Forest Management with Linear Programming

Lecture 4 (4/11/2016)

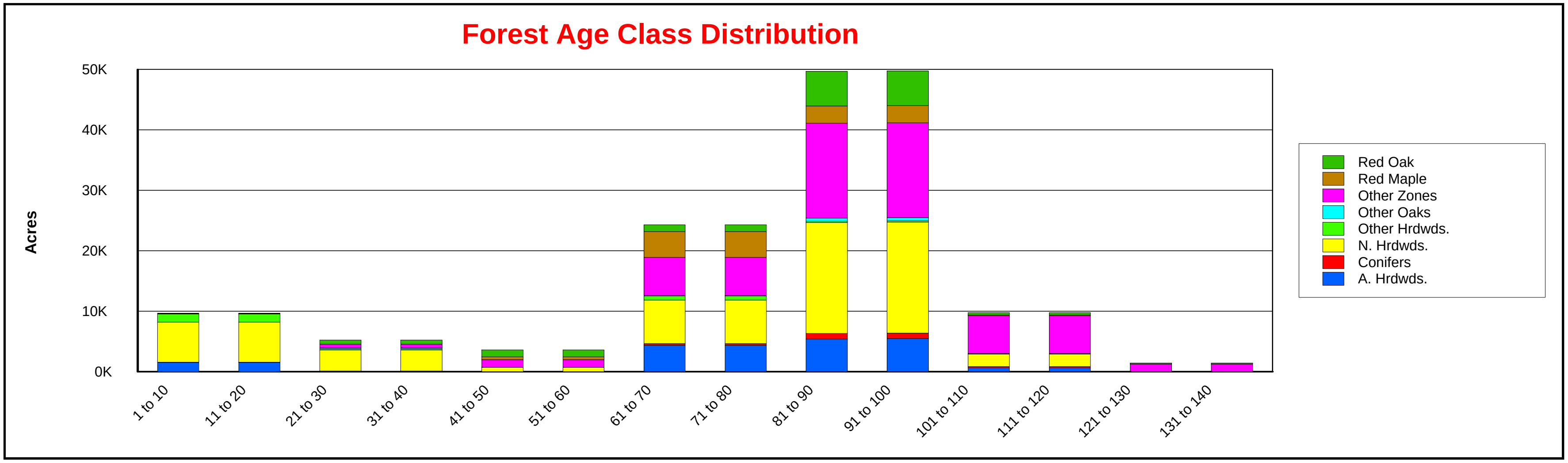
What is forest management?

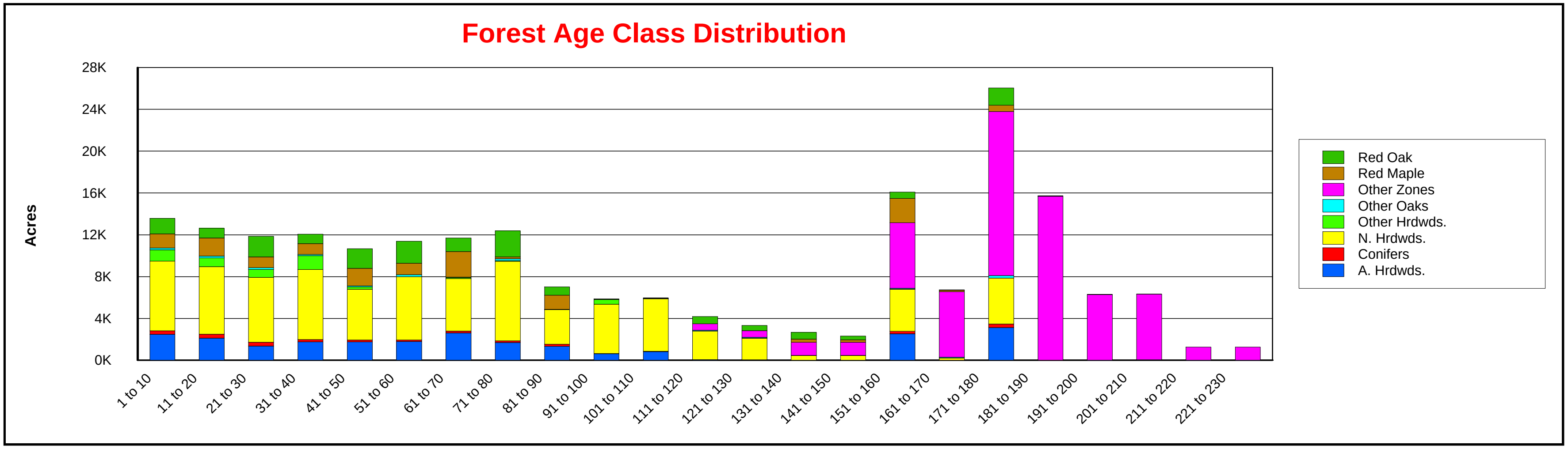
- “Identifying and selecting management alternatives for forested areas, large and small, to best meet landowner objectives within the constraints of the law and the ethical obligations of the landowner to be a responsible steward of the land.” (McDill’s Fores Resource Management)

Key points of the definition

- Forest Management must be driven by landowner objectives;
- Resource professionals can only determine how these objectives are best met, but they cannot define the objectives themselves;
- What is best for the forest? vs. What is best for the people?
- Landowners have property rights as well as moral/ethical obligations to be good stewards.

Balancing the Age-class Distribution



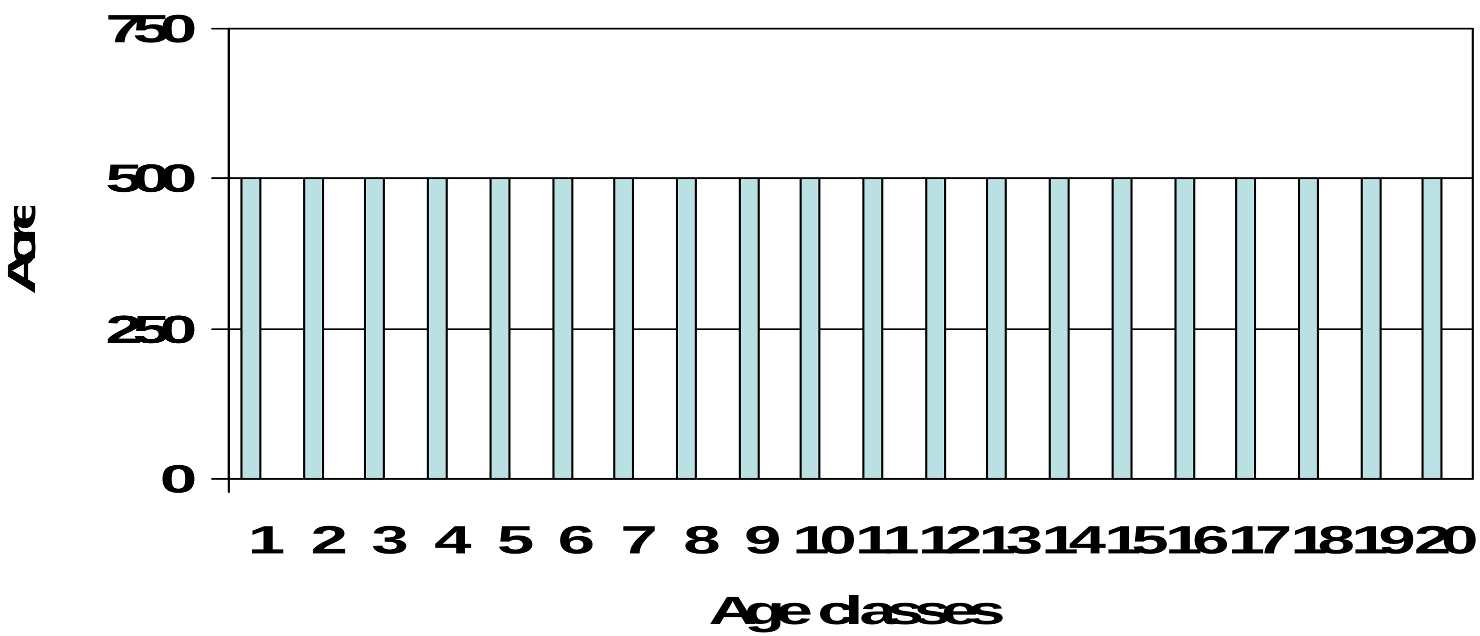


The regulated forest

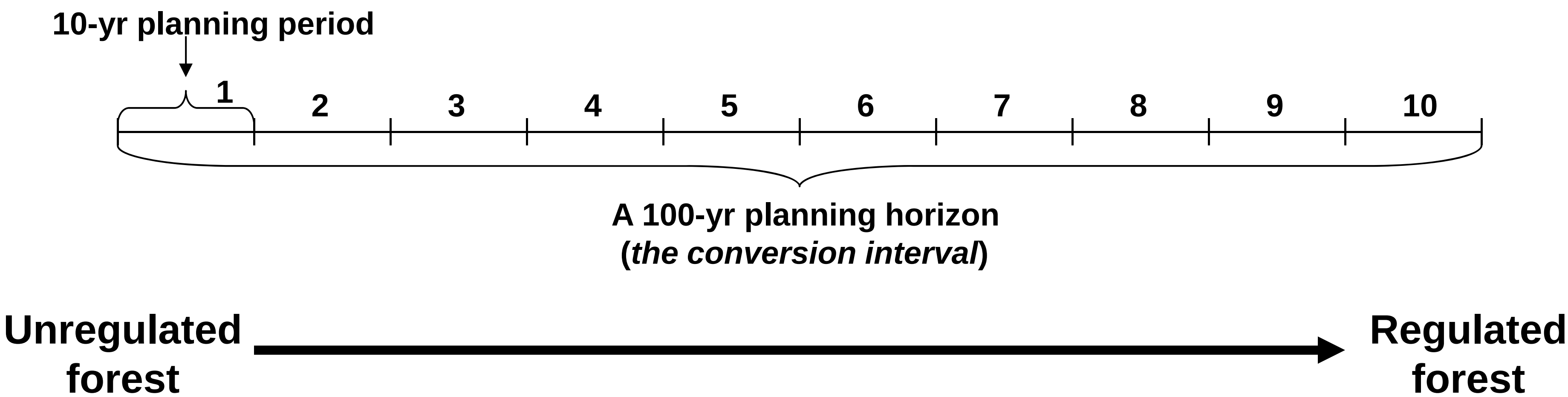
- Regulated forest: a forest with an equal number of acres in each age class
- Forest regulation: the process of converting a forest with an unbalanced age-class distribution into a regulated forest
- The purpose of forest regulation is to achieve a state where an even flow of products can be produced in perpetuity

The regulated forest cont.

- A regulated forest provides sustainability and stability
- In a regulated forest, the oldest age class is harvested each year, the age-class distribution is maintained



Planning periods and the planning horizon



$$Since\ A_1 = A_2 = A_3 = \ldots = A_R \quad \Rightarrow \quad A_i = \frac{A}{R} \times n$$

Period length

Rotation age

Area in each age class

Methods of Forest Regulation

- How to get from existing forest with an unbalanced age-class distribution to a regulated forest?
- The *area* and *volume control* focus on cutting a target area (or volume) in each period
- Linear Programming

Developing a harvest scheduling model: an example

Initial age-class distribution

Age Classes	Acres by site class	
	Site I	Site II
0-10	3,000	8,000
11-20	6,000	4,000
21-30	9,000	7,000
Total	18,000	19,000

Analysis area

Yield

Harvest Age	Volume (cds) by site class	
	Site I	Site II
10	2	5
20	10	14
30	20	27
40	31	38
50	37	47
60	42	54
70	46	60

If the acres that are initially in age-class 0-10 are to be cut in period 2, they will be 5+15=20 yrs old at the time of harvest.

Economic Data

Economic data

Item	Symbol	Amount
Wood Price	P	**$25/cd**
Planting Cost	E	**$100.00/ac**
Timber Sales Cost **-per acre** **-per cord**	 s_f s_v	 **$15.00/ac** **$0.20/cd**
Interest Rate	r	**4%**

LEV

Harvest Age	LEV	
	Site I	**Site II**
20	**$11.66**	**$94.94**
30	**$69.83**	**$147.21**
40	**$72.01**	**$117.68**
50	**$31.43**	**$72.04**
60	**-$2.66**	**$28.60**
R*(yr)	**40**	**30**

$$LEV_{R,s} = \frac{(P - s_v)Y_{R,s} - s_f - E(1+r)^R}{(1+r)^R - 1}$$

Formulating the example problem as a cost minimization LP

1. Define variables

$X_{sap} =$ the number of acres to cut from site class s (where s=1,2,3) and initial age-class a (where a=1,2 or 3) in period p (where p=0,1,2,3 and p=0 means no harvest during the planning horizon)

Example: $X_{231} =$ the number of acres from site class 2, initial age-class 3 to be cut in period 1.

2. Formulating the objective-function

$$Min\ Z=\sum_{s=1}^{2}\sum_{a=1}^{3}\sum_{p=0}^{3} c_{sap}\cdot X_{sap}$$

Where c_{sap}= the present value of the cost of assigning one acre to the variable X_{sap}.

$$\begin{aligned} Min\ Z = {} & c_{110}\cdot X_{110}+c_{111}\cdot X_{111}+c_{112}\cdot X_{112}+c_{113}\cdot X_{113}+ \\ & c_{120}\cdot X_{120}+c_{121}\cdot X_{121}+c_{122}\cdot X_{122}+c_{123}\cdot X_{123}+ \\ & c_{130}\cdot X_{130}+c_{131}\cdot X_{131}+c_{132}\cdot X_{132}+c_{133}\cdot X_{133}+ \\ & c_{210}\cdot X_{210}+c_{211}\cdot X_{211}+c_{212}\cdot X_{212}+c_{213}\cdot X_{213}+ \\ & c_{220}\cdot X_{220}+c_{221}\cdot X_{221}+c_{222}\cdot X_{222}+c_{223}\cdot X_{223}+ \\ & c_{230}\cdot X_{230}+c_{231}\cdot X_{231}+c_{232}\cdot X_{232}+c_{233}\cdot X_{233} \end{aligned}$$

- Calculating c_{sap}:

$$c_{231} = \frac{E + s_f + s_v \cdot v_{231}}{(1+r)^5} = \frac{\$100/ac + \$15/ac + \$0.20/cd \cdot 27cd/ac}{(1.04)^5} =$$

$$= \underline{\underline{\$98.96/ac}}$$

<u>The general formula</u>: $c_{sap} = \dfrac{E + s_f + s_v v_{sap}}{(1+r)^{10p-5}}$

<u>Note:</u> If p=0 then c_{sap}=0

Calculating the harvest ages and volumes

Initial Age Class	Harvest Age			Volume – Site I			Volume – Site II		
	Harvest period								
	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3
0-10	10	20	30	v_{111}=2	v_{112}=10	v_{113}=20	v_{211}=5	v_{212}=14	v_{213}=27
11-20	20	30	40	v_{121}=10	v_{122}=20	v_{123}=31	v_{221}=14	v_{222}=27	v_{223}=38
21-30	30	40	50	v_{131}=20	v_{132}=31	v_{133}=37	v_{231}=27	v_{232}=38	v_{233}=47

3. Formulating the constraints

a) Area constraints (for each analysis area): *you cannot manage more acres than what you have*

$$X_{110} + X_{111} + X_{112} + X_{113} \leq 3,000$$

$$X_{120} + X_{121} + X_{122} + X_{123} \leq 6,000$$

$$X_{130} + X_{131} + X_{132} + X_{133} \leq 9,000$$

$$X_{210} + X_{211} + X_{212} + X_{213} \leq 8,000$$

$$X_{220} + X_{221} + X_{222} + X_{223} \leq 4,000$$

$$X_{230} + X_{231} + X_{232} + X_{233} \leq 7,000$$

b) Harvest target constraints: *Require the production of some minimum timber output in each period*

The harvest target for the first decade with Hundeshagen's formula:

$$H_1 = \frac{375,500ac}{419,583ac} \times 31,050cd/yr = \underline{27,788cd/yr} \approx 280,000cd/period$$

The harvest target for the second decade: $(H_1+LTSY)/2$=295,000cd/period

The harvest target constraint for period 1:

$$\sum_{s=1}^{2}\sum_{a=1}^{3} v_{sa1} \cdot X_{sa1} \geq H_1$$

The specific harvest target constraints for periods 1-2:

$$2X_{111} + 10X_{121} + 20X_{131} + 5X_{211} + 14X_{221} + 27X_{231} \geq 280,000$$

$$10X_{112} + 20X_{122} + 31X_{132} + 14X_{212} + 27X_{222} + 38X_{232} \geq 295,000$$

b) Ending age constraints: *One way to ensure that the forest that is left standing at the end of the planning horizon will be of a desirable condition*

Several alternative constraints exist:

i. Target ending age-class distribution

ii. Target ending inventory

iii. <u>Average ending age constraints</u>

Calculating the average age of a forest (example):

Age Class	Avg. Age	Acres
0-10	5	300
11-20	15	100
21-30	25	250

$$\overline{Age} = \frac{300 \cdot 5 + 100 \cdot 15 + 250 \cdot 25}{300 + 100 + 250} = 14.23 years$$

<u>In general</u>: $\overline{Age} = \sum_{i=1}^{n} \frac{Area_i}{\sum_{j=1}^{n} Area_j} Age_i$

Ending age constraints cont.

Age_{sap}^{30} = the age in year 30 of acres in site class s, initial age class a that are scheduled to be harvested in period p (where p=0 implies no harvest during the planning horizon)

$$\overline{Age}^{30} = \frac{\sum_{s=1}^{2}\sum_{a=1}^{3}\sum_{p=0}^{3} Age_{sap}^{30} \cdot X_{sap}}{\sum_{s=1}^{2}\sum_{a=1}^{3}\sum_{p=0}^{3} X_{sap}} = \frac{\sum_{s=1}^{2}\sum_{a=1}^{3}\sum_{p=0}^{3} Age_{sap}^{30} \cdot X_{sap}}{\text{Total Area}}$$

Where: $\overline{Age}^{30}$ = the average age of the forest in year 30.

Ending age constraints cont.

This leads to the general form of the constraint:

$$\sum_{s=1}^{2}\sum_{a=1}^{3}\sum_{p=0}^{3}\mathrm{Age}_{\mathrm{sap}}^{30}\cdot X_{sap} \geq \overline{Age}^{30}\cdot TotalArea$$

Where $\overline{Age}^{30}$ = the target minimum average age of the forest in year 30

Ending age constraints cont.

Calculating the Age_{sap}^{30} coefficients:

Initial Age Class	Site I				Site II			
	Harvest Period							
	Not Cut	Period 1	Period 2	Period 3	Not Cut	Period 1	Period 2	Period 3
0-10	$Age_{110}^{30}=35$	$Age_{111}^{30}=25$	$Age_{112}^{30}=15$	$Age_{113}^{30}=5$	$Age_{210}^{30}=35$	$Age_{211}^{30}=25$	$Age_{212}^{30}=15$	$Age_{213}^{30}=5$
11-20	$Age_{120}^{30}=45$	$Age_{121}^{30}=25$	$Age_{122}^{30}=15$	$Age_{123}^{30}=5$	$Age_{220}^{30}=45$	$Age_{221}^{30}=25$	$Age_{222}^{30}=15$	$Age_{223}^{30}=5$
21-30	$Age_{130}^{30}=55$	$Age_{131}^{30}=25$	$Age_{132}^{30}=15$	$Age_{133}^{30}=5$	$Age_{230}^{30}=55$	$Age_{231}^{30}=25$	$Age_{232}^{30}=15$	$Age_{233}^{30}=5$

How should we set the minimum average ending age: $\overline{Age}^{30}$

What is the average age of the regulated forest? (R*+1)/2

Site I: (40+1)/2= 20.5
Site II: (30+1)/2=15.5
Average for the two sites: 17.93yrs

Ending age constraints cont.

$$35X_{110} + 25X_{111} + 15X_{112} + 5X_{113} + 45X_{120} + 25X_{121} + 15X_{122} + 5X_{123} +$$
$$55X_{130} + 25X_{131} + 15X_{132} + 5X_{133} + 35X_{210} + 25X_{211} + 15X_{212} + 5X_{213} +$$
$$45X_{220} + 25X_{221} + 15X_{222} + 5X_{223} + 55X_{230} + 25X_{231} + 15X_{232} + 5X_{233} \geq 629,000$$

Non-negativity Constraints

$X_{sap} \geq 0$ s=1,2 ; a=1,2,3 ; p=0,1,2,3

Harvest Scheduling with Profit Maximization

- No harvest targets are necessary
- More flexibility
- One can let the model tell how much to harvest
- Disadvantage: the price of wood must be projected for the duration of the planning horizon

The objective function:

$$Max\ \ Z = \sum_{s=1}^{2}\sum_{a=1}^{3}\sum_{p=0}^{2} c_{sap}^{p} \cdot X_{sap}$$

$where\ \ c_{sap}^{p} = the$ present value of the net revenue of assigning one acre to variable X_{sap}

$$c_{sap}^{p} = \begin{cases} \dfrac{P \cdot v_{sap} - [E + s_f + s_v \cdot v_{sap}]}{(1+r)^{10p-5}} = \dfrac{(P - s_v) \cdot v_{sap} - E - s_f}{(1+r)^{10p-5}} & \text{for p>0} \\ 0 & \text{for p=0} \end{cases}$$

Example:

$$c_{231}^{p} = \frac{(P - s_v)\cdot v_{231} - E - s_f}{(1+r)^{10\cdot 1-5}}$$

$$= \frac{(\$25/cd - \$0.2/cd)\cdot 27cd/ac - \$100/ac - \$15/ac}{1.04^5}$$

$$= \underline{\$455.84/ac}$$

$$\begin{aligned} Max\ \ Z = & -53.75X_{111} + 73.85X_{112} + 109.32X_{121} + 211.56X_{122} \\ & + 313.15X_{131} + 363.03X_{132} + 7.40X_{211} + 128.93X_{212} \\ & + 190.85X_{221} + 307.95X_{222} + 455.84X_{231} + 459.43X_{232} \end{aligned}$$

Constraints

- Area constraints (same as in the cost minimization problem):

$$\sum_{p=0}^{2} X_{sap} \leq A_{sa} \qquad \text{for s=1,2 and a=1,2,3}$$

$$X_{110} + X_{111} + X_{112} \leq 3,000$$
$$X_{120} + X_{121} + X_{122} \leq 6,000$$
$$X_{130} + X_{131} + X_{132} \leq 9,000$$
$$X_{210} + X_{211} + X_{212} \leq 8,000$$
$$X_{220} + X_{221} + X_{222} \leq 4,000$$
$$X_{230} + X_{231} + X_{232} \leq 7,000$$

Constraint cont.

- Harvest fluctuation constraints: limit the amount the harvest can go up or down from one period to the next and ensures an even flow of timber from the forest

$$H_2 \geq 0.85 \cdot H_1 \qquad \text{and} \qquad H_2 \leq 1.15 \cdot H_1$$

The harvest target constraint from cost minimization:

$$\sum_{s=1}^{2} \sum_{a=1}^{3} v_{sap} \cdot X_{sap} \geq H_p \qquad \text{for p=1,2}$$

The harvest accounting constraint for profit maximization:

$$\sum_{s=1}^{2} \sum_{a=1}^{3} v_{sap} \cdot X_{sap} - H_p = 0 \qquad \text{for p=1,2}$$

Constraints cont.

- Harvest fluctuation constraints cont.

$$0.85H_1 - H_2 \leq 0$$
$$-1.15H_1 + H_2 \leq 0$$

$$2X_{111} + 10X_{121} + 20X_{131} + 5X_{211} + 14X_{221} + 27X_{231} - H_1 = 0$$
$$10X_{112} + 20X_{122} + 31X_{132} + 14X_{212} + 27X_{222} + 38X_{232} - H_2 = 0$$

Constraints cont.

- Average ending age constraints (same as in cost minimization):

$$\sum_{s=1}^{2}\sum_{a=1}^{3}\sum_{p=0}^{2} Age_{sap}^{20} \times X_{sap} \geq \overline{Age}^{20} \times TotalArea$$

$$\begin{aligned}
&25X_{110} + 15X_{111} + 5X_{112} + 35X_{120} + 15X_{121} + 5X_{122} \\
&+45X_{130} + 15X_{131} + 5X_{132} + 25X_{210} + 15X_{211} + 5X_{212} \\
&+35X_{220} + 15X_{221} + 5X_{222} + 45X_{230} + 15X_{231} + 5X_{232} \geq 629,000
\end{aligned}$$

Constraints cont.

- Non-negativity constraints:

  $X_{sap} \geq 0$ for s=1,2 a=1,2,3 p=0,1,2

  and $H_p \geq 0$ for p=1,2

$$
\begin{aligned}
Max\ \ Z = {} & -53.75X_{111} + 73.85X_{112} + 109.32X_{121} + 211.56X_{122} \\
& + 313.15X_{131} + 363.03X_{132} + 7.40X_{211} + 128.93X_{212} \\
& + 190.85X_{221} + 307.95X_{222} + 455.84X_{231} + 459.43X_{232}
\end{aligned}
$$

subject to

$$X_{110} + X_{111} + X_{112} \le 3,000$$

$$X_{120} + X_{121} + X_{122} \le 6,000$$

$$X_{130} + X_{131} + X_{132} \le 9,000$$

$$X_{210} + X_{211} + X_{212} \le 8,000$$

$$X_{220} + X_{221} + X_{222} \le 4,000$$

$$X_{230} + X_{231} + X_{232} \le 7,000$$

$$0.85H_1 - H_2 \le 0$$

$$-1.15H_1 + H_2 \le 0$$

$$2X_{111} + 10X_{121} + 20X_{131} + 5X_{211} + 14X_{221} + 27X_{231} - H_1 = 0$$

$$10X_{112} + 20X_{122} + 31X_{132} + 14X_{212} + 27X_{222} + 38X_{232} - H_2 = 0$$

$$
\begin{aligned}
& 25X_{110} + 15X_{111} + 5X_{112} + 35X_{120} + 15X_{121} + 5X_{122} \\
& +45X_{130} + 15X_{131} + 5X_{132} + 25X_{210} + 15X_{211} + 5X_{212} \\
& +35X_{220} + 15X_{221} + 5X_{222} + 45X_{230} + 15X_{231} + 5X_{232} \ge 629,000
\end{aligned}
$$

$X_{sap} \ge 0$ for s=1,2 a=1,2,3 p=0,1,2

and $H_p \ge 0$ for p=1,2

ITERATIONS BY SIMPLEX METHOD = 14
ITERATIONS BY BARRIER METHOD = 0
ITERATIONS BY NLP METHOD = 0
TIME ELAPSED (s) = 0

OBJECTIVE FUNCTION VALUE

1) 7994986.451612903

VARIABLES	VALUE	REDUCED COST
X111	0.000000000	-128.447935484
X112	0.000000000	-68.854516129
X121	0.000000000	-42.622580645
X122	2831.854838710	-0.000000000
X131	5895.564516129	0.000000000
X132	3104.435483871	0.000000000
X211	0.000000000	-68.571290323
X212	0.000000000	-11.777096774
X221	0.000000000	-63.340870968
X222	4000.000000000	-0.000000000
X231	7000.000000000	0.000000000
X232	0.000000000	-39.823354839
X110	3000.000000000	0.000000000
X120	3168.145161290	0.000000000
X130	0.000000000	-83.113870968
X210	8000.000000000	0.000000000
X220	0.000000000	-99.885483871
X230	0.000000000	-222.832709677
H1	306911.290322581	0.000000000
H2	260874.596774193	0.000000000

CONSTRAINTS	SLACK OR SURPLUS	DUAL PRICES
AreaX11	0.000000000	184.622580645
AreaX12	0.000000000	258.471612903
AreaX13	0.000000000	415.434516129
AreaX21	0.000000000	184.622580645
AreaX22	0.000000000	358.357096774
AreaX23	0.000000000	555.153354839
H1	0.000000000	0.499354839
H2	92073.387096774	0.000000000
H1Ac	-0.000000000	0.424451613
H2Ac	-0.000000000	-0.499354839
EA	-0.000000000	-7.384903226

END OF REPORT