Connectivity in Spatial Optimization

Lecture 8 (4/26/2017)

The Conrad et al. (2012) Model

$$Max \sum_{i \in V} a_i x_i$$

Should parcel *i* be selected?

$s.t.:$

$$\sum_{i \in V} c_i x_i \le B$$

Variable to absorb residual flow

$$x_t = 1 \qquad \forall t \in T$$

$$z_0 + y_{0\hat{t}} = n$$

$$y_{ij} \le n x_j \qquad \forall (ij) \in E'$$

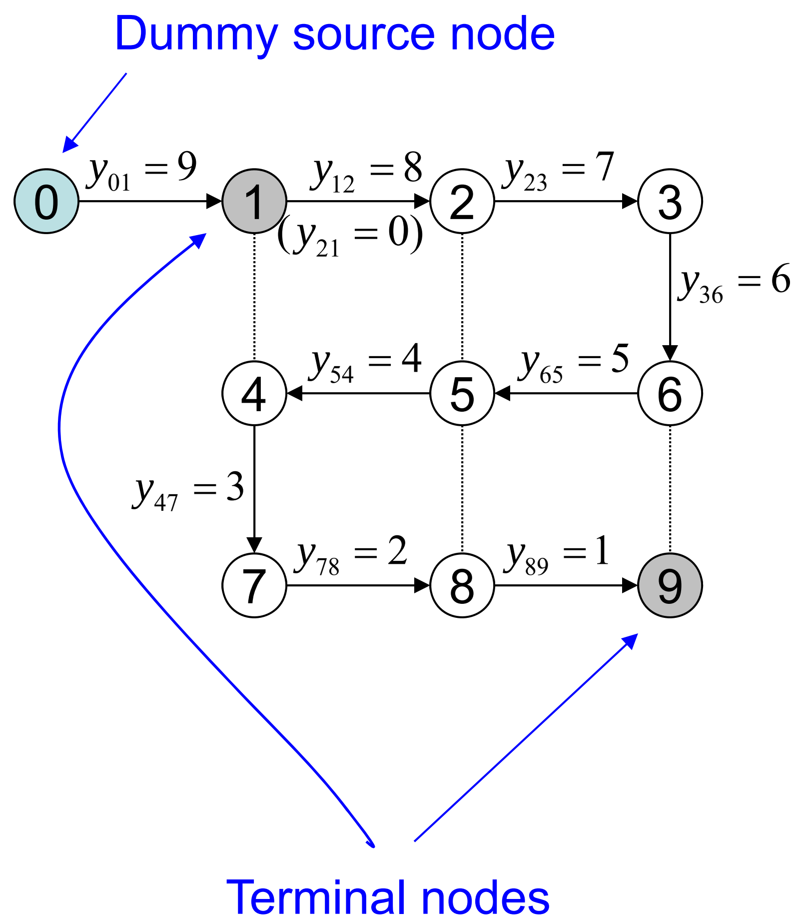
$$\sum_{i:(ij) \in E'} y_{ij} = x_j + \sum_{l:(jl) \in E'} y_{jl} \qquad \forall j \in V$$

Directional flow from *i* to *j*

$$\sum_{j \in V} x_j = y_{0\hat{t}}$$

$$x_i \in \{0,1\}, \; z_0 \in [0,n]$$

$$y_{ij} \in \mathbb{R}^+$$



The Önal and Briers (2006) Model

Should parcel i be selected?

$$Max \sum_{i \in V} a_i x_i$$

$$s.t.:$$

$$\sum_{i \in V} c_i x_i \le B$$

Should there be a flow from i to j ?

$$\sum_{i \in A_j} y_{ij} \le |A_j| x_j \qquad \forall j \in V$$

$$\sum_{j \in A_i} y_{ij} \le x_i \qquad \forall i \in V$$

$$\sum_{(ij) \in E} y_{ij} = \sum_{i \in V} x_i - 1$$

$$z_{ij} \ge w_i + 1 - m(1 - y_{ij}) \qquad \forall (ij) \in E \text{ with } i \ne j$$

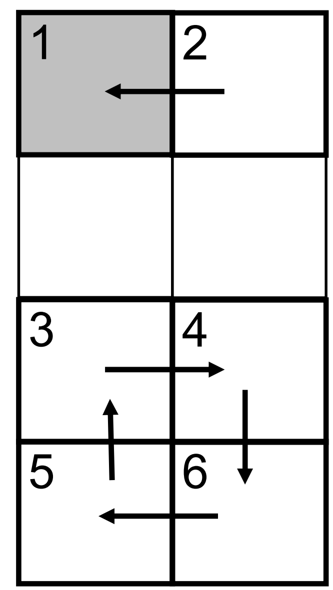
$$w_j = \sum_{i \in V} z_{ij} \qquad \forall j \in V$$

Tail function contribution from parcel i to j

$$x_i, y_{ij} \in \{0,1\}$$

$$w_i, z_{ij} \in \mathbb{R}^+$$

Tail function for parcel j



Linearizing Cross-product Terms

$$x_1 x_2 \ (\text{if } x_1 = 0 \vee x_2 = 0, \ \text{then } x_1 x_2 = 0)$$

0-1 variables

Step 1: Replace $x_1 x_2$ by a 0-1 variable y

Step 2: Impose the logical condition $y = 1 \leftrightarrow x_1 = 1 \ AND \ x_2 = 1$

$$y \le x_1,$$

$$y \le x_2,$$

$$x_1 + x_2 - y \le 1.$$

Linearizing Cross-product Terms cont.

$$\delta x$$

continuous

0-1

Step 1: Replace δx by a continuous variable y

Step 2: Impose the logical condition $\mathrm{x} = 0 \to y = 0,\ \mathrm{x} = 1 \to y = \delta$

$$y \leq Mx,$$

$$y \leq \delta,$$

$$\delta - y + Mx \leq M.$$

How much should *M* be?