches in consecutive periods overlap. We add temporal connectivity constraints to the Benchmark model to guarantee temporal connectivity among mature forest patches. Temporal connectivity constraints must enforce that mature patches in consecutive periods have at least one stand in common. In other words, the constraints must enforce that at least one stand of a mature patch in a planning period must be part of a mature patch in the next period. Extending the MPS model with temporal connectivity constraints results in a temporal connectivity model that allows for a dynamic change in the composition of mature forest habitat in the landscape and concurrently ensures a smooth transition between mature patches in consecutive periods. The first point is important from timber management perspectives, whereas the second one may be crucial for species with limited capabilities for relocation. Although our definition of temporal connectivity may be conservative, it provides less mobile species with the possibility to find new suitable habitat without leaving the mature forest.

Ensuring temporal connectivity in the MPS model requires that the K_p, the minimum mature habitat in period p, is nondecreasing over the planning horizon. Nevertheless, K_p

may increase from period to period. Hence, temporal connectivity in general allows the development of new mature forest patches in later periods, but guarantees that the new patches persist in the landscape via temporal connectivity.

Figure 2 illustrates the temporal connectivity problem between two periods on a hypothetical forest. In period 1, ten stands reached the age of maturity: N_1={10, 12, 16, 18, 19, 20, 27, 29, 49, 67}, and they form the following clusters that exceed the minimum patch size of 100 ha: $\Omega_1 = \{\{10, 49, 19, 20\}; \{10, 49, 16, 19, 20\}; \{12, 18, 27\}; \{12, 27\}; \{16, 20, 49, 19\}\}$. In period 2, fifteen stands reached the mature age: N_2 ={5, 7, 10, 12, 16, 18, 19, 20, 27, 29, 34, 48, 49, 54, 67}. The set of clusters just exceeding the MPS is: Ω_2 ={{5,12}; {5,27,12}; {5,34}; {7,67}; {10,49,19,20}; {10,49,16,19,20}; {12,27}; {12,18,5}; {12,18,27}; {16,20,49,10,19}; {16,20,19,54}; {16,20,49,54}; {19,49,54}; {27,5,34}; {29,34,5}; {29,34,48}; {34,48}; {54,20,19,49}}.

We use this example to illustrate how the Simple Temporal Connectivity model (TC.0) works, and how the first improvement – Merged Superclusters (TC.1) –, and the second improvement that one may apply only if K_p does not increase over time – Cluster Elimination (TC.2) – can reduce its size.

2.3.1 Simple Temporal Connectivity model (TC.0)

We can enforce temporal connectivity by requiring that each cluster that was a mature patch in some period must have an overlapping successor in the next period (i.e. a mature patch in the next period that has at least one stand in common with a current mature patch). Formally:

$$O_{M,p} \leq \sum_{L \in \Omega_{p+1}: M \cap L \neq \emptyset} O_{L,p+1} \quad M \in \Omega_p,\ p = 1, ...P-1 \tag{13}$$

where L is a cluster in the next period that contains at least one stand of cluster M. This formulation imposes a constraint for each cluster and each period except for the last one.

For the example on Figure 2, this would mean:

$$O_{\{10,49,19,20\},1} \leq O_{\{10,49,19,20\},2} + O_{\{10,49,16,19,20\},2} + O_{\{16,20,49,10,19\},2} + O_{\{16,20,19,54\},2} + O_{\{16,20,49,54\},2} + O_{\{19,49,54\}} + O_{\{54,20,19,49\},2} \quad (14)$$

$$O_{\{10,49,16,19,20\},1} \leq O_{\{10,49,19,20\},2} + O_{\{10,49,16,19,20\},2} + O_{\{16,20,49,10,19\},2} + O_{\{16,20,19,54\},2} + O_{\{16,20,49,54\},2} + O_{\{19,49,54\}} + O_{\{54,20,19,49\},2} \quad (15)$$

$$O_{\{12,18.27\},1} \leq O_{\{5,12\},2} + O_{\{5,27,12\},2} + O_{\{12,27\},2} + O_{\{12,18,14\},2} + O_{\{27,5,34\},2} \quad (16)$$

$$O_{\{12,27\},1} \leq O_{\{5,12\},2} + O_{\{5,27,12\},2} + O_{\{12,27\},2} + O_{\{12,18,14\},2} + O_{\{27,5,34\},2} \quad (17)$$

$$O_{\{16,20,49,19\},1} \leq O_{\{10,49,19,20\},2} + O_{\{10,49,16,19,20\},2} + O_{\{16,20,49,10,19\},2} + O_{\{16,20,19,54\},2} + O_{\{16,20,49,54\},2} + O_{\{19,49,54\}} + O_{\{54,20,19,49\},2} \quad (18)$$

2.3.2 Merged Superclusters (TC.1)

Some clusters in Ω, which are necessary for the problem formulation, are supersets or superclusters. Superclusters are clusters that have at least one subset (subcluster) that also qualifies as a cluster $M \in \Omega$. In mathematical terms: $\sum_{i \in M} a_i \geq A_{min}$ and $\sum_{i \in M \setminus \{j\}} a_i \leq A_{min}$ for some $j \in M$ and $\sum_{i \in M \setminus \{l\}} a_i \geq A_{min}$ for some $l \neq j \in M$.

For example, the cluster on the left hand side of Equation (16) is a superset of the left hand side of Equation (17); and the left hand side of Equation (15) is a superset of both

Equations (14) and (18). By constraint set (8), any indicator variable that corresponds to a cluster that is a subset of another cluster is satisfied if any of the indicator variables that correspond to the superclusters of the cluster have positive value. Therefore, Equations (14) and (18) together fully describe Equation (15).

Hence, we can write constraint set (13) for a subset of Ω only, that we call *minimal cluster* set and denote by Ω^*. Let the minimal cluster set contain all such clusters that are minimal with respect to a mature stand $i \in N_p$ in a period $p \in P$, meaning that if we removed *any stand other than* i from the cluster, its area would become lower than the threshold size. Formally:

$$M \text{ is a } \textit{minimal cluster} \text{ if and only if} \sum_{l \in M} a_l \geq A_{min}, \sum_{l \in M \setminus \{j\}} a_l \leq A_{min} \text{ for some } j \in M,\ i \neq j \tag{19}$$

Thus, if any cluster formed a mature patch in one period, there is at least one constraint to ensure a successor for that cluster in the following period. Furthermore, we omit no stands that can provide a link between two periods. Set Ω^* may be larger than the set of minimal covers (that can prohibit maximum clearcut size violations from occurring) for the same problem. For example, let $\hat{C}$ denote minimal covers in the minimal cover set $\hat{\Lambda}$ and let $\hat{M}$ denote clusters in minimal cluster set $\hat{\Omega}^*$ associated with forest and let $A_{max} = A_{min}$. Minimal covers $\hat{C} \in \hat{\Lambda}$ do not contain any stand v such that $\sum_{i \in C \setminus \{v\}} a_i \geq A_{min}$ $\forall \hat{C} \in \hat{\Lambda}$, $\forall i, v \in N$, assuming that $\sum_{i \in N} a_i \geq A_{min}$ (see Figure 3 for an example).

Thus, we propose the TC.1 formulation with the following constraints:

$$O_{M,p} \leq \sum_{L \in \Omega_{p+1} : M \cap L \neq \emptyset} O_{L,p+1} \quad \forall M \in \Omega_p^*, \quad p = 1, ...P-1 \tag{20}$$

This formulation requires a constraint for each minimal cluster and each period except the last one. For the earlier example, one would omit Equation (15), but would keep Equation

(16), because the cluster {12,18,27} is minimal with respect to stand 18.

2.3.3 Cluster Elimination (TC.2)

The temporal connectivity requirement provides us with an opportunity to reduce the number of eligible clusters if the minimum area of mature habitat is constant over the planning horizon ($K_p = \overline{K} = constant,\ p = 1, 2, ...P$). This condition is satisfied if forest managers prioritize the persistence of populations that already exist in the landscape and target their resources at protecting existing refugees of habitat.

If the minimum area of mature habitat is constant over time, new mature patches are not required to develop, but mature patches of the first period may 'float around' in the forest in consecutive planning periods. (Note, however, that the total area of mature forest habitat is not forced to be constant in this model.) Thus, clusters that cannot be connected to any cluster in the previous period may be omitted from the entire problem formulation. In mathematical terms, let Ω_p^- denote the set of clusters in period p that can potentially overlap with a cluster in the previous period. $\Omega_p^- = \{L \in \Omega_p : M \cap L \neq \emptyset,\ M \in \Omega_{(p-1)},\ p = 2, ...P\}$. Note that $\Omega_1 = \Omega_1^-$.

TC.2 is a preprocessing procedure that can simplify both TC.0 and TC.1 if $K_p = \overline{K}$ condition holds. If applied to TC.0, the resulting model is called TC.2; if applied to TC.1, the resulting model is called TC.1+2. The procedure may reduce the set of potential mature clusters, affecting Equations (8)-(12). In our example, clusters {5,34}, {7,67}, {29,34,48}, and {34,48} may be eliminated in period 2.

3 A computational experiment

We tested the TC.0, TC.1, TC.2, and TC.1+2 models in a computational experiment including four real forest planning problems by comparing formulation and solution characteristics of the four temporal connectivity approaches and the Benchmark model. We used constant minimum mature habitat requirement for each planning problem to test TC.2 and TC.1+2 in the same experiment. The purpose of the computational experiment was (1) to illustrate how the proposed models work in practice, (2) to show that real forest planning problems are tractable using these models, and (3) to evaluate the models by comparing their size and performance to the most similar model in the literature. Furthermore, we tested whether using the ADCE can improve the efficiency of model formulation.

The experiment was implemented in Java using IMB-ILOG CPLEX v. 12.2 Concert Technology (4-thread, 64-bit, released in 2010) (IBM ILOG CPLEX, 2009) as the optimization software. The experiment ran on a Power Edge 2950 server (four 3 GHz Intel Xeon 5160 processors, 16GB RAM, MS Windows Sever 2003 R2). We used a 0.05% optimality gap and 6 hours (21,600 seconds) time limit as stopping criteria for optimization and the default settings for other optimization parameters.

We used the software referred to in (Könnyű and Tóth, 2011) to generate harvest scheduling models with maximum clearcut size restrictions and the algorithms described in Section 2.2 to formulate MPS models.

3.1 Test problems

We used the following four real forest planning problems for the computational experiment: *Pack forest* (186 stands), *Shulkell* (1,039 stands), *El Dorado* (1,363 stands) and *NBCL5* (5,224 stands). The datasets are available in the FMOS repository (FMOS – Forest Management Optimization Site, 2011). Tables 1 and 2 summarize initial conditions of the test forests: stand and age class characteristics, respectively. These characteristics influenced the choice of planning parameters shown in Table 3.

4 Results

4.1 Computational results

4.1.1 Formulation characteristics

Table 4 compares model size and formulation time of TC.0, TC.1, TC2, TC.1+2, and the Benchmark model on the four test problems. The table shows that the TC.0 model always required more constraints than the Benchmark model. TC.1 always had fewer constraints than TC.0, but more than the Benchmark model. TC.2 and TC.1+2 reduced the number of eligible clusters and therefore the number of constraints and binary indicator variables relative to TC.0 in three of the four cases (Pack, Shulkell, NBCL5). The TC.2 and TC.1+2 models of Shulkell and El Dorado had fewer variables and fewer constraints than the Benchmark model of the same forests. There was no significant difference among the formulation time values of TC.0, TC.1, TC2, and TC.1+2. However, in one of the four cases (El Dorado), formulation time of the temporal connectivity model variations approximately doubled relative to the Benchmark model.

Comparing the cluster enumeration algorithms, the ADCE always generated fewer clusters in shorter time than the modified Path Algorithm. In Pack forest, where 31% of the area reached the mature age over the planning horizon, the proposed algorithm reduced the number of necessary clusters to 0.8%. In Shulkell, 65% of the area reached maturity, and the reduction in the number of clusters was 2.0%. In El Dorado, 73.8% of the area was one mature block; the ADCE reduced the number of necessary clusters to 47.4%. In NBCL5, 20% of the area reached maturity over the planning horizon and enumerating 0.5% of the clusters was sufficient. Moreover, based on four different MPS and three mature age values for each forest, we found that the higher the mature age and the larger the MPS, the better the ADCE performed.

4.1.2 Solution characteristics

Figure 4 compares solution time, and the upper and lower bounds on the objective functions of TC.0, TC.1, TC2, TC.1+2, and the Benchmark model on the four test problems. The figure shows that TC.0, TC.1, TC.2, and TC.1+2 provided a better quality solution within 6 hours than the Benchmark model in two cases (Pack, NBCL5), while they satisfy an additional requirement (temporal connectivity) that the Benchmark model does not.

TC.1. improved solution time and quality relative to the TC.0 model in two cases (Shulkell, El Dorado). Moreover, for few problem instances not shown on the figure (e.g. Shulkell: MPS = 20ha, K = 200ha, T = 80 years, and other parameters as defined in Table 3; El Dorado: MPS = 40 ha, K = 800 ha, T = 100 years, and other parameters as defined in Table 3), TC.1 and TC.1+2 found feasible solutions with non-zero objective function value within the time limit, but TC.0 and TC.2 did not. TC.2 improved solution time or solution quality relative to TC.0 in the cases of Shulkell and NBCL5. TC.1+2 improved solution time or solution quality relative to TC.0 in all four cases, improved solution time or quality relative to TC.2 in three cases (NBCL5 was an exception), and improved solution time relative to TC.1 in the case of NBCL5.

In Pack forest and NBCL5, the temporal connectivity requirement inferred lower objective function values. However, we observed no cost associated with the temporal connectivity constraints in the reported instance of Shulkell and El Dorado.

Values of the MPS, mature age (T), and minimum area of mature habitat (K) parameters had major effect on the solvability of the problems. The larger the MPS and lower the mature age, the larger the model was. Although changing the minimum area of mature habitat has no effect on model size, we observed an increase in solution time. For example, we found no feasible solution with positive objective function value for any of the temporal connectivity models within 50,000s (about 14 hours) for El Dorado: MPS = 40 ha, $K > 1200$ ha, T = 100 years; and NBCL5: MPS = 40 ha, $K > 120$ ha, T = 100 years; and other parameters as in Table 3.

4.2 Management plans

Figures 5 and 6 illustrate the first four periods of the forest management plans for El Dorado with and without temporal connectivity constraints. Maps show that the Benchmark model suggested a plan in which mature patches develop in different parts of the forest in consecutive planning periods. Temporal connectivity constraints ensured overlap between mature patches of consecutive periods, however, restricted their movement over time.

In some cases, (e.g. the Pack forest problem with MPS = 40 ha, K = 120 ha, T = 100 years and other parameters as defined in Table 3) we observed that the temporal connectivity requirement restricted the problem to a static one, in which the mature patches chosen in the first period constituted to the mature habitat over the planning horizon.

5 Discussion

Experimental results show that in most cases, the temporal connectivity requirement increased computational complexity of the MIP. Yet, in two out of four cases cases, the temporal connectivity model variations (TC.0, TC.1, TC.2, and TC.1+2) were smaller than the Benchmark model, and they provided a solution faster in one case. We found that both improvements (TC.1, TC.2), and their combination (TC.1+2) are useful in reducing the size of the simple temporal connectivity model (TC.0), and they can improve solution time and solution quality.

The temporal connectivity constraints allows for a dynamic change of mature patches over time. However, in some cases, the temporal connectivity condition limits the solution to be static. The more dispersed mature forest stands are in the forest, the larger the MPS is, and the fewer stands can meet the age requirement for maturity over the planning horizon, the less likely the composition of mature forest patches changes over time in the temporal connectivity model. This result is similar to that of Tóth and McDill (2008), who found that minimizing the perimeter of mature forest patches may also significantly restrict the movement of the patches over time.

The temporal connectivity model proposed in this study does not take the shape and compactness of the mature forest patches into consideration. This may be a limitation of the model, because the shape of a habitat patch is important for species persistence. However, there is no clear consensus in the ecology literature regarding the ideal shape of a habitat patch. While numerous ecological studies support compact patches with low or minimal edge-to-area ratio, some evidence supports elongated ones (Williams et al., 2005). If the compactness of the patches is important, further constraints can be incorporated in this temporal connectivity model based on Tóth and McDill (2008) or Öhman and Wikström (2008) to avoid unfavorable patch shapes.

Temporal connectivity constraints might not be always costly. We reported examples where temporal connectivity constraints inferred forgone timber revenues, and other examples, where the new constraints did not change the objective function value of the solution. We can think of two reasons why the new constraints would not change the objective function value. First, the Benchmark model might give a solution that satisfies the new conditions. Second, a different combination of mature patches might be possible without significantly changing the optimal harvest schedule. The latter case might happen if the properties of the forest allow, or other constraints (e.g. average ending age) otherwise require mature forest stands in the landscape.

Initial conditions of the forest, such as age class distribution and spatial configuration of mature patches, as well as planning parameters have key impact on the solvability of the temporal connectivity model. Choosing a larger MPS, lower mature age, longer planning horizon or even larger minimum area of mature habitat may have significant computational cost. In cases such as El Dorado, it might be more practical to apply a static reserve design model, as there are no potential mature patches that emerge later in the planning horizon.

Some might argue that the temporal connectivity model is overly restrictive about the definition of connectivity. We can relax this definition and require that mature patches of one period have a successor in the next period that is within a maximum distance. Both the TC.0 and the TC.1 models can be extended to satisfy this definition by appending the right hand side of constraint sets (13) and (20) with all clusters that are within the given maximum distance. The Floyd-Warshall all-pair shortest path algorithm (Floyd, 1962) can be used to find all stands that are within the maximum distance. The Cluster Elimination procedure can also be adapted for the new definition, using the same algorithm, to formulate TC.2 and TC.1+2.

6 Conclusion

This study introduced a model that ensures temporal connectivity of mature forest patches in spatially-explicit harvest scheduling models. It also proposed two improvements on the model. With the first improvement, we can reduce the number of constraints that are necessary to describe temporal connectivity relationships. With the second improvement, we can reduce the number of potential mature patches by eliminating those that cannot meet the temporal connectivity requirement if a special condition holds, i.e. the minimum area of mature habitat does not increase over time. We tested the proposed model and its improvements in an illustrative computational experiment with four real forest planning problems. Experimental results indicate that (1) it is possible to use exact programming to solve the MPS problem with temporal connectivity constraints for real, relatively large forest planning problems; (2) the temporal connectivity model might be smaller and easier to solve than the MPS model in some cases; and (3) the temporal connectivity requirement may not reduce timber revenues significantly.

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Table 1: Stand characteristics of the test forests (vertex degree is given by the ratio of the number of adjacencies and the number of stands in the forest).

Problem	Number of stands	Stand size distribution (ha)			Total area (ha)	Vertex degree
		min	max	average		
Pack	186	0.55	24.27	9.18	1,708	4.78
Shulkell	1,039	0.13	15.92	3.75	3,821	3,97
El Dorado	1,363	4.05	47.09	15.52	21,147	5.30
NBCL5	5,224	0.99	20.23	6.65	34,739	2.87

Table 2: Age characteristics of the test forests. (Data in bold represent the area of mature forest given the minimum mature age we chose for the experiment.)

Problem	Relative forest area in age class (%)					
	0-20	21-40	41-60	61-80	81-100	100+
Pack	41	27	2	20	2	8
Shulkell	0	23	18	49	10	0
El Dorado	25	1	0	0	0	74
NBCL5	16	15	18	32	15	4

Problem	Total forest area above age (ha)				
	70 yrs	80 yrs	90 yrs	100 yrs	120 yrs
Pack	501	**211**	147	147	138
Shulkell	1,539	449	**75**	18	0
El Dorado	15,612	15,612	15,612	**15,612**	0
NBCL5	17,457	12,046	6,510	3,253	**356**

Table 3: Planning parameters for the computational experiment. (Minimum rotation age and average ending age values differ by forest type in NBCL5.)

Problem	Pack	Shulkell	El Dorado	NBCL5
Planning horizon (P, yr)	7x5	5x5	5x5	4x10
Rotation age (R_k, yr)	45	40	40	40,50,60,70,80,100
Average ending age ($\overline{ET}$, yr)	50	40	40	20,25,30,35,40,50
Fluctuation bounds (L-U)	0.9-1.2	0.85-1.15	0.85-1.15	0.8-1.3
Max. clearcut size (A_{max}, ha)	40.5	16.2	48.6	21
Min. patch size (A_{min}, ha)	30	20	40	40
Mature habitat (K, ha)	120	20	400	120
Min. mature age (T, yr)	80	90	100	120

Table 4: Size and formulation characteristics of the temporal connectivity model variations

	Model	Clusters	Binary variables	Constraints	Total formulation time (s)
Pack forest	Benchmark	3,971	5,753	40,183	164,080.9
	TC.0	3,971	5,753	43,393	164,085.6
	TC.1	3,971	5,753	42,321	164,092.1
	TC.2	3,770	5,500	42,994	164,085.3
	TC.1+2	3,770	5,500	42,990	164,092.5
Shulkell	Benchmark	7,382	14,377	59,757	550.2
	TC.0	7,382	14,377	61,101	572.5
	TC.1	7,382	14,377	60,773	585.2
	TC.2	649	7,010	52,643	549.8
	TC.1+2	649	7,010	52,928	579.9
El Dorado	Benchmark	100,570	113,694	160,645	1,505.1
	TC.0	100,570	113,694	241,101	3,143.4
	TC.1	100,570	113,694	213,065	2,646.9
	TC.2	100,570	113,694	241,101	3,154.7
	TC.1+2	100,570	113,694	213,065	2,983.4
NBCL5	Benchmark	4,676	28,745	182.1	183.8
	TC.0	4,676	28,745	43,818	189.7
	TC.1	4,676	28,745	43,543	186.3
	TC.2	1,656	25,259	40,240	185.1
	TC.1+2	1,656	25,259	40,020	184.4

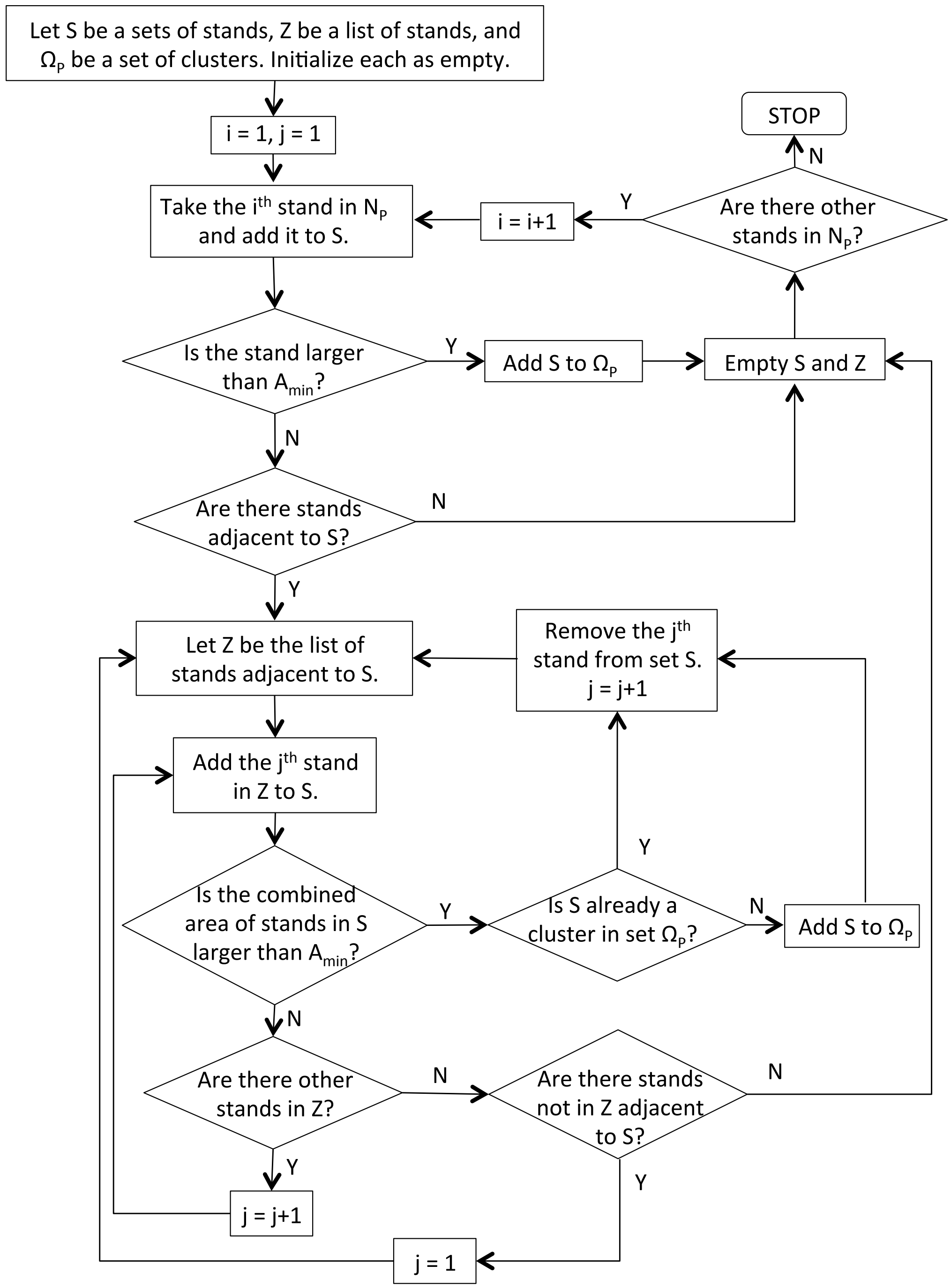

Figure 1: Age Discriminative Cluster Enumeration: $f(E_P, A_{min}) \mapsto \Omega_P$.

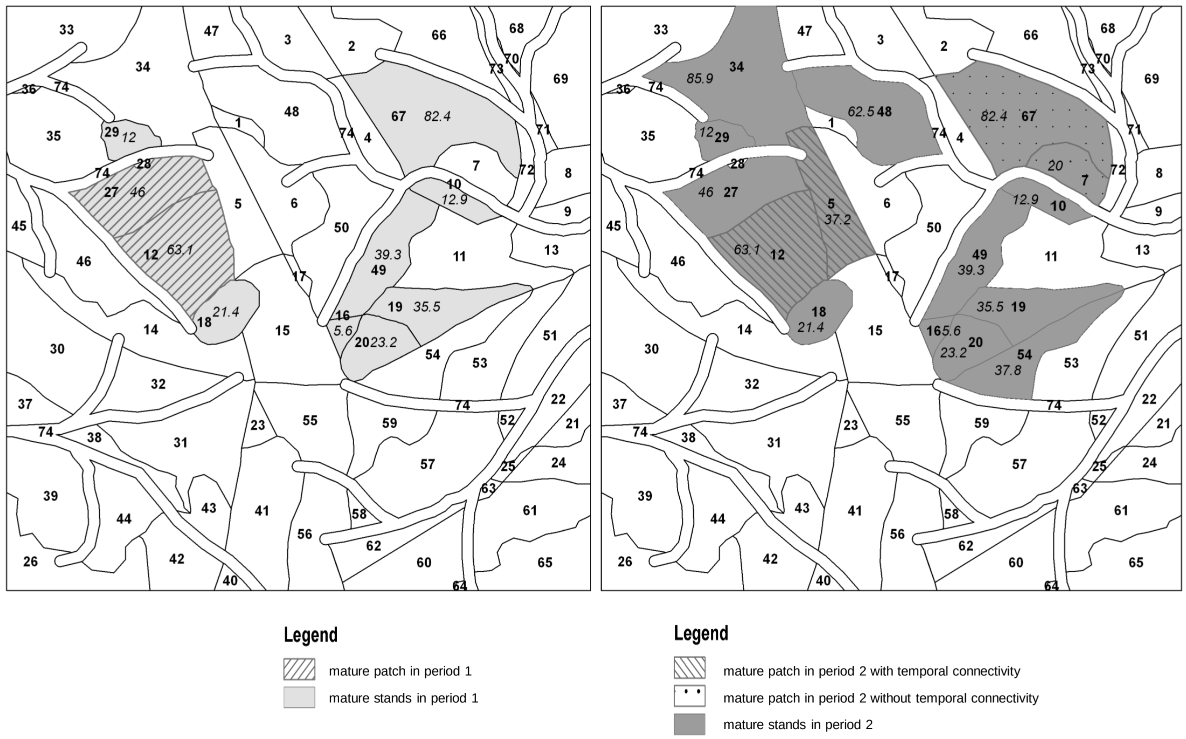

Figure 2: Connectivity of mature patches over time. (The bold number in each polygon represents the stand ID, the additional italic number in each highlighted polygon gives the area of the stand in hectares. We used the map of WLC forest (data available in the FMOS repository (FMOS – Forest Management Optimization Site, 2011)) to create this example.

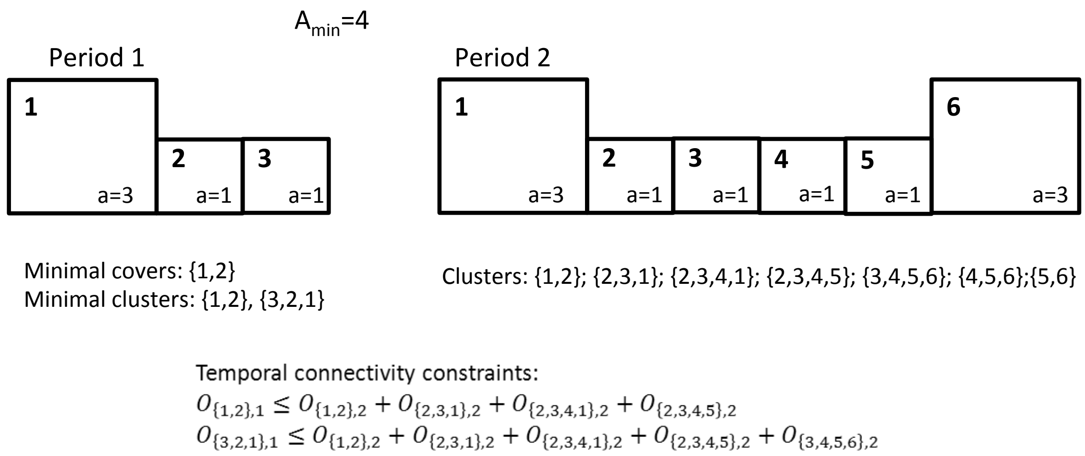

Figure 3: Difference between minimal covers and minimal clusters.

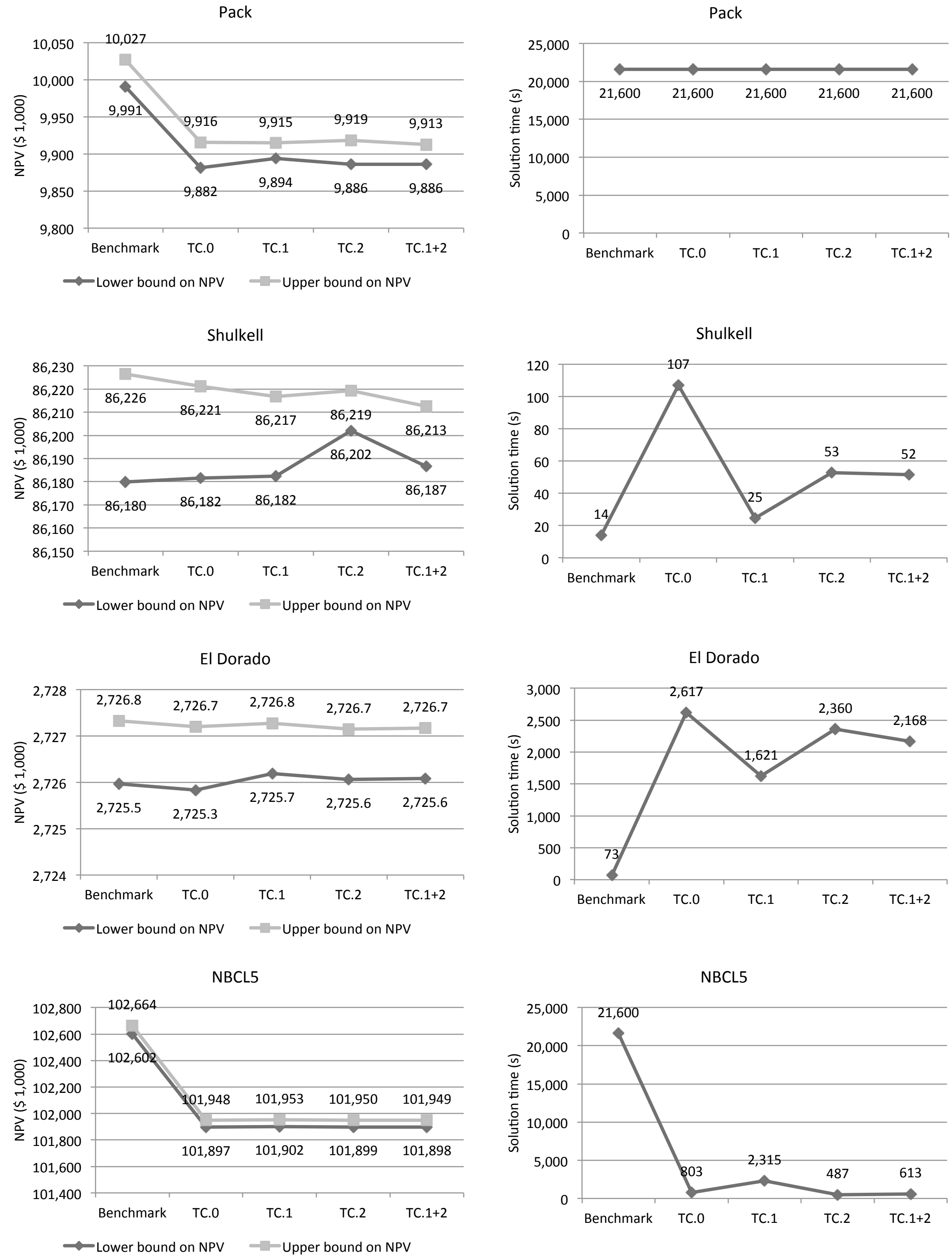

Figure 4: Size and formulation characteristics of the temporal connectivity model variations

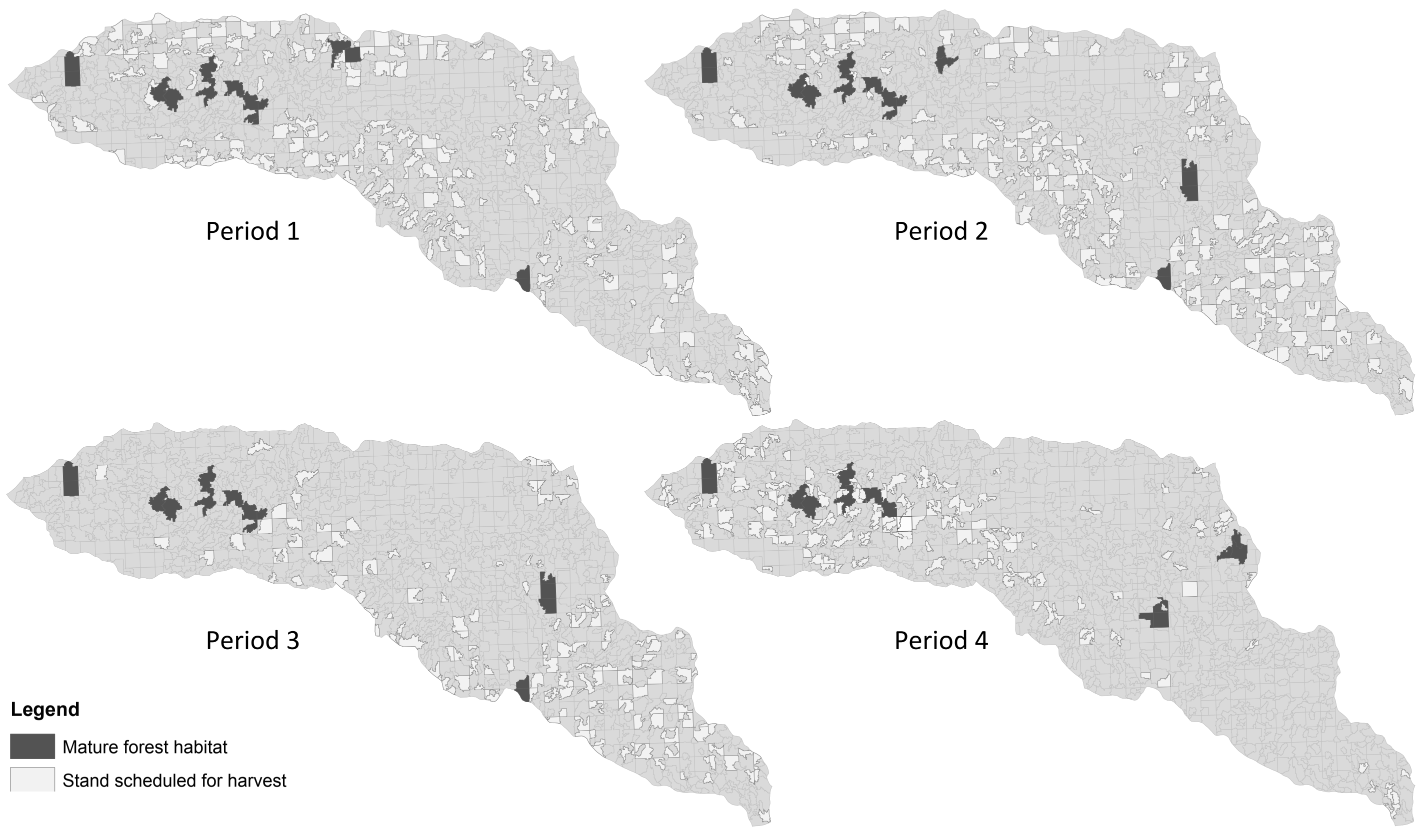

Figure 5: Forest management plan without temporal connectivity constraints

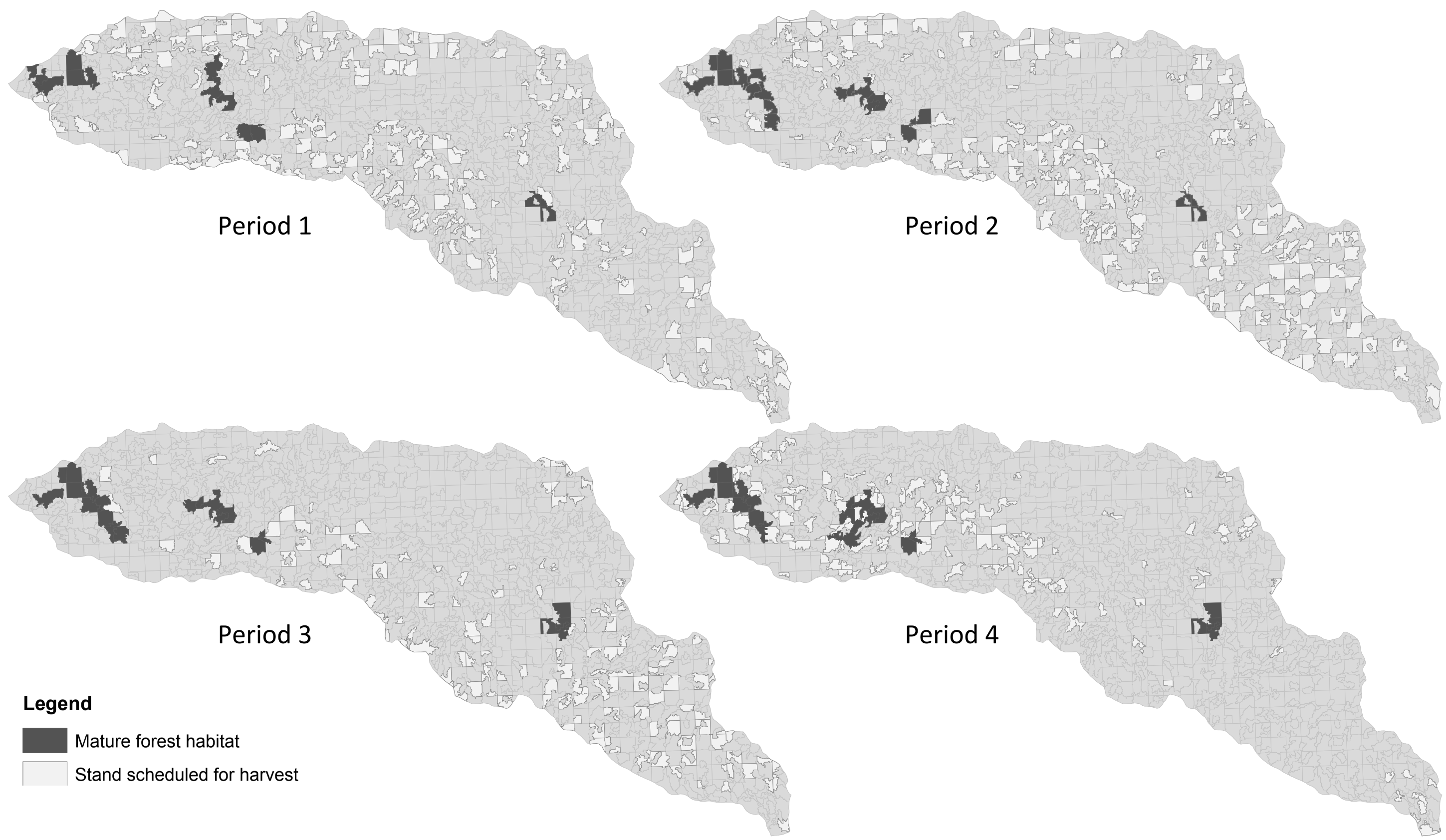

Figure 6: Forest management plan with temporal connectivity constraints