Financial Analysis

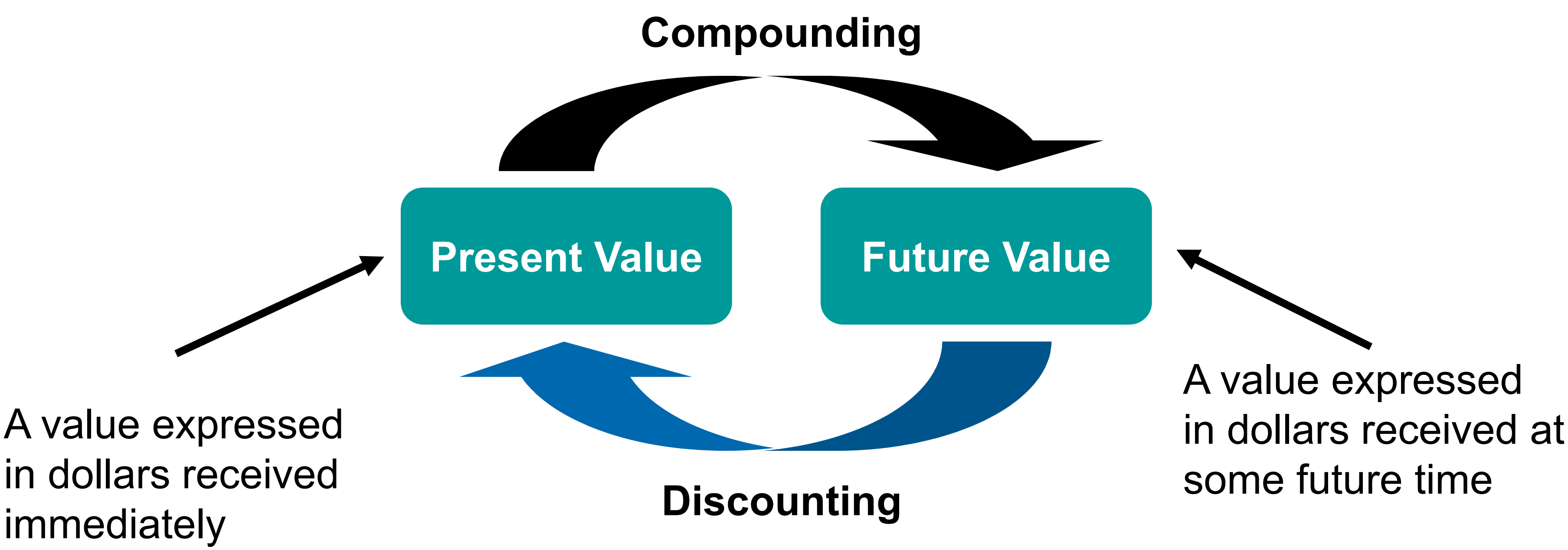
Lecture 4 (4/12/2017)

Financial Analysis

- Evaluates management alternatives based on financial profitability;
- Evaluates the opportunity costs of alternatives;
- Cash flows of costs and revenues;
- The timing of payments is important. Why?

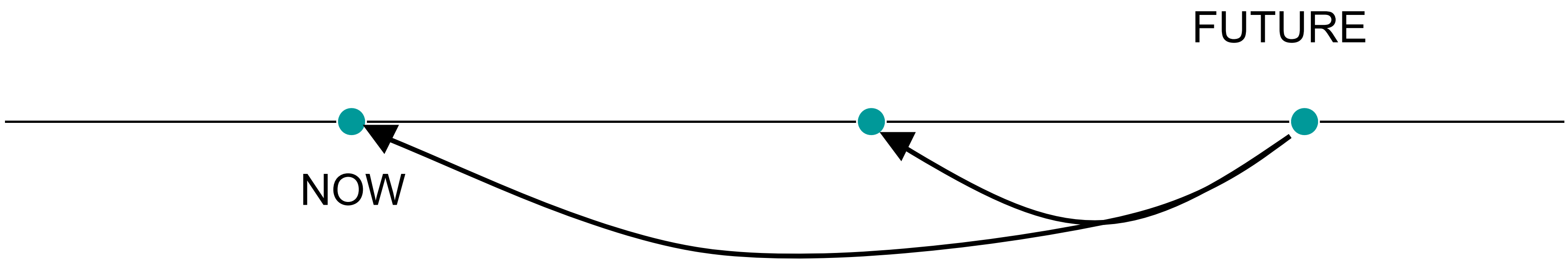
What is discounting?

- A process that accounts for time preferences
- Converts *future values* to *present values*



Definition of Discounting

- *The process of converting values expressed in dollars received at one point in time to an equivalent value expressed in dollars received at an earlier point in time*
- *Compounding is the reverse process)*

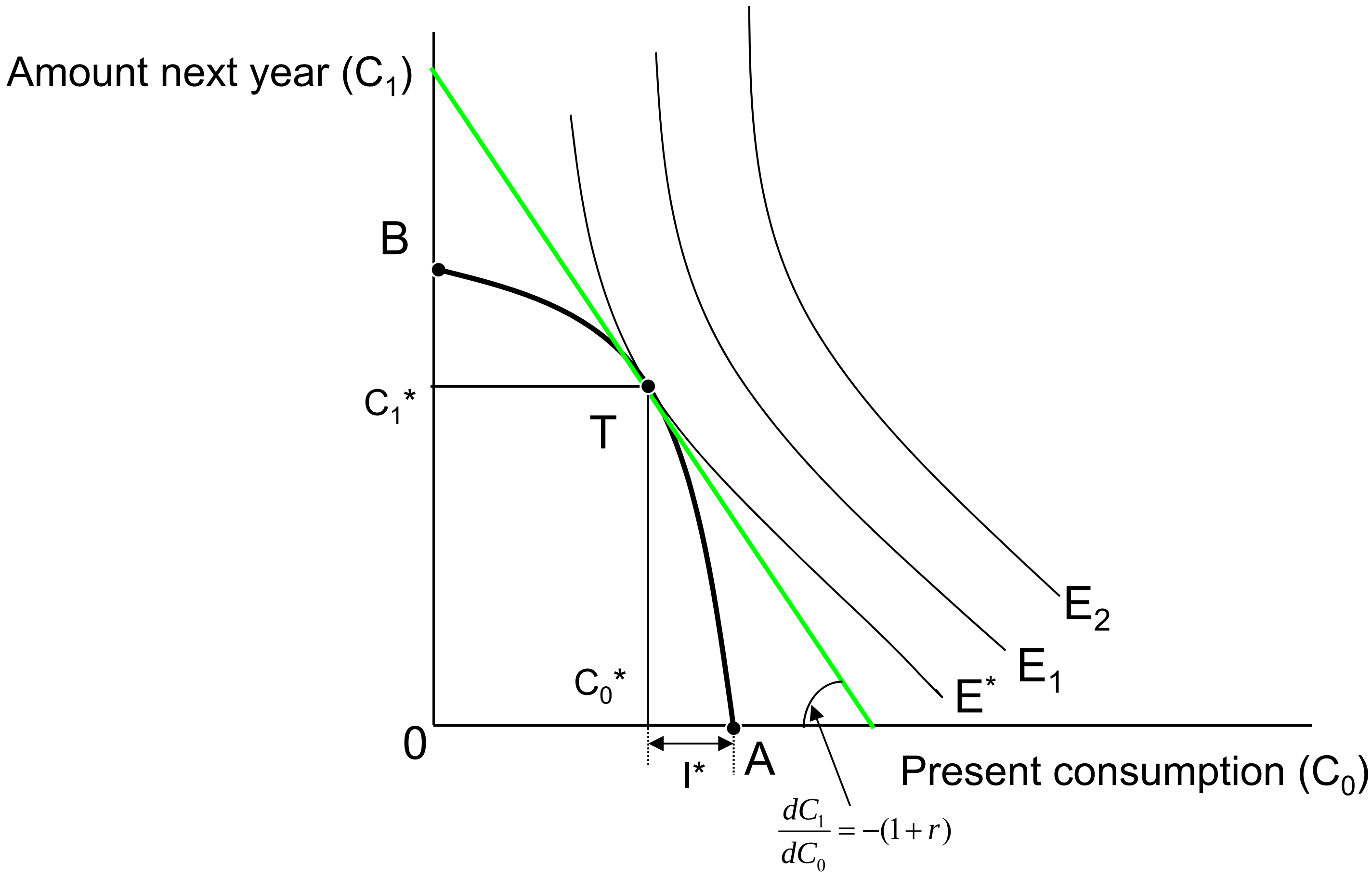


The interest rate

- Time preference: = human nature + potential investments
- Money can make money over time
- Corollary: using money costs money
- *The interest rate determines the relationship between present and future values*

Interest rate as a trade-off

(the economy of Robinson Crusoe, Buongiorno & Gilles 2003)



Source: Buongiorno and Gilles 2003, p. 374

The interest rate

- Also: *the interest rate is the percentage of the amount invested or borrowed that is paid in interest after one unit of time*

$$V_1 = V_0 + iV_0 = principal + interest$$

$$V_1 = V_0(1+i)$$

$$\boxed{Future\ Value:\ V_n = V_0(1+i)^n}$$

$$\boxed{Present\ Value:\ V_0 = V_n(1+i)^{-n} = \frac{V_n}{(1+i)^n}}$$

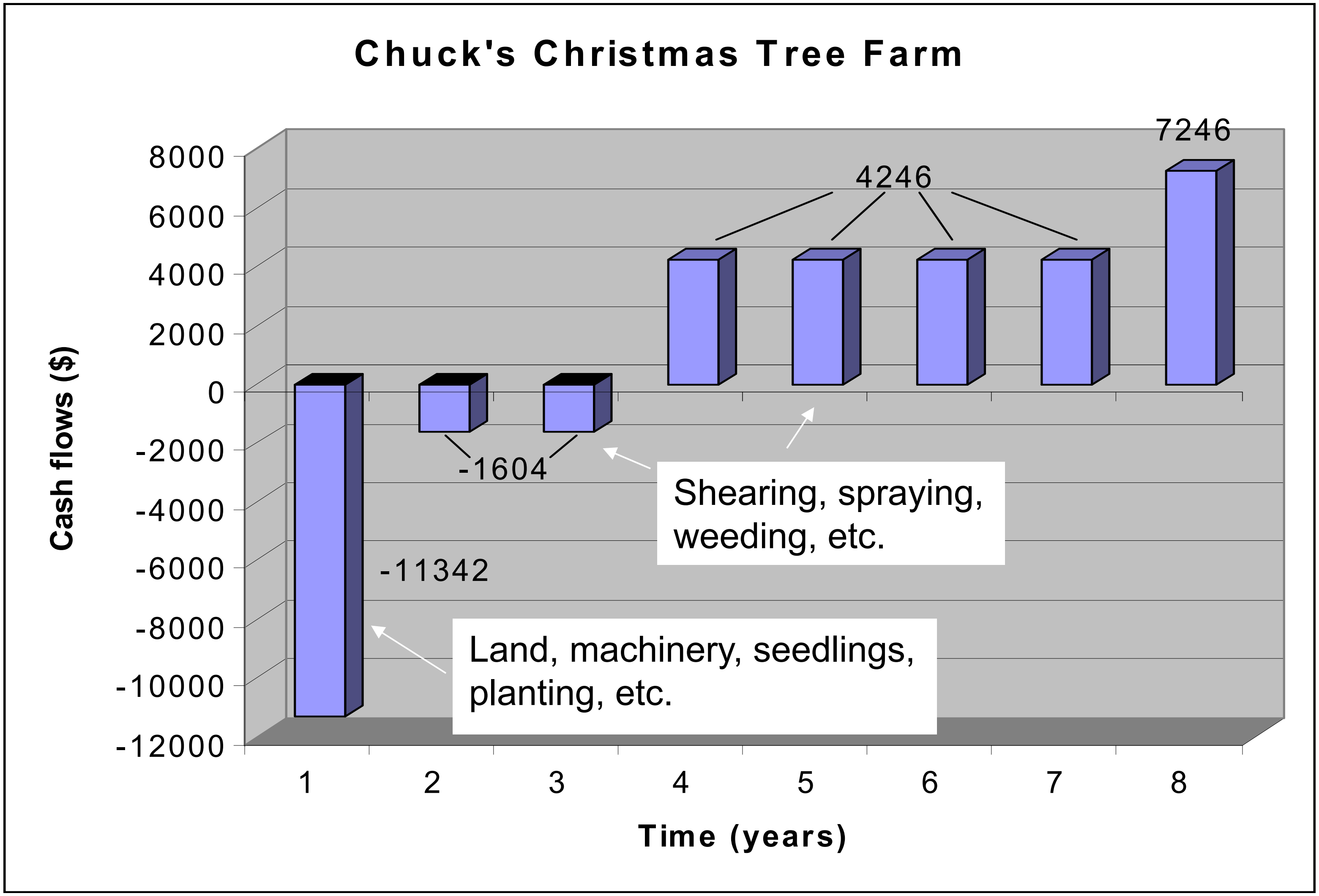
Discounting Multiple Payments/Costs

The Net Present Value (NPV)

- The NPV is the present value of revenues minus the present value of costs:

$$\boxed{\begin{aligned} NPV = {} & \frac{R_1}{(1+i)^1} + \frac{R_2}{(1+i)^2} + \ldots + \frac{R_n}{(1+i)^n} \\ & - \frac{C_1}{(1+i)^1} - \frac{C_2}{(1+i)^2} - \ldots - \frac{C_n}{(1+i)^n} \end{aligned}}$$

Cash flows



Chuck's Christmas Tree Farm
8000
6000
4000
2000
0
-2000
-4000
-6000
-8000
-10000
-12000
Cash flows ($)
7246
4246
-1604
-11342
Shearing, spraying, weeding, etc.
Land, machinery, seedlings, planting, etc.
1
2
3
4
5
6
7
8
Time (years)

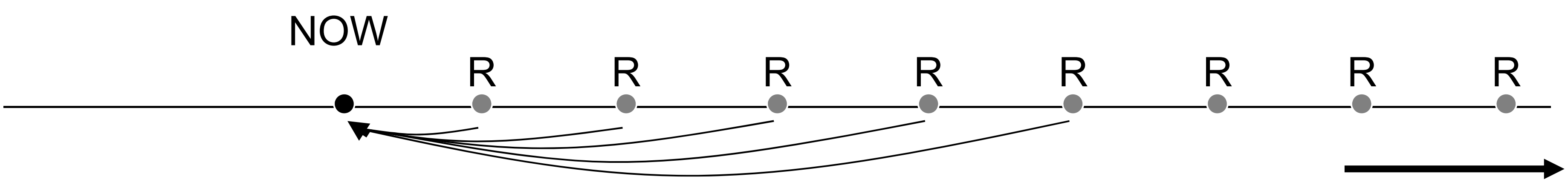
Derivation of the infinite annual series formula

$$V_0 = \frac{R}{1+i} + \frac{R}{(1+i)^2} + \frac{R}{(1+i)^3} + \ldots.$$

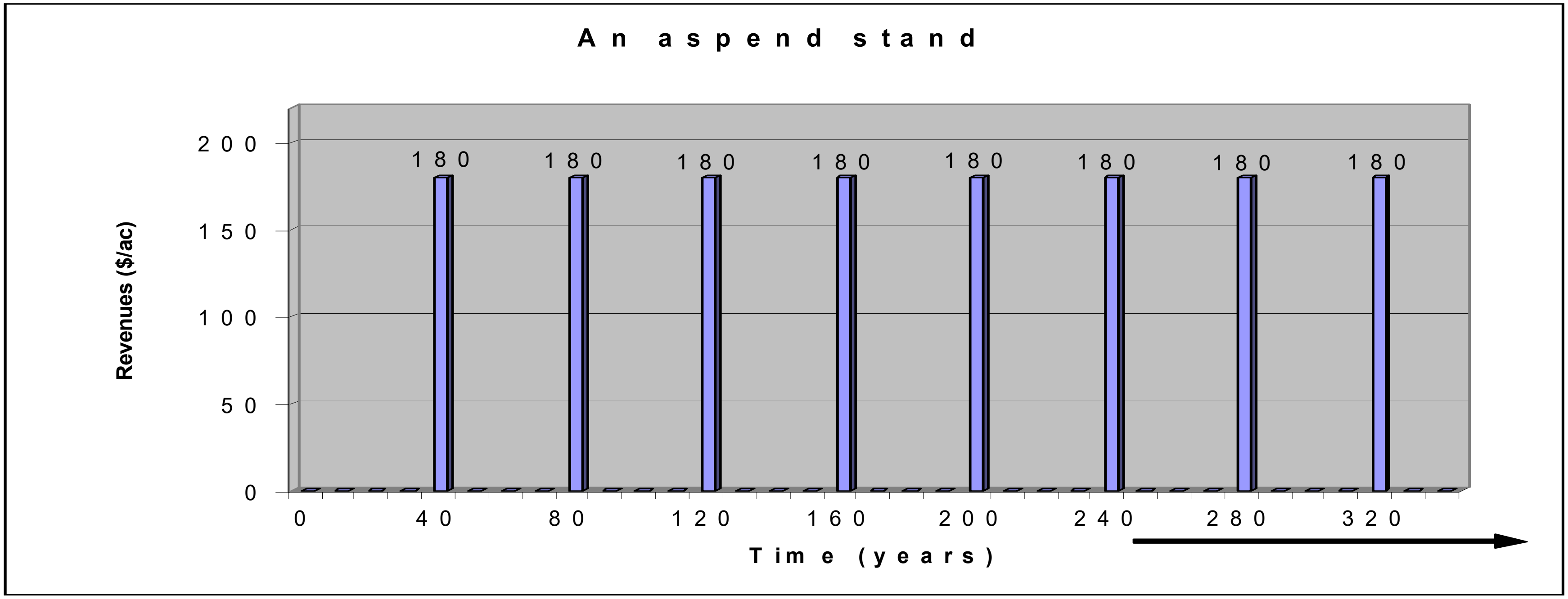
1. Leave $100 in a bank account forever at an interest rate of 5%. How much can you withdraw each year?
2. Answer: $100*0.05=$5/yr
3. In other words: $V_0 i = R \quad \rightarrow V_0 = R / i$

Infinite annual series

- Present value: $\boxed{V_0 = \frac{R}{i}}$
- The payment: $\boxed{R = iV_0}$
- The interest: $\boxed{i = \frac{R}{V_0}}$



Infinite series of periodic payments



Let's use the infinite annual payment formula, but substitute the annual interest rate with a 40 year compound interest rate:

$$V_0 = \frac{R}{i_{40}} = \frac{R}{\frac{R(1+i)^{40} - R}{R}} = \frac{R}{(1+i)^{40} - 1}$$

Infinite periodic series

- Present value: $$\boxed{V_0 = \frac{R}{(1+i)^t - 1}}$$
- The payment: $$\boxed{R = V_0\left[(1+i)^t - 1\right]}$$
- The interest: $$\boxed{i = \sqrt[t]{R/V_0 + 1} - 1}$$

Infinite series of periodic payments (the aspen example)

- So, how much is the present value of the revenues generated by the aspen stand at a 6% interest rate?

- <u>Solution:</u>

$$V_0 = \frac{R}{(1+i)^t - 1} = \frac{\$180}{(1+0.06)^{40} - 1} = \underline{\underline{\$19.38}}$$

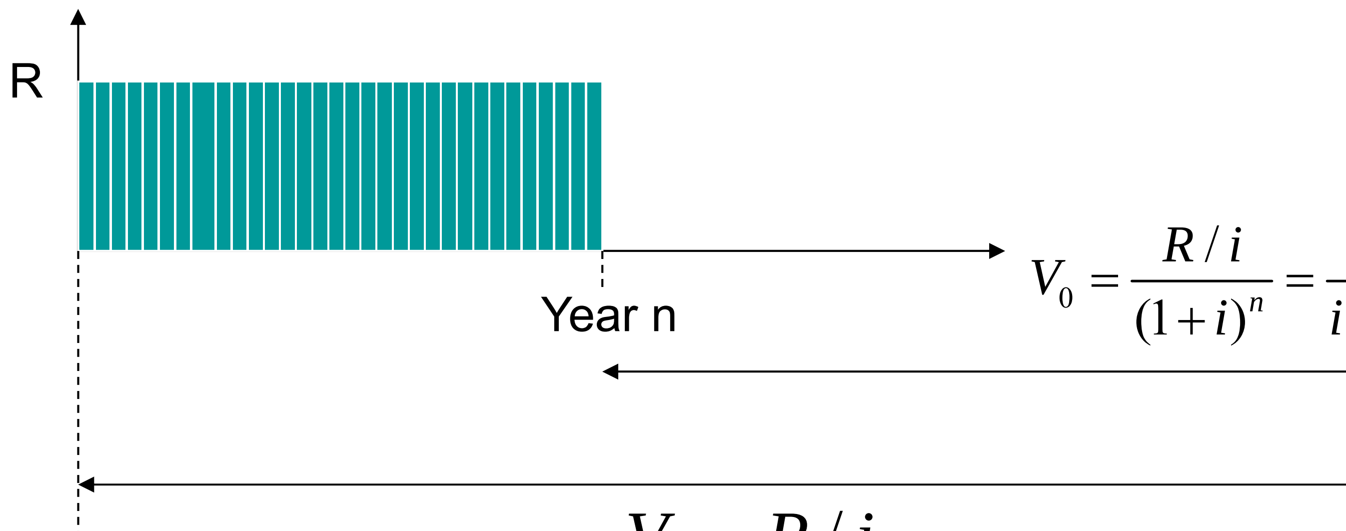
Finite series of annual payments

- Examples:
  - Calculating regular, annual payments on a loan for a fix period of time;
  - Calculating annual rent/tax payments or management costs for a fix period of time;
  - Or, calculating monthly payments.
- Calculating monthly interest rates:

$$i_m = [\sqrt[12]{i+1}] - 1 = (i+1)^{1/12} - 1 \approx \underline{i/12}$$

Finite series of annual payments

- Derivation of the formula:



$$V_0 = \frac{R/i}{(1+i)^n} = \frac{R}{i(1+i)^n}$$

$$V_0 = R/i$$

$$\boxed{V_0 = \frac{R}{i} - \frac{R}{i(1+i)^n} = \frac{R[(1+i)^n - 1]}{i(1+i)^n}}$$

Finite series of annual payments

Present Value: $V_0 = \dfrac{R[(1+i)^n - 1]}{i(1+i)^n}$

Future Value (in year n): $V_n = \dfrac{R[(1+i)^n - 1]}{i}$

Payment to achieve a given Present and Future Value:

$$R = \frac{V_0 i(1+i)^n}{(1+i)^n - 1} = \frac{V_n i}{(1+i)^n - 1}$$

Example

- You want to buy a house in Seattle for \$500,000. You have \$100,000 to put down, so you get a loan with a 5.0% interest. How much would you have to pay each month to pay the loan off within 30 years?

Solution procedure cont.

1. Convert the annual interest rate of 5% to a monthly interest rate

$$i_m = [\sqrt[12]{i+1}] - 1 = [\sqrt[12]{0.05+1}] - 1 =$$

$$= 0.004074 = \underline{0.4074\%}$$

2. Plug in the monthly interest rate in the finite annual payments formula:

$$R_m = \frac{V_0 i_m (1+i_m)^n}{(1+i_m)^n - 1} = \frac{\$400,000 \cdot 0.004074 \cdot (1+0.004074)^{360}}{(1+0.004074)^{360} - 1} =$$

$$= \frac{\$7043.2517}{3.32194} = \underline{\underline{\$2,120.22}}$$

Finite series of periodic payments

- There is a fixed amount (*R*) that you receive or pay every *t* years for *n* years (where *n* is an integer multiple of *t*);
- <u>Example:</u> An intensively managed black locust stand (*Robinia pseudoacacia*) is coppiced three times at 20-year intervals. After the third coppice (at age 60), the stand has to be replanted. At ages 20, 40 and 60 yrs the stand produces $1,000 per acre. Using a 5% discount rate, what would the present value of these harvests be?

Solution procedure

- What do we know?
  1. R_{20}=\$1,000
  2. n=60 yrs, t=20 yrs
  3. i=5%=0.05
- What do we need to know?
  - Present Value (V_0)
- What formula to use?
  - Use the finite annual payment formula with a 20-year compound interest rate.

Solution procedure cont.

- First let's calculate the 20 year compound interest rate:

$$i_{20} = (1+0.05)^{20} - 1 = \underline{165.3298\%}$$

- Plug in the 20-yr interest rate into the finite annual series formula:

$$V_0 = \frac{R[(1+i)^n - 1]}{i(1+i)^n} = \frac{\$1,000[(1+1.6533)^3 - 1]}{1.6533(1+1.6533)^3} =$$

$$= \frac{\$17,679.23436}{30.88238} = \underline{\underline{\$572.47}}$$

Finite periodic payments formula

- In general:

$$\boxed{V_0 = \frac{R[(1+i)^n - 1]}{[(1+i)^t - 1](1+i)^n}}$$

- The payment to achieve a given present value:

$$\boxed{R = \frac{V_0[(1+i)^t - 1](1+i)^n}{[(1+i)^n - 1]}}$$

Discounting with Inflation

Definition

- Inflation: an increase in average price level, reducing the purchasing power of a unit currency (deflation is the reverse process)
- Inflation rate: average annual rate of increase in the price of goods

Measuring Inflation

- Consumer Price Index (CPI)*: measures the average increase in the cost of a standard collection of consumer goods (market basket)
- Producer Price Index (PPI): measures the average increase in the cost of a standard collection of production inputs

*CPI: *the Consumer Price Index for All Urban Consumers (CPI-U) for the U.S. City Average for All Items, 1982-84=100.*

The Average Annual Inflation Rate

$$k = \sqrt[(t_2 - t_1)]{\frac{CPI_{t_2}}{CPI_{t_1}}} - 1$$

- Example: Calculate the average annual inflation rate for the last 30 years (1985-2015)
- Solution: Use the website at http://stats.bls.gov to get CPIs:

$$k = \sqrt[(2015-1985)]{\frac{CPI_{2015}}{CPI_{1985}}} - 1 = \sqrt[30]{\frac{207.8}{105.5}} - 1 = 0.02285 = \underline{\underline{2.285\%}}$$

Components of the Interest Rate

- The nominal rate: includes both the cost of capital and inflation;
- The real rate: is the rate earned on an investment after accounting for inflation. This is the real return for investing one's money.

the nominal rate ≈ the inflation rate + the real rate

$$\underline{i \approx k + r}$$

Combining Interest Rates

- Let i = the nominal rate;
- r = the real rate; and
- k = the inflation rate.

$$i = \frac{\left[R(1+r)\right](1+k) - R}{R} = (1+r)(1+k) - 1 =$$

$$= \underline{\underline{r + k + rk}} \approx r + k$$

$$\boxed{i = r + k + rk; \qquad r = \frac{(1+i)}{(1+k)} - 1; \qquad k = \frac{(1+i)}{(1+r)} - 1}$$

Combining Interest Rates

- <u>Example:</u> You bought a house in 1985 for $120,000. In 2015 it was appraised at $450,000. How much was your real rate of return on this house if the average annual inflation rate between 1985 and 2015 was 2.285%?

Solution:

- Which formula to use? $$r = \frac{(1+i)}{(1+k)} - 1$$

- How do we calculate *i* ?

$$i = \sqrt[(2015-1985)]{\frac{V_{2015}}{V_{1985}}} - 1 = \sqrt[11]{\frac{\$450,000}{\$120,000}} - 1 = \underline{0.045044}$$

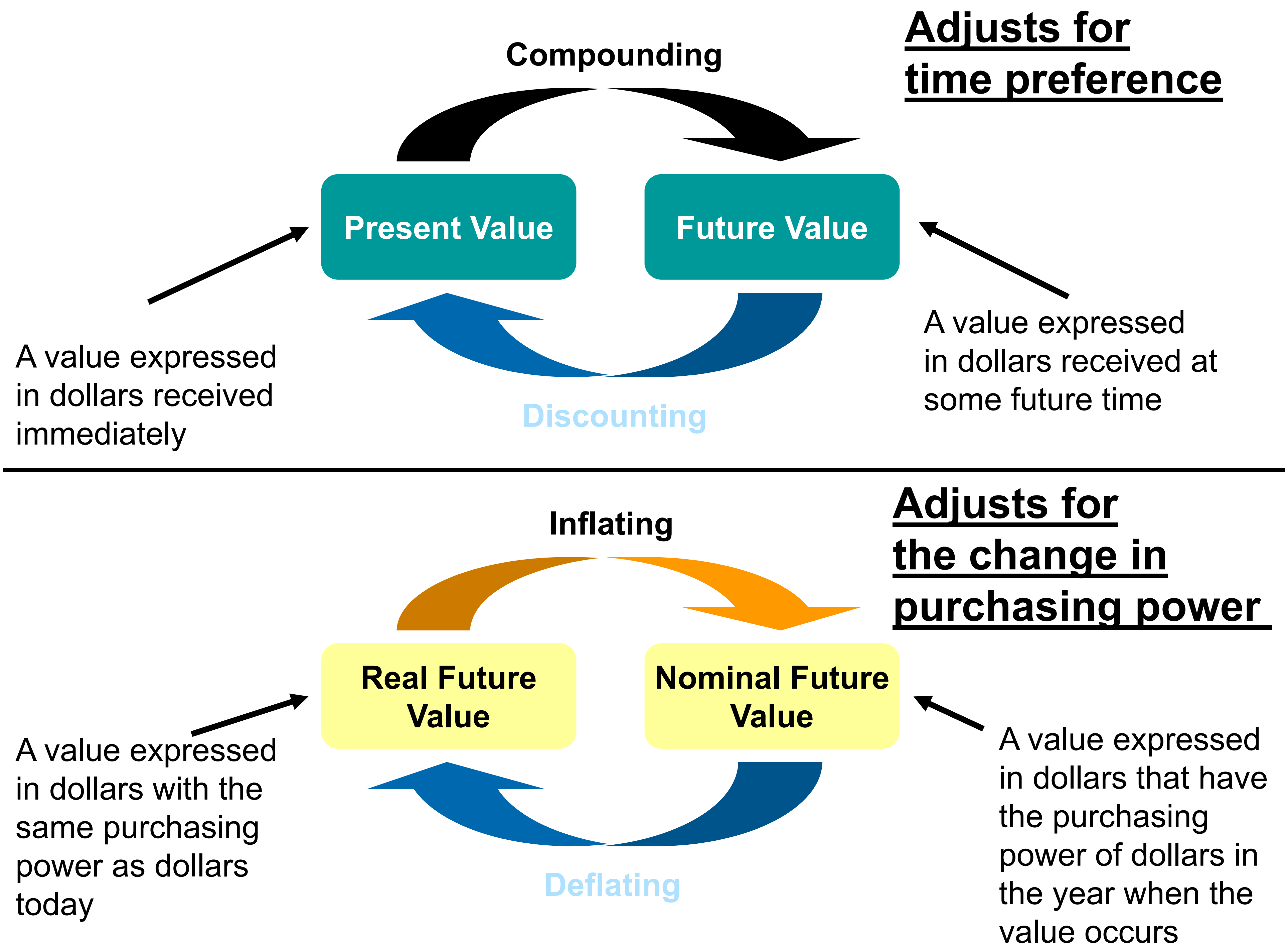
- Calculate *r*:

$$r = \frac{1+0.045044}{1+0.02285} - 1 = 0.0217 \approx \underline{\underline{2.2\%}}$$

Deflating and Inflating

- Deflating: The process of converting a value expressed in the currency of a given point in time into a value expressed in the currency of an earlier time with the same purchasing power ;
- Inflating: is the reverse process.

Note: Historical inflation rates are available to inflate past values to the present.


Adjusts for time preference
Compounding
Present Value
Future Value
Discounting
A value expressed in dollars received immediately
A value expressed in dollars received at some future time
Adjusts for the change in purchasing power
Inflating
Real Future Value
Nominal Future Value
Deflating
A value expressed in dollars with the same purchasing power as dollars today
A value expressed in dollars that have the purchasing power of dollars in the year when the value occurs

Deflating and Inflating

$V_n^* =$ nominal value occuring in year n,

$V_n =$ real value occuring in year n, and

$\mathrm{n}_0 =$ reference year.

$$\boxed{\underline{\text{Real value:}}\;\; V_n = (1+k)^{-(n-n_0)} V_n^*}$$

$$\boxed{\underline{\text{Nominal value:}}\;\; V_n^* = (1+k)^{(n-n_0)} V_n}$$

Note: Deflating/inflating is mathematically same as discounting/compounding but conceptually very different.

Example 1

- How much would a salary of \$69,000 in 2020 be worth in current (2014) dollars if the forecasted average annual inflation rate is 4%?
- <u>Solution:</u>

1. What do we know? $V^*_{2020} = \$69,000,\ n_0 = 2014$
2. What do we need to know? $V_{2020} = ?$
3. Which formula to use?

$$V_{2020} = (1+0.04)^{-(2020-2014)} V^*_{2020} =$$

$$= 1.04^{-6} \cdot \$69,000 = \underline{\underline{\$54,531.70}}$$

Example 2

- How much would a salary of $15,000 in 1976 be worth in current dollars (2014)?
- <u>Solution:</u>

1. What do we know? $V_{1976}^{*} = \$15,000, \ \mathrm{n}_0 = 2014$

2. What do we need to know? $V_{1976} = ?$

3. Which formula to use?

$$V_{1976} = (1+k)^{-(1976-2014)} V_{1976}^{*},$$

$$where \ \ k = \sqrt[2014-1976]{\frac{CPI_{2014}}{CPI_{1976}}} - 1$$

Example 2

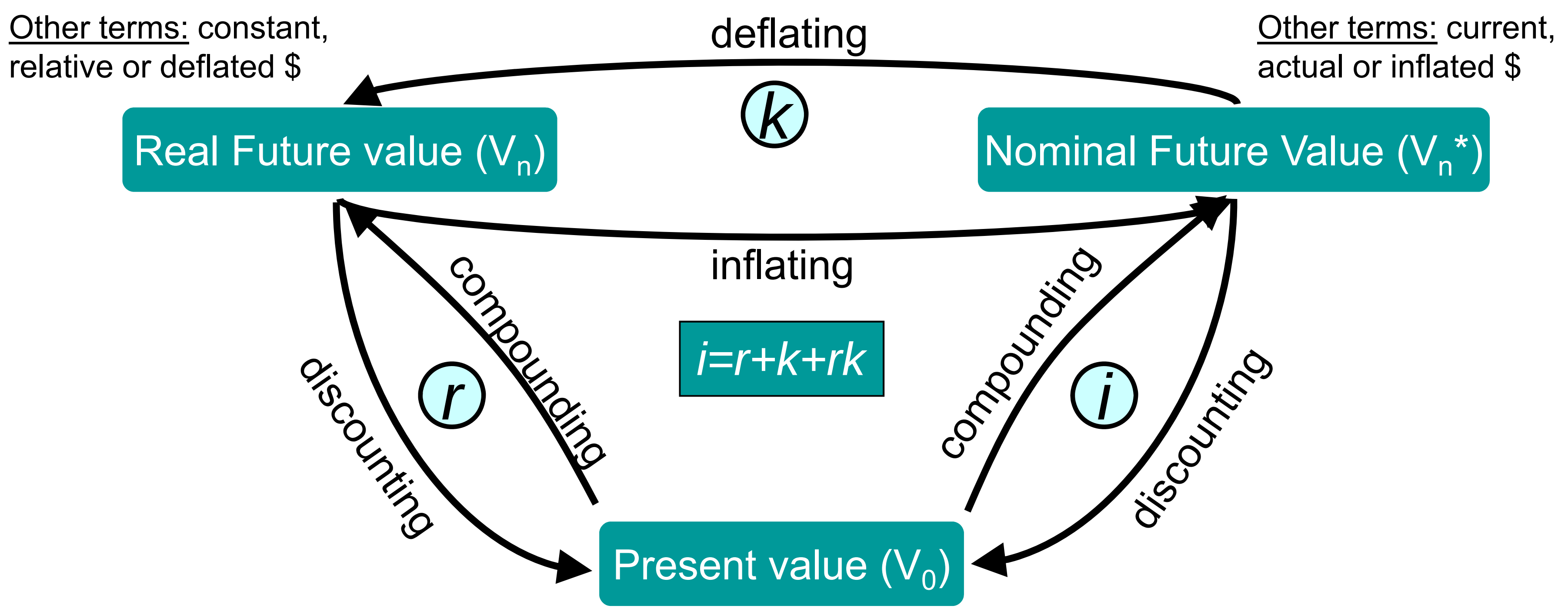
$$V_{1976} = \left(1 + \sqrt[38]{\frac{CPI_{2014}}{CPI_{1976}}} - 1\right)^{38} V_{1976}^{*} = \left(\sqrt[38]{\frac{CPI_{2014}}{CPI_{1976}}}\right)^{38} V_{1976}^{*} =$$

$$= \frac{CPI_{2014}}{CPI_{1976}} \cdot V_{1976}^{*} = \frac{236.3}{56.9} \cdot \$15,000 = \underline{\underline{\$62,214.41}}$$

Rules of discounting with inflation

- Discount nominal future values with a nominal rate and discount real future values with a real rate;
- When a present value is compounded by a real rate, then the result is a real future value;
- When a present value is compounded by a nominal rate, then the result is a nominal future value.

Discounting with inflation

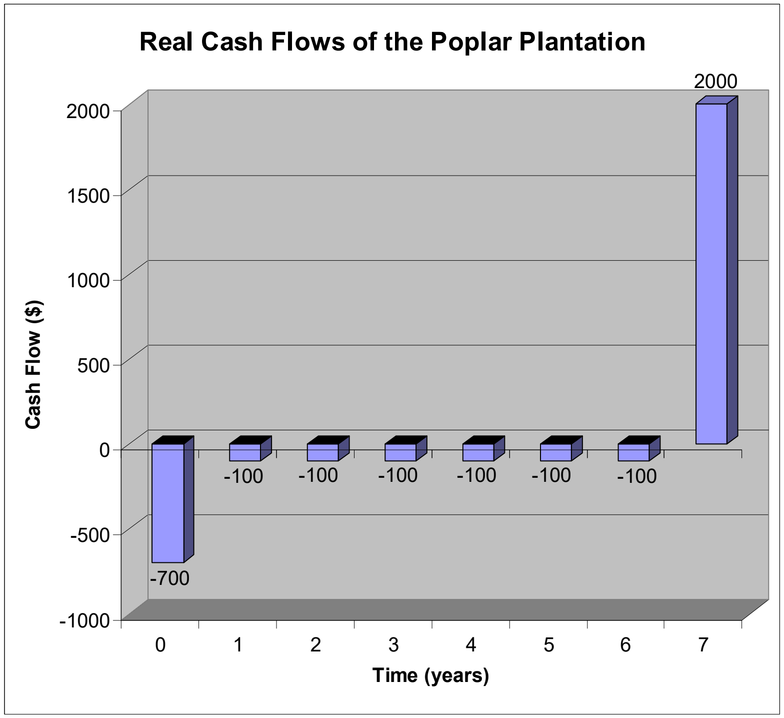


Note: It is often hard to tell if a future value is real or nominal

A hybrid poplar plantation

- The plantation can be established for $600/ac on a land that can be rented for $100/ac/year. You expect the land rent to go up at about the same rate as the inflation rate (=4%/year). After 7 years, the plantation will produce 20 tons of chips per acre. The current price for chips is $100/ton and you expect this price to go up at the rate of the inflation. What is the present value of the poplar project at an 8% real interest rate?

➢ The cash flows



Year	Future Values		Present Values ($)
	Real (constan values)	Nominal (current values)	
0	-700		
1	-100		
2	-100		
3	-100		
4	-100		
5	-100		
6	-100		
7	2000		
NPV	N/A	N/A	

k = 4%

r = 8%

$$i = r + k + rk = 0.08 + 0.04 + 0.08 \cdot 0.04 =$$

$$= 0.1232 = \underline{12.32\%}$$