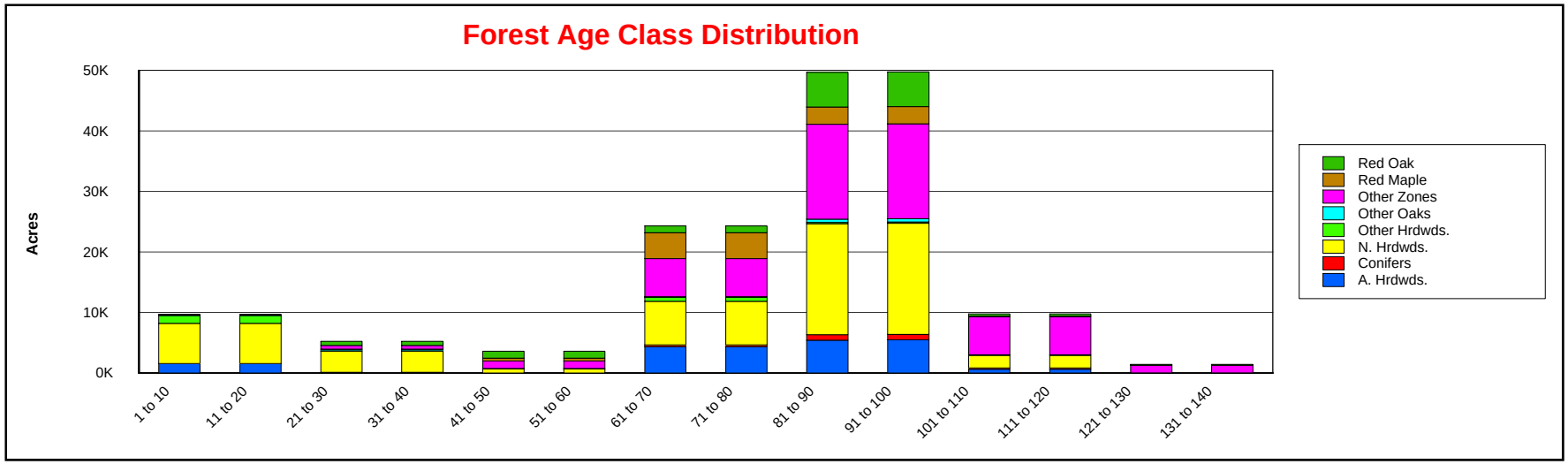
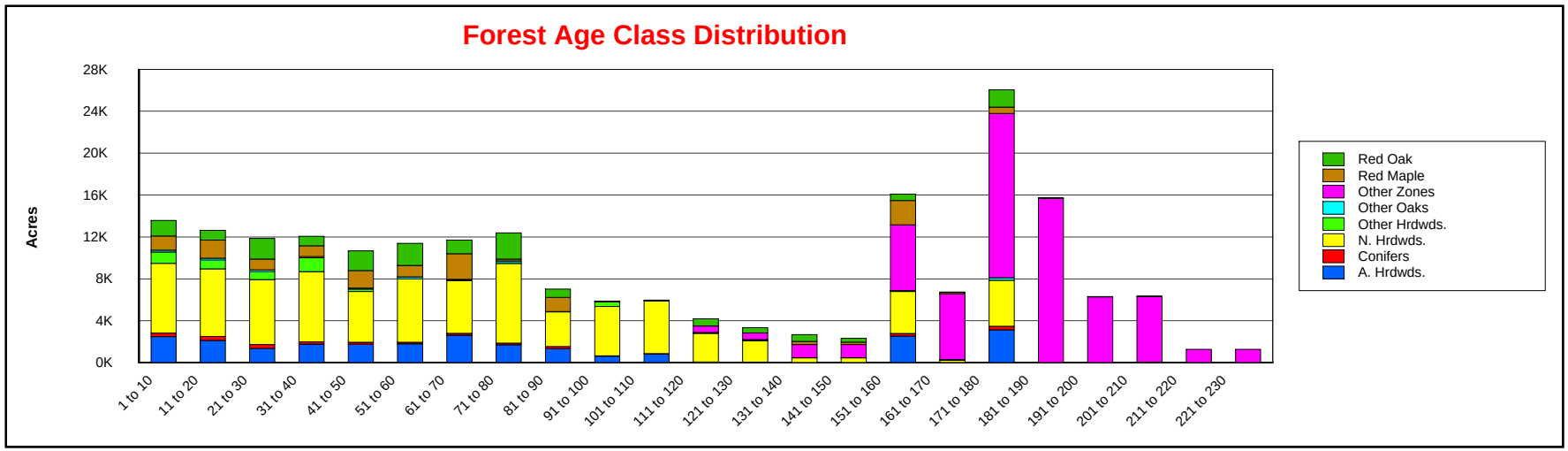
Modeling Non-Timber Objectives in Harvest Scheduling with Linear Programming

Lecture 4 (4/8/2014)

- Extended rotations: a minimum of 1,500ac will be required from each Site I and II to be older than 40 and 30 yrs, respectively;
- Wildlife openings: set aside 500ac of wildlife openings – clearcut these in pd. 1 and maintain them for browse and forage;
- Stream-side management zones: no harvests allowed in 8% of the forest (4% in each site class)

Balancing the Age-class Distribution





Harvest scheduling prescriptions with up to two possible harvests within a 40-year planning horizon and a minimum rotation of 20 yrs

Prescription	Planning Period				
	1	2	3	4	
1	Harvest				X_{sa10}
2	Harvest		Harvest		X_{sa13}
3	Harvest			Harvest	X_{sa14}
4		Harvest			X_{sa20}
5		Harvest		Harvest	X_{sa24}
6			Harvest		X_{sa30}
7				Harvest	X_{sa40}
8	No Harvest				X_{sa00}

$X_{sap_1p_2}$ = the number of acres from site class s, initial age-class a assigned to be harvested first in period p_1 and again in period p_2.

Extended Rotations

Period	Variable							
	X_{sa10}	X_{sa13}	X_{sa14}	X_{sa20}	X_{sa24}	X_{sa30}	X_{sa40}	X_{sa00}
1	0	0	0	a	a	a	a	a
2	1	1	1	0	0	a+1	a+1	a+1
3	2	0	2	1	1	0	a+2	a+2
4	3	1	0	2	0	1	0	a+3

Initial age-class distribution

Age Classes	Acres by site class	
	Site I	Site II
0-10	3,000	8,000
11-20	6,000	4,000
21-30	9,000	7,000
Total	18,000	19,000

$X_{2330} + X_{2340} + X_{2300} \geq 1,500$ (ext. rot. constraint for site class II, pd.2)

$X_{2240} + X_{2200} + X_{2340} + X_{2300} \geq 1,500$ (ext. rot. constraint for site class II, pd.3)

$X_{1340} + X_{1300} \geq 1,500$ (ext. rot. constraint for site class I, pd.3)

$X_{2100} + X_{2200} + X_{2300} \geq 1,500$ (ext. rot. constraint for site class II, pd.4)

$X_{1200} + X_{1300} \geq 1,500$ (ext. rot. constraint for site class I, pd.4)

Modifying the Average Ending Age Constraint

$$\sum_{a=1}^{3}\left[\mathrm{Age}_{\mathrm{sa}00}^{40}\cdot X_{sa00}+\sum_{p_1=1}^{4}\left(\mathrm{Age}_{\mathrm{sap}_1 0}^{40}\cdot X_{\mathrm{sap}_1 0}+\sum_{p_2=p_1+2}^{4}\mathrm{Age}_{\mathrm{sap}_1\mathrm{p}_2}^{40}\cdot X_{\mathrm{sap}_1\mathrm{p}_2}\right)\right]\geq \overline{Age}^{40}\cdot TotalArea_s$$

$$TotalArea_s=\sum_{a=1}^{3}\left[X_{sa00}+\sum_{p_1=1}^{4}\left(X_{sap_1 0}+\sum_{p_2=p_1+2}^{4}X_{sap_1 p_2}\right)\right]$$

$$\overline{Age}^{40}\times TotalArea_s=\overline{Age}^{40}\times\sum_{a=1}^{3}\left[X_{sa00}+\sum_{p_1=1}^{4}\left(X_{sap_1 0}+\sum_{p_2=p_1+2}^{4}X_{sap_1 p_2}\right)\right]=$$

$$\sum_{a=1}^{3}\left[\overline{Age}^{40}X_{sa00}+\sum_{p_1=1}^{4}\left(\overline{Age}^{40}X_{sap_1 0}+\sum_{p_2=p_1+2}^{4}\overline{Age}^{40}X_{sap_1 p_2}\right)\right]$$

Modifying the Average Ending Age Constraint (continued)

$$\sum_{a=1}^{3}\left[\text{AgeDif}_{sa00}^{40}\cdot X_{sa00}+\sum_{p_1=1}^{4}\left(\text{AgeDif}_{sap_10}^{40}\cdot X_{sap_10}+\sum_{p_2=p_1+1}^{4}\text{AgeDif}_{sap_1p_2}^{40}\cdot X_{sap_1p_2}\right)\right]\geq 0$$

$$\text{AgeDif}_{1100}^{40}\cdot X_{1100}+\sum_{a=1}^{3}\sum_{p_1=1}^{4}\left(\text{AgeDif}_{1ap_10}^{40}\cdot X_{1ap_10}+\sum_{p_2=p_1+2}^{4}\text{AgeDif}_{1ap_1p_2}^{40}\cdot X_{1ap_1p_2}\right)\geq 0$$

$$\sum_{a=1}^{3}\sum_{p_1=1}^{4}\left(\text{AgeDif}_{2ap_10}^{40}\cdot X_{2ap_10}+\sum_{p_2=p_1+2}^{4}\text{AgeDif}_{2ap_1p_2}^{40}\cdot X_{2ap_1p_2}\right)\geq 0$$

Implementing Wildlife Openings

- Forest management activity scheduling model;
- Creating 500 acres of wildlife browse habitat;
- The openings are cleared in Pd. 1 and will be maintained over time by planting browse species at a cost of $10/ac/yr

1. Defining the wildlife opening variables:

   W_{sa}= the number of acres from site class *s*, initial age class *a* assigned to be cleared in pd. 1 and maintained as wildlife openings for the remainder of the planning horizon

2. Modifying the Area Constraints

$$X_{sa00} + \sum_{p=1}^{4} \left[X_{sap_1 0} + \sum_{p_2 = p_1 + 2}^{4} X_{sap_1 p_2} \right] + W_{sa} \leq A_{sa} \qquad \text{for } s=1,2 \text{ and } a=1,2,3$$

Note: wildlife openings as the ninth prescription

3. Specifying the target area for wildlife openings

$$\sum_{s=1}^{2}\sum_{a=1}^{3} W_{sa} \geq 500$$

or,

$$\sum_{a=1}^{3} W_{sa} \geq 500 \qquad s=1,2$$

4. Modifying the objective function (accounting for the costs and revenues that are associated with the new activities)

$$Max\ Z = \sum_{s=1}^{2}\sum_{a=1}^{3}\left[\sum_{p_1=1}^{4}\left(c^{p}_{sap_1 0}\cdot X_{sap_1 0} + \sum_{p_2=p_1+2}^{4} c^{p}_{sap_1 p_2}\cdot X_{sap_1 p_2}\right) + c^{wp}_{sa}\cdot W_{sa}\right]$$

where

$c^{p}_{sap_1 p_2}$ = the discounted net revenue (profit) from assigning one acre from site class *s*, initial age class *a* to be harvested in periods p_1 and p_2, and

c^{wp}_{sa} = the discounted net revenue from assigning an acre from site class *s*, initial age class *a* to be managed as a wildlife opening.

5. The objective function coefficients

$$c_{sa}^{wp} = \frac{(P - s_v)v_{sa10}^1 - s_f}{(1+r)^5} - \frac{c_w[(1+r)^{35} - 1]}{r(1+r)^{35}(1+r)^5}$$

where

$P =$ the wood price,

$s_v =$ the variable (per cord) timber sale administration cost,

$s_f =$ the fixed (per acre) timber sale administration cost,

$v_{sa10}^1 =$ the volume of wood that will be harvested in period 1 for each acre assigned to the variable W_{sa}.

$c_w =$ the annual, per-acre cost of maintaining the wildlife openings, and

$r =$ the real interest rate.

Example Objective Function Coefficient Calculation

Economic data

Item	Symbol	Amount
Wood Price	***P***	**$25/cd**
Planting Cost	***E***	**$100.00/ac**
Timber Sales Cost		
-per acre	s_f	**$15.00/ac**
-per cord	s_v	**$0.20/cd**
Interest Rate	***r***	**4%**

Yield

Harvest Age	Acres by site class	
	Site I	Site II
10	**2**	**5**
20	**10**	**14**
30	**20**	**27**
40	**31**	**38**
50	**37**	**47**
60	**42**	**54**
70	**46**	**60**

$$C_{23}^{wp} = \frac{(25-0.2)\cdot 27-15}{(1.04)^5} - \frac{10\left[(1.04)^{35}-1\right]}{0.04(1.04)^{40}} = \underline{\$384.62}$$

6. Modifying the harvest accounting constraints

$$\sum_{s=1}^{2}\sum_{a=1}^{3}\left[v^{1}_{sa10}X_{sa10}+v^{1}_{sa13}X_{sa13}+v^{1}_{sa14}X_{sa14}+v^{1}_{sa10}W_{sa}\right]-H_{1}=0$$

7. Average ending age constraints

The wildlife opening variables should be excluded from these constraints

Implementing Stream-side Management Zones (SMZs)

- It is assumed that the area assigned to SMZs has already been calculated, and
- 8% of each analysis area will be reserved for SMZs.

$$A^{*}_{sa} = A_{sa} \times (1 - smz_{sa})$$

$$X_{sa00} + \sum_{p=1}^{4} \left[X_{sap_1 0} + \sum_{p_2 = p_1 + 2}^{4} X_{sap_1 p_2} \right] \leq A^{*}_{sa} \qquad \text{for s=1,2 and a=1,2,3}$$