CHAPTER 6: THE LAND EXPECTATION VALUE (LEV)

In this chapter, we begin considering the financial analysis of stand-level forest management decisions. We begin with decisions related to even-aged stands because they are simpler. An **even-aged stand** is defined as having one age-class, with the range of ages in the stand being no more than 20% of a rotation. Even-aged stands are regenerated by removing all or most of the stand; thus, there is little to no overlap among generations of stands on a given site. Even-aged management favors shade intolerant species, which includes many of the most valuable commercially managed species, including, for example, most of the pines and oaks, Douglas fir, ash, black cherry, and aspen. Because most of the stand is harvested at once, harvest costs are relatively low with even-aged management. For these and other reasons, even-aged management tends to be favored when the production of wood fiber for industrial needs is the primary management objective.

Uneven-aged management, the most common alternative to even-aged management, will be discussed in Chapter 9. The analysis of even-aged management decisions is covered in three chapters. This chapter considers the value of bare land at the start of an even-aged forest rotation. The next chapter extends the concepts developed in this chapter to apply to even-aged stands at any stage of development. Chapter 8, the third chapter on even-aged management, considers the analysis of thinning and other intermediate stand treatment decisions.

The subject of this chapter, the **Land Expectation Value** (**LEV**) (also known as the **Soil Expectation Value** (**SEV**) or the **bare land value**), is one of the most important financial concepts in timberland management – perhaps the most important. There are at least two reasons for this:

- When the assumptions used in calculating the LEV are realistic, the LEV gives an estimate of the value of forest land (excluding the value of the standing timber) for land that is used primarily for growing timber.
- When the primary objective of the landowner is to maximize financial return, the LEV – or generalizations of it – is the main tool used to identify optimal even-aged management regimes, including rotation decisions, thinning regimes, stand establishment effort and intermediate treatments.

In managing forests, it is obviously important to be able to determine the value of forest land. The **transactions evidence** approach is the most common alternative to the LEV for timberland valuation. The transactions evidence approach is based on identifying a few recent sales (transactions) of similar properties. This is the method used most often by Realtors in valuing properties. This approach is difficult to apply to timberland valuation, however, because it is often difficult to find a sufficient number recent transactions of sufficiently similar properties. There are many reasons for this. First, forest land does not change hands as often as residential or other urban properties. Like any other real estate transaction, the location of a forest property will greatly affect its value, and values can change rapidly. Thus, only sales that have occurred recently and in close proximity to the property under consideration should be regarded as comparable. Second, forest land typically comes bundled with many other assets whose values must be un-bundled from the transaction price in order to assess the value of the timberland alone. Obviously, timberland frequently comes with some timber on it. In order to calculate the value of the land itself, the value of the timber must be subtracted from the transaction price. However, timber values can also be difficult to determine. Timber values depend on a host of factors. In many forest land sales, the value of the standing timber is based on the value of the timber if it were sold immediately – i.e., its **liquidation value**. However, unless one actually plans to remove the timber soon, the liquidation value can substantially understate the true value of the timber. Furthermore, timberland also often comes bundled with agricultural land, lakefront or streamside property, roads, and cabins or other buildings.

Because of the complications discussed above, the LEV is frequently the best available method for estimating the value of timberland. However, the LEV is not the most appropriate method for valuing forest land if the main value of the land is not timber related. For example, many non-industrial private forest land owners own forest land primarily for recreational or development purposes. In such cases, the LEV will be a poor predictor of forest land values. However, LEV calculations can still provide important information about forest land values, even in these cases.

Forest management is the process of identifying, assessing and selecting management options for forested properties. When the primary ownership objective is to maximize the financial return from growing timber, the LEV is the primary tool for assessing and selecting management options for even-aged stands. [1]Essentially, the management option that maximizes the LEV is financially optimal. For example, the financially optimal rotation can be identified by maximizing the LEV. Similarly, the financially optimal thinning program can be determined by calculating the LEV for each thinning alternative and selecting the alternative with the highest LEV. While the LEV applies only to even-aged management options, the concepts and methods used to analyze uneven-aged management options is similar to the concepts and methods used to calculate the LEV. Thus, the basic concept of the LEV has very broad applicability in forest management.

When the primary management objectives for an area are not financial, the LEV will obviously be less directly applicable. Costs and benefits that are difficult to measure in financial terms are difficult to include in a LEV calculation. However, the management option which maximizes the LEV can be used as a benchmark against which the **opportunity costs** of selecting alternative management options can be measured. If, for example, the financially optimal management alternative for a stand would produce an LEV of $350/ac, and the LEV

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[1] Many simplifying assumptions are made in calculating a LEV. More sophisticated analyses will use generalizations of the LEV in analyzing management alternatives. Thus, the following discussion would be more accurate if every instance of the acronym "LEV" was replaced with the phrase "LEV, or a generalization of the LEV."

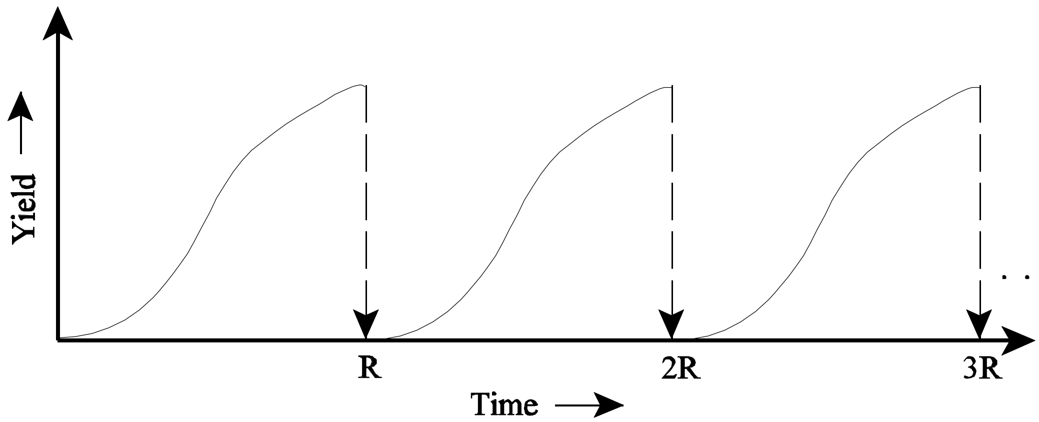

Figure 6.1. A series of identical even-aged rotations, illustrating the fundamental assumptions underlying the Land Expectation Value (LEV).

of the chosen alternative is $250/ac, then the opportunity cost of implementing the chosen alternative is $100/ac. This does not mean that it would be a mistake to choose this alternative. Rather; it suggests that the unquantified benefits associated with the chosen alternative must be worth at least $100/ac.

1. The Definition and Assumptions of the LEV

Figure 6.1 illustrates the growth-harvest pattern of an even-aged forest stand. The figure shows a sequence rotations, each *R* years long, in which the stand is grown to age *R* and then harvested. While the figure shows only 3 such rotations, this sequence of rotations is expected to continue in perpetuity. The LEV is simply the present value of the costs and revenues resulting from such a sequence of rotations, as indicated in the following definition:

> The **Land Expectation Value (LEV)** is the present value, per unit area, of the projected costs and revenues from an infinite series of identical even-aged forest rotations, starting initially from bare land.

In the simplest case, an LEV calculation assumes that:

i) each rotation is of equal length,
ii) the sequence of events within each rotation is the same, and
iii) the net revenue associated with each event within a rotation is the same for all rotations.

In essence, these assumptions say that each rotation will be exactly the same. These assumptions clearly are not likely to hold in most situations. It is unlikely that any timber stand will be managed identically for each rotation. It is even more unlikely that the net revenues associated with a given event will be the same from one rotation to the next – that would require constant (real) costs and prices and constant yields. These assumptions are made to simplify the analysis. In any analysis, it is necessary to make some simplifying assumptions. The key question to ask is whether better assumptions could be made, and, if so, whether the analysis would be sufficiently improved to warrant the increased complexity and cost of acquiring additional information. Generally, we do not know how things are likely to change in the future. Without specific information about how things might change, assuming that things will not change is often the most reasonable thing to do. If there is information that can be used to predict how future costs or benefits are likely to change, the concept of the LEV can be generalized to accommodate such expectations. An important point to keep in mind is that in most cases the assumptions made about rotations beyond the initial rotation will not greatly affect the value of the LEV. Because future rotations are far in the future, their discounted value typically makes up only a small portion of the overall LEV value. Usually, the critical assumptions are the ones that are made about the initial rotation.

2. Calculating the LEV

Calculating a LEV is a straightforward application of the financial analysis techniques that were discussed in Chapters 2 and 3. The main difficulty is keeping track of all of the different cash flows associated with a single rotation of the stand. Since each rotation in the infinite time horizon of the management unit is assumed to be the same, the LEV calculation deals first with a single, typical rotation.

The four basic types of costs and revenues associated with most even-aged forest rotations are 1) an establishment cost, 2) a final net revenue, 3) annual costs, and 4) miscellaneous intermediate costs or revenues that occur in the middle of the rotation. Figure 6.2 depicts

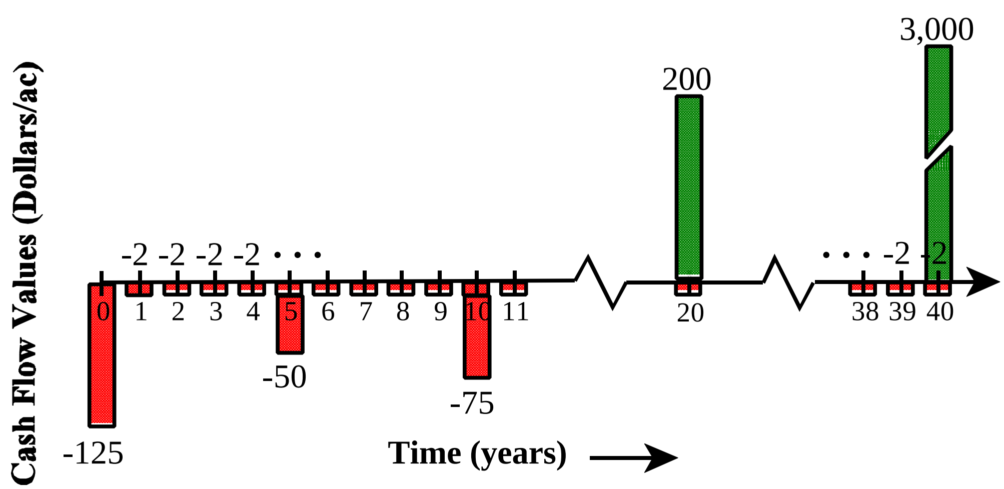

Figure 6.2. Cash flow diagram for an example LEV problem.

each of these. In the example shown in the figure, the stand establishment cost is \$125. The final net revenue is \$3,000. There is an annual cost of \$2, which occurs every year, starting with year one and ending in year 40. Finally, there are three intermediate costs and/or revenues: a cost of \$50 in year 5, a cost of \$75 in year 10, and a revenue of \$200 in year 20. Each of these four types of cash flows must be either discounted or compounded in order to combine them into a single present or future value representing all of the values for this single rotation. Three ways to calculate the LEV are discussed below. These methods differ primarily in how the values within the typical rotation are combined.

After the costs and revenues associated with a typical rotation are combined into a single present or future value, the infinite periodic series formula is applied to account for the fact that the LEV is the present value of an infinite series of these typical rotations.

Notation

It is generally useful to start by defining the notation that will be used. Let:

R = the length of a rotation (in years),
E = the stand establishment cost per unit area,
A = the net cost or revenue per unit area from all annual costs and benefits,
I_t = an intermediate cost or revenue per unit area occurring at a time t larger than 0 but less than R,
$Y_{p,R}$ = the expected yield per unit area of product p at age R,
P_p = the price of product p,
n = the number of products in the final harvest,
C_h = the cost of selling the timber, and
r = the real interest rate.

Note that the letter R is used here to represent the rotation length. This is the same notation that was used in Chapter 2 to represent the payment in the discounting formulas for series of payments. This can be confusing, so be aware of it. Perhaps it would be better to use some other letter to represent the rotation length, but it's hard to think of one more appropriate than R. Alternatively, a Greek letter could be used, but that probably wouldn't be much of an improvement. Sometimes there just aren't enough letters in the alphabet. The best solution, apparently, is for you to recognize what R represents based on the context of the problem.

Note also that no assumption is made here whether the annual net revenues and the intermediate net revenues are positive net revenues – i.e., revenues – or negative net revenues – i.e., costs. For example, when the annual net revenue is a cost the value of A would simply be negative. However, the establishment cost is specifically assumed to be a cost – i.e., when E is positive, it represents a cost.

In general, the establishment cost (E) includes any cost that occurs at time 0 in the rotation. Typically, this will include site preparation and/or planting costs. In the example in Figure 6.2, E = \$125/ac. Annual revenues are not common in forestry, but would include such things as hunting or mineral lease revenues. Annual costs include such things as property taxes and annual management fees. In the example in Figure 6.2, A = -\$2/ac. Miscellaneous intermediate costs include such things as herbicide treatments, prescribed burning costs, and precommercial thinnings. Miscellaneous intermediate costs in the example in Figure 6.2 include the \$50/ac cost in year 5 (I_5 = -\$50/ac) and the \$75/ac cost in year 10 (I_{10} = -\$75/ac). Miscellaneous intermediate revenues include such things as commercial thinnings, firewood sales, and sales of pine needle mulch. In Figure 6.2, the thinning revenue at age 20 is a miscellaneous intermediate revenue (I_{20} = \$200/ac).

A real interest rate should almost always be used in a LEV calculation. Recall that one of the basic assumptions of the LEV is that prices and costs are assumed to be constant. As discussed in Chapter 3, it generally does not make sense to assume that nominal prices or costs will stay constant. While the assumption of constant real prices is also somewhat questionable, it is far more tenable than assuming that nominal prices will be constant. Thus, inflation should be removed from the interest rate and from the prices and costs used in the analysis. Since prices are assumed to be constant, it is not necessary to use a time subscript on the price variable.

Calculation Method 1: Calculating the Present Value of the First Rotation First

We are now ready to develop a formula for calculating the LEV. As discussed earlier, the LEV calculation uses the formula for an infinite periodic series. That is,

$$V_0 = \frac{R}{(1+i)^t - 1}$$

(Note that here R represents the periodic payment.) In the LEV calculation, the period between payments will be the length of the rotation – i.e., the time between each clearcut. Thus, the t in the above formula will be replaced with R (for *rotation*). The numerator of this formula, also R (for *periodic payment*), is assumed to be earned at the *end* of each period. Thus, the R in the numerator should be replaced by the *future value* of each rotation – i.e., the accumulated value at the *end* of the rotation. Since, for a given stand, all rotations are assumed to be the same, the future value of the cost and revenue streams from all rotations are assumed to be the same as the future value of the cost and revenue streams from the first rotation. Thus, the first step will be to calculate the future value of the first rotation. This value will replace the R in the numerator of the infinite periodic series formula.

Even though the equation for the infinite periodic series requires the future value of the first rotation, you may find it useful to first calculate the present value of the first rotation and then convert this to a future value. This is the basic approach in this first method for calculating the LEV. One advantage of this method is that calculating the present value of the first rotation provides a good opportunity for a consistency check to make sure all of the calculations have been done right. Since the LEV is the sum of the present values of all future rotations, including the first one, the present value of the first rotation should be less than the

LEV. However, the present value of the first rotation usually represents a sizeable proportion of the LEV – usually between 70 and 99 percent. Thus, if you calculate a LEV value that is less than the present value of the first rotation, or one that is a lot bigger than the present value of the first rotation, you should suspect that you have probably done something wrong.

The following steps describe the procedure for calculating the LEV using method 1.

1. First, calculate the present value of the first rotation (PV_{R1}). This is given by the following somewhat complicated-looking formula:

$$PV_{R_1} = -E + \sum_{t=1}^{R-1} \frac{I_t}{(1+r)^t} + \frac{A[(1+r)^R - 1]}{r(1+r)^R} + \frac{\sum_{p=1}^{n} P_p Y_{p,R} - C_h}{(1+r)^R}$$

   This formula is not really is not as complicated as it may seem at first glance. It contains four terms corresponding to the four types of costs and revenues generally found in a forest rotation that were discussed earlier. The first term, -*E*, is just the present value of the establishment cost. The next term, with the I_t in the numerator, includes the present value of each of the intermediate costs and returns. The summation sign (Σ) sums all of the possible intermediate net revenues that might occur in any year from year 1 to year *R*-1. Of course I_t will be zero for most of these years. The third term, with the *A* in it, uses the finite annual series formula to give the present value of the costs and revenues that occur annually. The final term gives the present value of the final harvest. The summation sign in the numerator of this last term allows for the possibility of up to *n* – in other words, as many as are needed – products in this harvest. The value of each product is calculated by multiplying the stumpage price for that product by the yield. The term C_h recognizes that there often are costs associated with harvests, in addition to revenues.

2. Next, convert the present value of the first rotation into a future value:

$$FV_{R_1} = (1+r)^R \, PV_{R_1}$$

3. Now, apply the infinite periodic payment formula:

$$LEV = \frac{FV_{R_1}}{(1+r)^R - 1}$$

Combining these equations gives:

$$LEV = \frac{\left[-E + \sum_{t=1}^{R-1} \frac{I_t}{(1+r)^t} + \frac{A[(1+r)^R - 1]}{r(1+r)^R} + \sum_{p=1}^{n} \frac{P_p Y_{p,R}}{(1+r)^R} - \frac{C_h}{(1+r)^R} \right](1+r)^R}{(1+r)^R - 1}$$

This formula may seem somewhat overwhelming and complex. However, it just represents the combined result of the three steps described above. In practice, it is generally best to do each of the three steps separately. In fact, it is often best to break up the calculations in the first step using a table, as illustrated in the example later in this section.

Calculation Method 2: Calculating the Future Value of the First Rotation Directly

Since the numerator in the infinite periodic series is supposed to be a future value, you may have been wondering why you should bother calculating the present value of the first rotation when you are just going to convert it to a future value anyway. This is a good point, and you can calculate LEVs without first calculating the present value of the first rotation if this logic appeals to you. The following steps outline the procedure of calculating the LEV by calculating the future value of the first rotation directly, rather than by calculating the present value of the first rotation first.

1. The following formula is used to calculate the future value of the first rotation:

$$FV_{R1} = -E(1+r)^R + \sum_{t=1}^{R-1} I_t(1+r)^{(R-t)} + \frac{A[(1+r)^R - 1]}{r} + \sum_{p=1}^{n} P_p Y_{p,R} - C_h$$

   Again, while this formula looks complicated at first glance, it is really just the sum of four relatively easily interpreted terms. The first term, $-E\,(1+r)^R$, gives the future value of the establishment cost. The next term, with I_t in it, gives the future value of each of the intermediate costs and returns. (Why are they compounded for R-t years?) The third term, with the A in it, gives the future value of the costs and benefits that occur annually. The final term is just the future value of the final harvest.

2. Now, apply the infinite periodic payment formula:

$$LEV = \frac{FV_{R1}}{(1+r)^R - 1}$$

Combining these two equations gives the following formula for the LEV.

$$LEV = \frac{-E(1+r)^R + \sum_{t=1}^{R-1} I_t(1+r)^{(R-t)} + \frac{A[(1+r)^R - 1]}{r} + \sum_{p=1}^{n} P_p Y_{p,R} - C_h}{(1+r)^R - 1}$$

You can verify for yourself that this formula is equivalent to the final formula given for method 1 by multiplying the individual terms in the brackets in the numerator of the combined LEV formula for method 1 by the term $(1+r)^R$. As with method 1 it is usually best to break down the LEV calculation into steps, rather than trying to plug all of your values into this formula at once.

Calculation Method 3: Separating Out Annual Costs and Revenues

You may have noticed that the most complicated part of the LEV formula is the annual net revenue component. This term can actually be simplified considerably by recognizing that, while the above methods treat it as a finite annual series, the net annual revenue is really an infinite annual series since it is a finite annual series imbedded in an infinite periodic series. Recognizing this simplifies the calculations because the formula for the present value of an infinite annual series is much simpler than the formula for a finite annual series. This third method of calculating the LEV uses this insight to simplify the calculations in method 2. A similar approach could be used to simplify method 1.

1. Calculate the future value of the first rotation directly, ignoring the annual net revenue (this modified future value is denoted here by FV'_{R1}):

$$FV'_{R_1} = -E(1+r)^R + \sum_{t=1}^{R-1} I_t (1+r)^{(R-t)} + \sum_{p=1}^{n} P_p Y_{p,R} - C_h$$

2. Apply the infinite periodic payment formula for the future value of the first rotation calculated in step 1, and use the infinite annual series formula for the net annual revenue:

$$LEV = \frac{FV'_{R_1}}{(1+r)^R - 1} + \frac{A}{r}$$

Combining these equations gives:

$$LEV = \frac{-E(1+r)^R + \sum_{t=1}^{R-1} I_t (1+r)^{(R-t)} + \sum_{p=1}^{n} P_p Y_{p,R} - C_h}{(1+r)^R - 1} + \frac{A}{r}$$

Note that this is the simplest-looking of the three formulas for the LEV – and the easiest to type into a calculator. All three formulas are mathematically equivalent, and all should give the correct LEV. You can use any of these formulas – whichever one you find works best for you.

Example: A Northern Hardwood Stand with Natural Regeneration

A northern hardwood stand regenerates naturally, with no regeneration cost. Every 70 years, it can be harvested to yield 10 mbf/ac of hardwood sawtimber at \$350/mbf and 15 cd/ac of pulpwood at \$6 per cord. The annual property taxes are \$2/ac, and annual management expenses are \$1.50/ac. Assume that the discount rate is 3%. What is the value of the land (per acre)?

Answer: Note that there is no establishment cost and there are no intermediate costs or revenues. All we have are the annual costs and the final harvest revenue. Applying Method 3, the LEV can be calculated as follows:

$$FV'_{R_1} = 10\,mbf/ac \times \$350/mbf + 15\,cd/ac \times \$6/cd = \$3{,}590/ac$$

Now, apply the infinite periodic payment formula:

$$LEV = \frac{\$3{,}590/ac}{(1.03)^{70} - 1} - \frac{(\$2/ac + \$1.5/ac)}{0.03} = \$402.28/ac$$

If the assumptions about interest rates, yields, and prices are correct, and if the sole value of the land is for timber production, the bare-land value of this northern hardwood timberland is $402.28 per acre.

Example: A Southern Pine LEV

Consider the following per-acre expected incomes and costs associated with managing a hypothetical stand of southern pine:

Table 6.1. Costs and returns for a hypothetical southern pine plantation.

	Amount	Year
Reforestation cost	$125.00	0
Brush control cost	$50.00	5
Thinning cost	$75.00	10
Property tax	$3.00	annual
Hunting revenue	$1.00	annual
Thinning revenue	$200.00	20
Final harvest	$3,000.00	40

Calculate the LEV for this stand assuming that the real alternative rate of return is 6%. (You may have noticed that this is the problem depicted in Figure 6.2.)

Answer: First, calculate either the present value of the first rotation or the future value of the first rotation. This is easiest to do if we organize the information in a table as shown in Table 6.2

Table 6.2. Present and future values of costs and revenues associated with the hypothetical southern pine plantation.

	Amount	Year	Present Value	Future Value
Reforestation cost	\$125.00	0	-\$125.00	-\$1,285.71
Brush control cost	\$50.00	5	-\$37.36	-\$384.30
Thinning cost	\$75.00	10	-\$41.88	-\$430.76
Property tax	\$3.00	annual	-\$45.14	-\$464.29
Hunting revenue	\$1.00	annual	\$15.05	\$154.76
Thinning revenue	\$200.00	20	\$62.36	\$641.43
Final harvest	\$3,000.00	40	\$291.67	\$3,000.00
Total			\$119.69	\$1,231.12

Note that if the present value of the first rotation is compounded forward 40 years the result is equal to the future value of the first rotation that was calculated directly.

$$FV_{R_1} = (1+r)^R PV_{R_1} = (1.06)^{40}\$119.69 = \$1{,}231.12$$

Now, the LEV can be calculated using the formula for an infinite periodic series.

$$LEV = \frac{FV_{R_1}}{(1+r)^R - 1} = \frac{\$1{,}231.12}{(1.06)^{40} - 1} = \$132.58$$

The problem can also be solved using Method 3, as follows:

$$LEV = \frac{-125(1.06)^{40} - 50(1.06)^{35} - 75(1.06)^{30} + 200(1.06)^{20} + 3{,}000}{(1.06)^{40} - 1} - \frac{2}{0.06}$$

$$= \frac{-1{,}285.71 - 384.30 - 430.76 + 641.43 + 3{,}000}{9.28572} - 33.33 = 132.58$$

3. The Financially Optimal Rotation

As discussed in the introduction, one of the key uses of the LEV is to determine which of several alternatives for managing an even-aged stand is best – assuming, as we often do in this book, that the landowner's objective is to maximize financial return. One of the most fundamental decisions that must be made in managing an even-aged stand is the rotation age, i.e., the age when the stand is to be harvested and a new stand established.

As stated earlier, the financially optimal rotation is the one that maximizes the LEV. One way, then, to identify the best rotation for a stand is to calculate the LEV for a variety of rotation ages and then pick the rotation age corresponding to the highest LEV. Consider the following example:

1. Assume that the sawtimber yield (*Y*) of the stand, in thousands of board feet per acre, is given as a function of age (*A*) by the following equation:

$$Y(A) = Me^{-k/(A-d)} = 40e^{-90/(A-10)} \quad \textit{for } A > 10,\ 0\ \textit{otherwise}.$$

2. Assume that
   - ! the price of sawtimber is \$600/mbf;
   - ! the cost of establishing the stand is \$200/ac;
   - ! the annual property tax is \$2/ac;
   - ! the annual management cost is \$1/ac;
   - ! the marginal tax rate on income is 22%; and
   - ! the real alternate rate of return is 3%.
3. Finally, assume that all of these values are unchanging and that the landowner is interested in maximizing the financial return on the forest land.

Applying Method 3 for calculating the LEV (with an additional term to account for the income tax[2]), the formula for the LEV in this example – as a function of the rotation age – is:

$$LEV(R) = \left[\frac{-200(1.03)^R + 600 \times Y(R)}{(1.03)^R - 1} - \frac{3}{0.03}\right](1 - 0.22)$$

This equation, combined with the above yield equation, can be used to calculate the value of the LEV for the example stand for any rotation age. Figure 6.3 shows the yield curve for this example and a curve showing the LEV over a range of rotation ages. Note that the yield curve is for sawtimber; thus, very little volume accrues before age 40. There would, of course, be pulpwood volume at earlier ages, but the value of the pulpwood would be small and has been ignored here to simplify the example.

Of course, our primary interest in Figure 6.3 is the LEV curve. It should be fairly clear from the figure that the LEV – for this example, at least – is quite sensitive to the choice of a rotation age. The LEV rises sharply from 0\$/ac at a rotation age of 38 to a maximum of \$367.90/ac with a 62-year rotation. After peaking at age 62, the LEV drops sharply,

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[2] In accounting for the income tax, it has been assumed that the establishment cost, the annual property tax, and the annual management cost are all expensed against current income in the year the costs occurred. The tax factor, (1-.22), therefore applies to all terms – including the final revenue – and can be factored out of the equation as a single term.

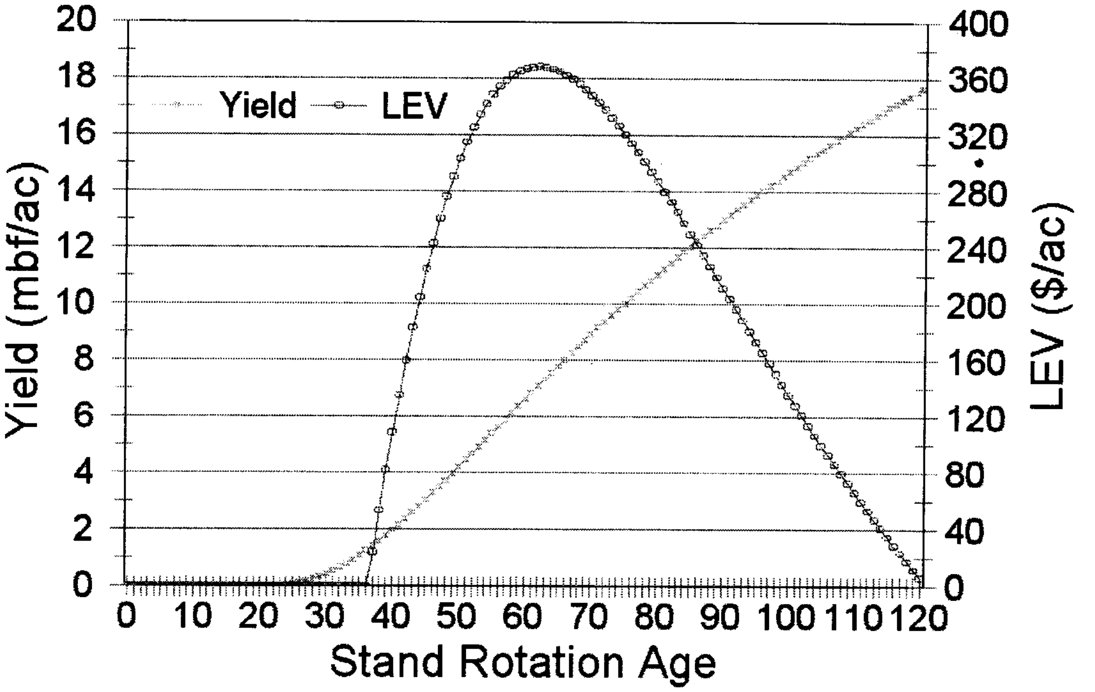

Figure 6.3. Yield and Land Expectation Value for the example problem over a range of rotation ages.

dropping to $286.62/ac with an 80-year rotation (a 22% reduction in the value of the LEV), and dropping further to $138.44 with a 100-year rotation (a reduction in the value of the LEV of 63%). The LEV drops to zero again at a rotation age of about 120 years.

You may recall from Chapter 5 that the age where the **mean annual increment (MAI)** is maximized (**culmination of mean annual increment**, or CMAI) is often considered the optimal biological rotation. This is because the average volume production over the life of the stand is maximized at the CMAI. (The MAI measures the average volume production over the life of the stand, so maximizing the MAI maximizes the average volume production over the life of the stand.) The CMAI for this yield function is at age 109. At this rotation age, the LEV is only $72.78. In fact, the financially optimal rotation age – 62 years – is almost a half a century shorter than the CMAI. Many people are surprised to learn that financially optimal rotations are so short. Why is the financially optimal rotation age so different from the rotation age that maximizes volume production? It does not seem to make sense.

First, the point should be made that the financially optimal rotation age is not always shorter than the CMAI. It is possible to construct examples where the financially optimal rotation age is longer than the CMAI. Even in this example, if the interest rate is lowered sufficiently (i.e., very close to zero) the financially optimal rotation age becomes longer than the CMAI. However, these are the exceptions: usually the rotation age that maximizes the LEV is shorter than the CMAI. To understand why the financially optimal rotation age can be so different from the CMAI, it may help to consider the rotation age decision using a technique called marginal analysis.

Marginal Analysis of the Rotation Decision

A **marginal analysis** considers the costs and benefits of doing just one more unit of something. In this case, consider the financial costs and benefits of allowing a stand of trees to grow for just one more year. The primary benefit is that the stand will grow, adding value to the stock that will eventually be harvested. The value of this growth – the price of wood times the volume of growth, minus income taxes – is the **marginal benefit** at age *a* of allowing the stand to grow one more year:

$$MB_a = P \cdot \Delta Y_{a+1}(1 - t_{inc})$$

where P = the price of wood,
ΔY_{a+1} = the growth of wood in the stand between age a and age $a+1$, and
t_{inc} = the marginal income tax rate.

Now, what are the marginal costs of letting a stand grow for just one more year? Obviously, if there are annual property taxes or annual management costs, these should be included. The other costs of waiting to harvest a stand are less intuitive. First, consider the cost of using the land for one more year. If the land were rented, then this cost would be the **rent** for one more year. But what about someone who owns the land – what is the cost of using the land for them? Even if the land is not rented, a landowner *could* rent the land to someone else. Even for the landowner, the potential rent that could be earned is an **opportunity cost**[3] of using the land. Thus, the concept of rent applies whether the land is actually rented or not.

So, the rent is a part of the opportunity cost of owning the land. How much should the rent on a piece of even-aged forest land be? To answer this question, think of the land as a financial asset. If someone borrows a financial asset, how much do they pay for the use of that asset? People pay interest to use financial assets, which is calculated by multiplying the value of the asset times the interest rate. The value of an even-aged forest property is given by the highest LEV that can be earned by the land, so the rent should be the interest rate times the optimal LEV – i.e., LEV^*.[4]

$$Rent = r \cdot LEV^*$$

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[3] An **opportunity cost** is the value of benefits that are forgone by *not* doing something.

[4] If the land was to be rented, the owner would probably want to charge this *true rent* plus the cost of the annual property taxes. That is, the rent would be:

$$Rent = r \cdot LEV^* + t_{prop}(1 - t_{inc})$$

So far, we have considered three items to include in the marginal cost of waiting one year to harvest an even-aged stand of timber: 1) annual property taxes, 2) annual management costs, and 3) the land rent – equal to the interest rate times the LEV. There is one more component which is also an opportunity cost: the opportunity cost of not having the use of the money that would have been earned if the stand had been harvested. This cost is called the **inventory cost**. It is interest that could have been earned on the value of the standing inventory volume, if the volume were harvested and sold and the proceeds invested in the best alternative investment. The value of the inventory volume is the volume times the stumpage price minus taxes. The inventory cost at any given age *a* is the interest rate times the value of the inventory volume at that age – i.e., the price times the inventory volume, minus any taxes that would have to be paid. Thus,

$$Inventory\,Cost_a = r \cdot P \cdot Y_a (1 - t_{inc})$$

All of the components of the marginal cost of waiting to harvest an even-aged forest stand can now be summarized as follows:

$$MC_a = A(1 - t_{inc}) + t_{prop}(1 - t_{inc}) + r \cdot LEV^* + r \cdot P \cdot Y_a (1 - t_{inc})$$

where t_{prop} = the annual property tax and all other variables are as previously defined.

Note that both the annual management cost and the annual property tax have been reduced by the income tax rate. This is because we have assumed that these costs can be expensed – i.e., deducted from net income to reduce the income taxes in the year in which they occur.

Figure 6.4 shows the marginal benefits and the marginal costs of waiting to harvest the example stand. Both the marginal benefit and the marginal cost of waiting to harvest the stand vary with the stand age because the growth rate (in MB_a) and the yield (in MC_a) are functions of the stand age. Before the **inflection point** in the yield curve, the growth rate – and therefore the marginal benefit of waiting to harvest the stand – is increasing with stand age. However, as the stand ages, the growth rate reaches a maximum and then slows down, causing the marginal benefit of waiting to eventually decrease as the stand ages. On the other hand, the marginal cost of waiting one more year to harvest the stand increases as the stand ages because the inventory cost rises. Early in the life of a stand, the marginal benefit of waiting to harvest the stand will be greater than the marginal cost of waiting. Thus, the net benefits of waiting will be positive, and the stand should not be harvested. As the stand matures, however, the marginal benefit of waiting will decrease and the marginal cost will increase until at some point the marginal cost of waiting will become greater than the marginal benefit. When this happens, the net benefits of waiting to harvest the stand will become negative and the stand should be harvested. The optimal rotation is given by the age where the marginal benefit curve crosses the marginal cost curve from above. In Figure 6.4, this occurs at age 62 – the same age where the LEV curve reaches its peak in Figure 6.3.

Reconsider now the question why the financially optimal rotation age is often so much shorter than the age that maximizes the mean annual increment. Maximizing the mean annual increment clearly maximizes the average annual volume production of the stand. However,

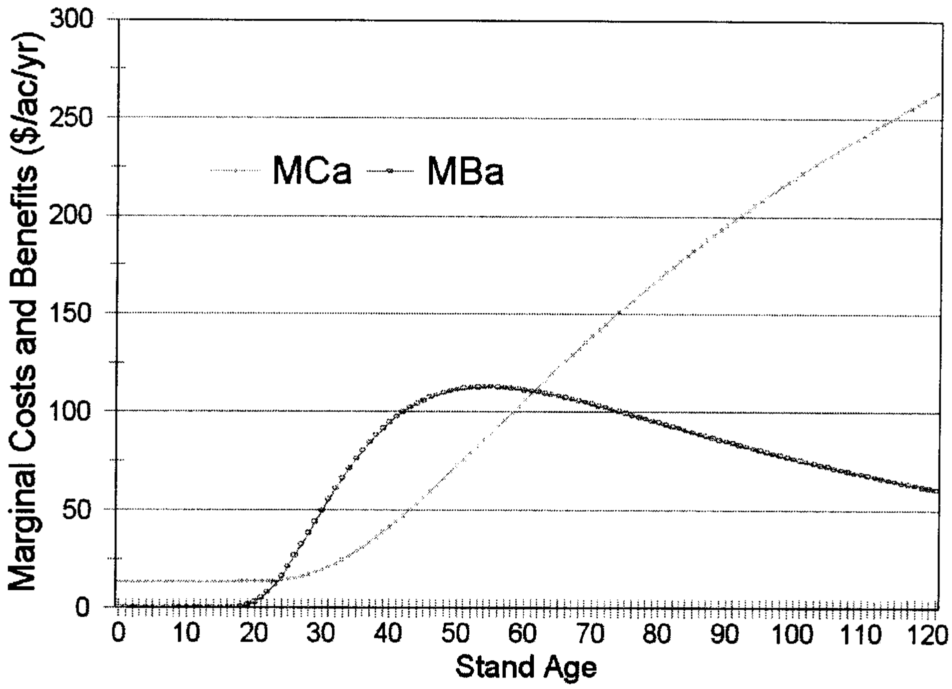

Figure 6.4. Marginal analysis of the optimal rotation – marginal costs and marginal benefits of waiting one year to harvest.

selecting this as the optimal rotation age ignores the costs of maximizing volume production – especially the inventory cost. In fact, selecting the rotation to maximize the MAI ignores all economic factors, as it is based solely on the yield curve. The financially optimal rotation, on the other hand, accounts for costs and benefits and is a variable target which will change with changes in economic conditions. The next section explores how changes in prices, costs, taxes, and the interest rate affect the optimal rotation.

The Effect of Changing Economic Conditions on the Financially Optimal Rotation

The last section demonstrated how the financially optimal rotation is determined by the intersection of two curves representing, respectively, the marginal costs and marginal benefits of waiting to harvest an even-aged stand. The marginal cost of waiting includes four components: 1) annual property taxes, 2) annual management costs, 3) the land rent, and 4) the inventory cost. The marginal benefit of waiting is equal to the value of the additional growth that will occur in the next year. In addition to varying with the age of the stand, the marginal cost and marginal benefit of waiting to harvest a stand depend on the stumpage price, the establishment cost, annual management costs, taxes, and the interest rate. This section considers each of these variables and their effect on the marginal cost and marginal benefit curves, and, hence, their effect on the financially optimal rotation for the stand.

First, consider the effect on the optimal rotation of changing the stumpage price. Increasing the stumpage price will increase the marginal benefit curve in proportion to the increase in the price – that is, a 25 percent increase in the stumpage price will shift the marginal benefit of waiting curve up by 25 percent. The effect on the marginal cost curve of changing the stumpage price is a bit more complicated. Neither of the terms reflecting annual costs (i.e., the annual management cost and the annual property tax) will be affected. However, increasing the stumpage price will increase the LEV and the inventory cost. As with the marginal benefit curve, the increase in the inventory cost will be proportionate to the increase in the stumpage price. The proof is a bit hard for an undergraduate text, but it can be shown that the increase in the LEV resulting from an increase in the stumpage price will be such that

1. if there is no establishment cost, the marginal benefit curve will shift up proportionate to the increase in the stumpage price, and
2. if there is a positive establishment cost, the shift in the marginal benefit curve will be *more* than proportionate to the increase in the stumpage price.

In the first case, when there is no establishment cost, both the marginal benefit and the marginal cost curves shift up proportionately, and the age at which the curves intersect will not change. Thus, when there is no establishment cost, changing the stumpage price will have no effect on the financially optimal rotation. On the other hand, if there is a stand establishment cost, the shift in the marginal cost curve will be more than proportionate to the change in the stumpage price, while the shift in the marginal benefit curve will be proportionate to the change in the stumpage price. Thus, for a price increase, the marginal cost curve will shift up more than the marginal benefit curve, resulting in the two curves crossing at an earlier age than before the stumpage price increase. Thus, the financially optimal rotation will be reduced if the price increases. In other words, when there is a positive stand establishment cost, the financially optimal rotation will be inversely affected by changes in the stumpage price; i.e., increases in the price will reduce the financially optimal rotation and vice versa.

What about the establishment cost? How will changes in the establishment cost affect the financially optimal rotation? Again, consider the effect of a change in the establishment cost on the marginal benefit and marginal cost curves.[5] The only term in either curve that would be affected by a change in the stand establishment cost is the LEV. An increase in the stand establishment cost will decrease the LEV. This will shift the marginal cost curve down and move the point where the marginal cost curve intersects the marginal benefit curve to the right – i.e., to a longer optimal rotation age. Thus, the financially optimal rotation age will move in the same direction as the change in the establishment cost – i.e., increasing the stand establishment cost will increase the financially optimal rotation and vice versa. The intuitive explanation of this is straightforward. When the stand establishment cost is increased, one

[5] Note that we are not talking here about a change in the establishment effort. We are considering only a change in the cost of a given level of effort, which is held constant in the analysis. This is important because we are assuming that the yield curve does not change. If the establishment effort were changed, one would expect the yield curve to change also.

way to reduce the impact of the cost increase is to re-establish the stand less often. This is accomplished by increasing the rotation.

Now, consider annual management costs and property taxes. Increasing these would seem to increase the marginal cost curve since annual costs and property taxes appear directly in that equation. However, remember that increasing either the annual costs or the property taxes will also reduce the LEV. In fact, the reduction in the LEV will just offset the increase in the annual costs or property taxes, and the net result will be that the marginal cost curve will not change. Thus, changes in the annual management costs or property taxes will have no effect on the financially optimal rotation.

Under the assumption that all costs are expensed (i.e., deducted against the current year's income), the income tax will also have no effect on the optimal rotation. All of the terms in both the marginal cost and marginal benefit functions will change in inverse proportion to the change in the taxes. Thus, both curves will shift up or down by the same proportion, and they will intersect at the same age. However, a **severance tax**, which is a tax paid on the value of harvested products, *will* affect the financially optimal rotation age. The case of a severance tax is different because costs are not expensed against income. A change in the severance tax is therefore like a change in the stumpage price. Increasing the severance tax is just like decreasing the stumpage price. Thus, as with price changes, the effect of a change in the severance tax will depend on whether there is a stand establishment cost. When there is no stand establishment cost, changing the severance tax will have no effect on the financially optimal rotation. When there is a stand establishment cost, increasing the severance tax will increase the optimal rotation and vice versa.

So far, we have considered the direction of change in the optimal rotation in response to changes in the interest rate, prices, stand establishment costs, annual management costs, and taxes. What we have not considered is the magnitude of these effects. Of course, the exact magnitude of these changes will depend on the specific details of the problem. However, there are a few general conclusions that can be drawn. The financially optimal rotation typically changes only slightly, if at all, as a result in changes in stumpage prices, the establishment cost, annual management costs, and taxes. By far, the financially optimal rotation is the most sensitive to the interest rate. Changing the interest rate has no effect on the marginal benefit curve. It affects the marginal cost curve in three ways. The two last terms in the marginal cost equation directly involve the interest rate. When the interest rate goes up, these terms will tend to go up and vice versa. However, the LEV is also affected by the interest rate, and the LEV is inversely related to the interest rate (i.e., when the interest rate goes up the LEV goes down and vice versa). Thus, the change in the rent in response to a change in the interest rate is ambiguous. However, the effect of changing the interest rate on the inventory cost will generally be quite large and will always outweigh the effect on the rent term. Thus, the marginal cost curve will shift in the same direction as the interest rate. Increasing the interest rate will shift up the marginal cost curve, shifting back the intersection of the two curves, and shortening the optimal rotation. Similarly, decreasing the interest rate will shift the marginal cost curve down, moving the point of intersection to the right and increasing the financially optimal rotation.

Table 6.3 summarizes the effect of each of these variables on the financially optimal rotation and the LEV. "Positive" in the table means that an increase in the variable will increase the optimal rotation or LEV. "Negative" means that an increase in the variable will decrease the optimal rotation or LEV. A zero means the variable has no effect on the optimal rotation or LEV. In general, the LEV decreases with increasing costs and increases with increasing prices.

Table 6.3. Effects of changes in economic variables on the financially optimal rotation and the LEV.

	R^*		LEV
r	Negative		Negative
P	C=0 0	C>0 Neg.	Positive
C	Positive		Negative
A	0		Negative
t_{prop}	0		Negative
t_{inc}	0		Negative
t_{sever}	C=0 0	C>0 Pos.	Negative

4. <u>Study Questions</u>

1. Why is the LEV an important concept in forest management? How is it used?

2. What is the LEV? What assumptions are made when calculating a LEV?

3. Why are so many unlikely assumptions made in calculating the LEV?

4. When calculating the LEV, why are the assumptions regarding future rotations less critical than assumptions regarding the initial rotation?

5. What proportion of the value of the LEV typically comes from the first rotation?

6. Why is a real interest rate generally used when calculating a LEV?

7. What is a marginal analysis?

8. Explain why the land rent is a cost of growing an even-aged stand of timber whether or not the land is actually rented.

9. Why is the rotation that maximizes the mean annual increment (MAI) often called the biologically optimal rotation?

10. Explain why the financially optimal rotation is often quite different – and usually shorter – than the rotation that maximizes the mean annual increment.

11. When considering whether to harvest an even-aged stand, what are the marginal benefits of waiting one more year before harvesting the stand? What are the marginal costs of waiting?

12. Why does the financially optimal rotation occur at the age where the marginal benefits of waiting one more year before harvesting the stand just equal the marginal costs of waiting?

13. How are the financially optimal rotation and the LEV affected by an increase in the interest rate?

14. Replace "interest rate" in question 13 by any of the following:
a. the stumpage price
b. the stand establishment cost
c. the annual management cost
d. the annual property tax
e. the income tax rate
f. the severance tax rate

5. <u>Exercises</u>

*1. Consider the following yield and economic data.

Age	Yield (N. Hardwoods) Pulpwood (cd/ac)	Yield (N. Hardwoods) Sawtimber (mbf/ac)	LEV ($/ac)
50	26	3	
60	24	6.5	
70	20	11	
80	19	15	

- Real interest rate = 4%
- N. hardwood pulp price = $20/cd
- Establishment costs = $180/ac
- N. hardwood sawtimber price=$425/mbf

- Precommercial thin = $60/ac in year 40
- Annual taxes = $2/ac
- Annual management costs = $3/ac
- Annual hunting lease revenue = $3.50/ac

Assume that real prices and costs will remain constant. Calculate the LEV for each of the four rotation ages. Show an example calculation with your answer.

*2. You own 20 acres of southern pine timberland on which you would like to grow timber. You figure that such an enterprise would have the following per acre costs and returns:

- Planting Cost: $150
- Interest Rate: 5%

Revenue at final harvest:

Age	Net stumpage revenue	LEV	Net stumpage revenue (w/ thin)	LEV with thinning
20	$ 880			
25	$1,200			
30	$1,500			

a. Fill in column 3 of the table. Show an example calculation for each column. What is the rotation age with the maximum land expectation value?

b. If you precommercially thinned, you believe that you could raise the net revenue at rotation age by 20 percent at a cost of $100/ac in year 12. (I.e., assume that the net revenues are 20% higher, but that there is an additional cost of $100/ac in year 12.) Fill in column 4 of the table. By how much will precommercial thinning raise or lower your maximum land expectation value? Show your calculations.

c. Does thinning change your optimal rotation age in this case? How?

*3. You have a 100-acre tract of aspen in northern Maine. You have just clearcut the stand (before they make clearcutting illegal in Maine), and you are wondering whether to sell the land. To give you an idea what the land is worth, you decide to calculate the LEV for the tract, assuming it will be managed for timber production forever. The site has some residual trees that should be killed. This will cost about $25 per acre. (This is a typical cost that you can expect each time you harvest.) In addition, the taxes on the property are $4 per acre per year. Your estimates of the pulpwood yield for several rotation ages are given in the table below. Aspen pulp currently sells for about $20/cd on the stump. However, this price has been rising, and you want to consider some cases where the price continues to rise for a period of time. First, assume that the aspen pulp price will rise at about the same rate as inflation. Next, assume that the price will increase at 1% above the rate of inflation for 20 years before it levels out. Finally, assume that the real price will increase 2% per year for the next 20 years before leveling out. Note: in all cases, assume that the price will remain constant in real terms after 20 years have passed.

Calculate the future aspen stumpage price for each scenario. Then, use a real alternate rate of return of 4% to calculate LEVs for rotations of 30, 35, and 45 years for each of the three price assumptions.

Price Scenario ==>		0% real price inc.	1% real price inc.	2% real price inc.
Future aspen pulp price				
Rotation	Yield (cd/ac)	LEVs		
30	35			
35	46			
40	55			

*4. a. Fill in the table below using the following column-by-column instructions. Assume that the alternate rate of return is 3%, that the price of wood is $10 per cord for all rotation ages, that there is an establishment cost of $50, that the Land Expectation Value is $42.90 per acre, that the annual taxes are $1 per acre, and use the following yield equation (cords/acre) to fill in the table below:

$$Y(a) = \frac{70}{1 + 699(A-10)^{-1.9}} \quad \textit{for } A > 10,\ 0 \textit{ otherwise.}$$

Column 2: Calculate the yield at the age given in column 1.
Column 3: Calculate the yield one year before the age given in column 1.
Column 4: Calculate the value of the annual increment (Price * Growth).
Column 5: Calculate the interest on the inventory (Price * Yield * *r*).
Column 6: Calculate the interest on the inventory plus the annual land cost (Col. 4 + L.E.V * *r* + *tax*).

Age	Yield (Age)	Yield (Age-1)	Annual Increment Value	Interest on Inventory	Interest on Inventory plus Rent
25					
30					
35					
40					
45					
50					

b. On a separate piece of paper (graph paper if by hand), graph the last four columns of data in the table. The graph should be appropriately labeled -- i.e. title, each series clearly identified, axes labeled, units given, etc.

c. Which curve shows the marginal benefit of waiting one more year before harvesting?

d. Which curve shows the marginal cost of waiting one more year before harvesting?

e. Estimate the optimal rotation for this stand if managed for purely financial objectives.

f. Check whether the LEV with this rotation is really $42.90 per acre, as you assumed in a.

g. In your figure, which curve(s) would change if the interest rate was increased to 4%? How would it (they) change? How would the optimal rotation change (increase, decrease, no change)?

h. Which curve(s) would change if the price per cord of wood was increased to $15 per cord? How would it (they) change? How would the optimal rotation change?

i. Which curve(s) would change if the stand establishment cost was only $25? How would it (they) change? How would the optimal rotation change?

5. Establishing an oak plantation will cost you $0.30 per tree and $60 per acre to put up an electric fence to keep the deer out. The stand will also need a precommercial thin after 20 years of growth. At age 85, you expect to do a shelterwood cut, followed by an overstory removal at age 90. Your yields will depend on the number of trees you plant. Prices will depend on the rates of inflation for oak sawtimber and harvest costs. Harvest costs, based on current costs, are given in the following table, which also indicates yields for different initial planting spacings.

Number of trees planted per acre	Precommercial Thinning Cost ($/per ac)	Shelterwood Cut Yield (mbf/acre)	Shelterwood Cut Cost ($/mbf)	Final Cut Yield (mbf/acre)	Final Cut Cost ($/mbf)
300 trees/acre	190	12	60	31	40
500 trees/acre	240	15	67	36	38

a. For each planting intensity fill in the tables on the following page assuming first that there will be no real change in harvest costs (thinning, shelterwood and final) or in the price of oak sawtimber. Then assume that—over the next 90 years—thinning costs will decrease at a real rate of 0.5% per year and that oak sawtimber prices will increase at a real rate of 1% per year (real price changes case). Assume that current oak prices are $480 per mbf, that the general rate of inflation will be 3% for the next 80 years, and use a real alternate rate of return of 5%.

Table 6.4. Results for 300 trees per acre.

Item	Future Value Year	No Real Price Change			Real Price Changes		
		Real Future Value	Nominal Future Value	Present Value	Real Future Value	Nominal Future Value	Present Value
Planting & Fence Cost							
Precommercial thin							
Shelterwood cost							
Shelterwood revenue							
Overstory cut cost							
Overstory cut revenue							
Total							

Table 6.5. Results for 500 trees per acre.

Item	Future Value Year	No Real Price Change			Real Price Changes		
		Real Future Value	Nominal Future Value	Present Value	Real Future Value	Nominal Future Value	Present Value
Planting & Fence Cost							
Precommercial thin							
Shelterwood cost							
Shelterwood revenue							
Overstory cut cost							
Overstory cut revenue							
Total							

b. Calculate the LEV for each planting density and for each of the two price scenarios.

Planting Density	No Real Price Change	Real Price Changes
300 trees per acre		
500 trees per acre		

c. If you believe that real prices will not change, what is the optimal planting density?

d. If you believe that real harvest costs will go down at 1/2 percent per year and that real oak sawtimber prices will go up at 1 percent per year, what is the optimal planting density?

*6. Your boss has asked you to analyze two management regimes for loblolly pine plantations. The first involves planting 600 trees per acre, thinning at ages 15 and 23, and conducting a final harvest at age 33. The second involves planting 700 trees per acre and thinning at ages 15 and 24, with a final harvest at age 35. In both cases, prescribed burns will be conducted 1 year before each thin and before the final harvest. Trees are marked just prior to thinning (in the same year as the thin). A chemical release will be applied in both cases in year 3. Stand establishment costs include site preparation and tree planting. The following tables give the economic assumptions and yield assumptions you are to use.

Table 6.6. Economic assumptions for problem 1.

Treatment	Cost/acre	Item	Price/Cost
Site preparation	\$70	Seedling purchase & planting	\$0.17/tree
Chemical release	\$67	Pulpwood price	\$24/cord
Prescribed burn	\$15	Sawtimber price	\$310/mbf
Mark for thinning	\$15	Interest rate	5%
Property tax	\$6/ac·yr	Hunting Lease Revenue	\$4/ac·yr

Table 6.7. Yield assumptions for problem 1.

Harvest	Prescription 1 (600 tpa)			Prescription 2 (700 tpa)		
	Year	Pulp Yield	Sawtimber Yield	Year	Pulp Yield	Sawtimber Yield
Thin 1	15	8.6	0	15	10.0	0
Thin 2	23	7.3	1.0	24	8.1	1.2
Final harvest	33	2.3	8.5	35	2.0	9.1

On a separate sheet of paper (attach), calculate the present value of the first rotation (excluding the annual costs and revenues) and the LEV (including the annual costs and revenues) for each of these prescriptions. Report your answers in the table below.

	Present Value of 1st Rotation	Land Expectation Value
Prescription 1		
Prescription 2		

*7. Calculate the LEV for a loblolly pine stand which will be thinned twice, at ages 14 and 24, and clearcut at age 33. The first thin is expected to yield 5 cords per acre, the second thin is expected to yield 10 cords per acre, and the final harvest should yield 13 cords per acre plus 7 mbf per acre. Use the following price and cost data (assume all prices and costs will increase at about the same rate as inflation):

Planting cost: $185/ac.
Release cost at age 3: $67/ac.
Prescribed burn costs at ages 12, 17, 22, 27 and 32: $12/ac.
Annual taxes: $6/ac.·yr.
Annual hunting revenue: $4/yr.
Marking trees for thinning: $15/ac.
Pine pulp price: $25/cd.
Pine sawtimber price: $310/mbf.
Real interest rate: 4%.