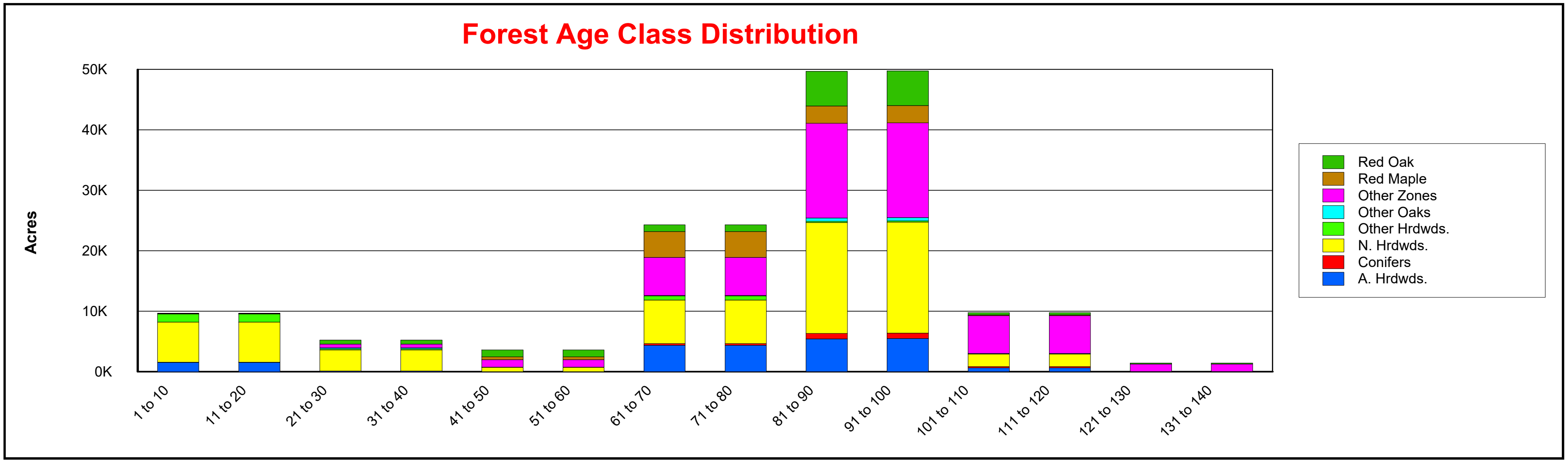
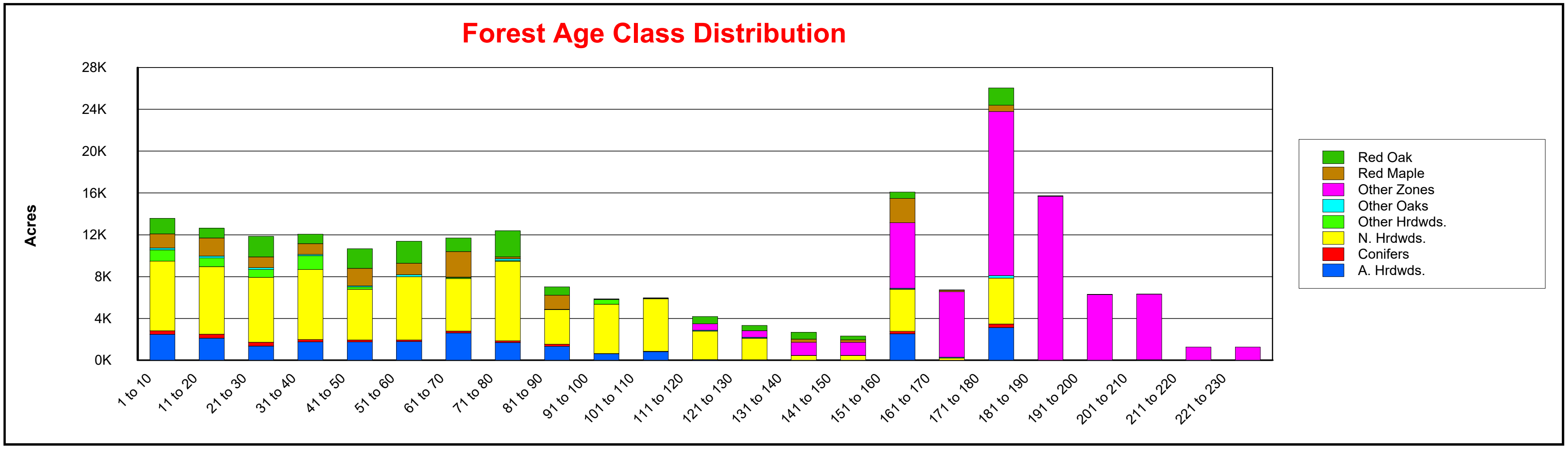
Modeling Non-Timber Objectives in Harvest Scheduling with Linear Programming

Lecture 5 (4/19/2017)

- <u>Extended rotations</u>: a minimum of 1,500ac will be required from each Site I and II to be older than 40 and 30 yrs, respectively;
- <u>Wildlife openings</u>: set aside 500ac of wildlife openings – clearcut these in pd. 1 and maintain them for browse and forage;
- <u>Stream-side management zones:</u> no harvests allowed in 8% of the forest (4% in each site class)

Balancing the Age-class Distribution





Harvest scheduling prescriptions with up to two possible harvests within a 40-year planning horizon and a minimum rotation of 20 yrs

Prescription	Planning Period				
	1	2	3	4	
1	Harvest				X_{sa10}
2	Harvest		Harvest		X_{sa13}
3	Harvest			Harvest	X_{sa14}
4		Harvest			X_{sa20}
5		Harvest		Harvest	X_{sa24}
6			Harvest		X_{sa30}
7				Harvest	X_{sa40}
8	No Harvest				X_{sa00}

$X_{sap_1p_2}$ = the number of acres from site class s, initial age-class a assigned to be harvested first in period p_1 and again in period p_2.

Extended Rotations

Period	Variable							
	X_{sa10}	X_{sa13}	X_{sa14}	X_{sa20}	X_{sa24}	X_{sa30}	X_{sa40}	X_{sa00}
1	0	0	0	a	a	a	a	a
2	1	1	1	0	0	a+1	a+1	a+1
3	2	0	2	1	1	0	a+2	a+2
4	3	1	0	2	0	1	0	a+3

Initial age-class distribution

Age Classes	**Acres by site class**	
	Site I	**Site II**
0-10	**3,000**	**8,000**
11-20	**6,000**	**4,000**
21-30	**9,000**	**7,000**
Total	**18,000**	**19,000**

$$X_{2330} + X_{2340} + X_{2300} \geq 1,500 \quad \text{(ext. rot. constraint for site class II, pd.2)}$$

$$X_{2240} + X_{2200} + X_{2340} + X_{2300} \geq 1,500 \quad \text{(ext. rot. constraint for site class II, pd.3)}$$

$$X_{1340} + X_{1300} \geq 1,500 \quad \text{(ext. rot. constraint for site class I, pd.3)}$$

$$X_{2100} + X_{2200} + X_{2300} \geq 1,500 \quad \text{(ext. rot. constraint for site class II, pd.4)}$$

$$X_{1200} + X_{1300} \geq 1,500 \quad \text{(ext. rot. constraint for site class I, pd.4)}$$

Modifying the Average Ending Age Constraint

$$\sum_{a=1}^{3}\left[\mathrm{Age}_{\mathrm{sa00}}^{40}\cdot X_{sa00}+\sum_{p_1=1}^{4}\left(\mathrm{Age}_{\mathrm{sap_1 0}}^{40}\cdot X_{\mathrm{sap_1 0}}+\sum_{p_2=p_1+2}^{4}\mathrm{Age}_{\mathrm{sap_1 p_2}}^{40}\cdot X_{\mathrm{sap_1 p_2}}\right)\right]\geq \overline{Age}^{40}\cdot TotalArea_s$$

$$TotalArea_s=\sum_{a=1}^{3}\left[X_{sa00}+\sum_{p_1=1}^{4}\left(X_{sap_1 0}+\sum_{p_2=p_1+2}^{4}X_{sap_1 p_2}\right)\right]$$

$$\overline{Age}^{40}\times TotalArea_s=\overline{Age}^{40}\times\sum_{a=1}^{3}\left[X_{sa00}+\sum_{p_1=1}^{4}\left(X_{sap_1 0}+\sum_{p_2=p_1+2}^{4}X_{sap_1 p_2}\right)\right]=$$

$$\sum_{a=1}^{3}\left[\overline{Age}^{40}X_{sa00}+\sum_{p_1=1}^{4}\left(\overline{Age}^{40}X_{sap_1 0}+\sum_{p_2=p_1+2}^{4}\overline{Age}^{40}X_{sap_1 p_2}\right)\right]$$

Modifying the Average Ending Age Constraint (continued)

$$\sum_{\mathrm{a}=1}^{3}\left[\mathrm{AgeDif}_{\mathrm{sa00}}^{40}\cdot X_{sa00}+\sum_{p_1=1}^{4}\left(\mathrm{AgeDif}_{\mathrm{sap_1 0}}^{40}\cdot X_{\mathrm{sap_1 0}}+\sum_{p_2=p_1+1}^{4}\mathrm{AgeDif}_{\mathrm{sap_1p_2}}^{40}\cdot X_{\mathrm{sap_1p_2}}\right)\right]\geq 0$$

$$\mathrm{AgeDif}_{1100}^{40}\cdot X_{1100}+\sum_{a=1}^{3}\sum_{p_1=1}^{4}\left(\mathrm{AgeDif}_{\mathrm{1ap_1 0}}^{40}\cdot X_{\mathrm{1ap_1 0}}+\sum_{p_2=p_1+2}^{4}\mathrm{AgeDif}_{\mathrm{1ap_1p_2}}^{40}\cdot X_{\mathrm{1ap_1p_2}}\right)\geq 0$$

$$\sum_{a=1}^{3}\sum_{p_1=1}^{4}\left(\mathrm{AgeDif}_{\mathrm{2ap_1 0}}^{40}\cdot X_{\mathrm{2ap_1 0}}+\sum_{p_2=p_1+2}^{4}\mathrm{AgeDif}_{\mathrm{2ap_1p_2}}^{40}\cdot X_{\mathrm{2ap_1p_2}}\right)\geq 0$$

Implementing Wildlife Openings

- Forest management activity scheduling model;
- Creating 500 acres of wildlife browse habitat;
- The openings are cleared in Pd. 1 and will be maintained over time by planting browse species at a cost of $10/ac/yr

1. Defining the wildlife opening variables:

   W_{sa}= the number of acres from site class *s*, initial age class *a* assigned to be cleared in pd. 1 and maintained as wildlife openings for the remainder of the planning horizon

2. Modifying the Area Constraints

$$X_{sa00} + \sum_{p=1}^{4}\left[X_{sap_1 0} + \sum_{p_2=p_1+2}^{4} X_{sap_1 p_2} \right] + W_{sa} \leq A_{sa} \qquad \text{for } s\text{=1,2 and } a\text{=1,2,3}$$

Note: wildlife openings as the ninth prescription

3. Specifying the target area for wildlife openings

$$\sum_{s=1}^{2}\sum_{a=1}^{3} W_{sa} \geq 500$$

or,

$$\sum_{a=1}^{3} W_{sa} \geq 500 \qquad s=1,2$$

4. Modifying the objective function (accounting for the costs and revenues that are associated with the new activities)

$$Max\ Z = \sum_{s=1}^{2}\sum_{a=1}^{3}\left[\sum_{p_1=1}^{4}\left(c^{p}_{sap_1 0}\cdot X_{sap_1 0} + \sum_{p_2=p_1+2}^{4} c^{p}_{sap_1 p_2}\cdot X_{sap_1 p_2}\right) + c^{wp}_{sa}\cdot W_{sa}\right]$$

where

$c^{p}_{sap_1 p_2}$ = the discounted net revenue (profit) from assigning one acre from site class s, initial age class a to be harvested in periods p_1 and p_2, and

c^{wp}_{sa} = the discounted net revenue from assigning an acre from site class s, initial age class a to be managed as a wildlife opening.

5. The objective function coefficients

$$c_{sa}^{wp} = \frac{(P - s_v)v_{sa10}^{1} - s_f}{(1+r)^5} - \frac{c_w[(1+r)^{35} - 1]}{r(1+r)^{35}(1+r)^5}$$

where

$P =$ the wood price,

$s_v =$ the variable (per cord) timber sale administration cost,

$s_f =$ the fixed (per acre) timber sale administration cost,

$v_{sa10}^{1} =$ the volume of wood that will be harvested in period 1 for each acre assigned to the variable W_{sa}.

$c_w =$ the annual, per-acre cost of maintaining the wildlife openings, and

$r =$ the real interest rate.

Example Objective Function Coefficient Calculation

Economic data

Item	Symbol	Amount
Wood Price	P	**$25/cd**
Planting Cost	E	**$100.00/ac**
Timber Sales Cost		
-per acre	s_f	**$15.00/ac**
-per cord	s_v	**$0.20/cd**
Interest Rate	r	**4%**

Yield

Harvest Age	Acres by site class	
	Site I	Site II
10	2	5
20	10	14
30	20	27
40	31	38
50	37	47
60	42	54
70	46	60

$$c_{23}^{wp} = \frac{(25-0.2)\cdot 27 - 15}{(1.04)^5} - \frac{10\left[(1.04)^{35}-1\right]}{0.04(1.04)^{40}} = \underline{\$384.62}$$

6. Modifying the harvest accounting constraints

$$\sum_{s=1}^{2}\sum_{a=1}^{3}\left[v_{sa10}^{1}X_{sa10}+v_{sa13}^{1}X_{sa13}+v_{sa14}^{1}X_{sa14}+v_{sa10}^{1}W_{sa}\right]-H_{1}=0$$

7. Average ending age constraints

The wildlife opening variables should be excluded from these constraints

Implementing Stream-side Management Zones (SMZs)

- It is assumed that the area assigned to SMZs has already been calculated, and
- 8% of each analysis area will be reserved for SMZs.

$$A^*_{sa} = A_{sa} \times (1 - smz_{sa})$$

$$X_{sa00} + \sum_{p=1}^{4} \left[X_{sap_1 0} + \sum_{p_2 = p_1 + 2}^{4} X_{sap_1 p_2} \right] \leq A^*_{sa} \qquad \text{for s=1,2 and a=1,2,3}$$