CHAPTER 13: A PROFIT MAXIMIZING HARVEST SCHEDULING MODEL

The previous chapter introduced harvest scheduling with a model that minimized the cost of meeting certain harvest targets. These harvest targets could be based on mill requirements, or on the estimated productive capacity of the forest. Often, however, there are no particular harvest requirements to meet. Even when there are – for example, when a mill requires a certain amount of wood – it may be better to meet these wood requirements by buying some of the wood on the open market. This chapter presents an alternative formulation that maximizes the discounted profits from the forest. This profit maximization formulation of the problem gives much more flexibility in how the harvest targets must be specified in the problem. As you will see, it is not even necessary to designate specific harvest targets with the profit-maximizing formulation, so if you don't know what harvest levels would be appropriate for an area, you can let the model tell you how much to harvest. The main disadvantage of the profit maximization formulation, however, is that it requires you to project the price of wood for the duration of the planning horizon. In practice, it will often be easier to predict future wood needs or the productive capacity of the forest than to predict future wood prices.

The profit-maximization formulation is very similar to the cost-minimization formulation. The definitions of the basic variables — the X_{sap}'s — will not change. Obviously, the objective function coefficients will change. Also, while the minimum harvest target constraints that were used in the cost-minimization problem could be used here, an alternative way to constrain harvests that is more suitable for a profit-maximization problem will be presented. Finally, the area constraints and the average ending age constraint will not change in this formulation.

1. The Profit-Maximization Objective Function

The objective in this case is to maximize the present value of the profits, or net revenue, from managing the forest over the 20-year planning horizon. The general form of the objective function will be:

$$Max\ Z = \sum_{s=1}^{2}\sum_{a=1}^{3}\sum_{p=0}^{2} c_{sap}^{p} \cdot X_{sap}$$

where c_{sap}^{p} = the present value of the net revenue of assigning one acre to the variable X_{sap} (the superscript p here is for "profit").

This objective function looks a lot like the cost-minimization objective function. There are two key differences. First, we are maximizing this function. Second, the coefficients here give the discounted net revenue, or profit, per acre for each variable, rather than the discounted cost per acre. The net revenue is just the revenue minus the cost, so the formula for the coefficients should look familiar. These coefficients, however, include an expression for the revenue, which is just the price times the harvested volume, and the costs now are negative. The general formula for the net revenue coefficients is:

$$c_{sap}^{p} = \begin{cases} \dfrac{P \cdot v_{sap} - [E + s_f + s_v v_{sap}]}{(1+r)^{10 \cdot p-5}} = \dfrac{(P - s_v) \cdot v_{sap} - E - s_f}{(1+r)^{10 \cdot p-5}} & for\ p > 0 \\ 0 & for\ p = 0 \end{cases}$$

where P = the wood price,
v_{sap} = the harvest volume per acre for acres assigned to the variable X_{sap},
s_v = the variable (per cord) timber sale cost,
s_f = the fixed (per acre) timber sale cost,
E = the stand establishment (regeneration) cost, and
r = the real interest rate.

Recall that the expression $10 \cdot p$ - 5 just gives the midpoint of the period p. As an example, the net revenue coefficient for acres from site class 2, initial age class 3, assigned to be harvested in period 1 (i.e., acres assigned to the variable X_{231}) will be:

$$c_{231}^{p} = \frac{(P - s_v) \cdot v_{231} - E - s_f\,]}{(1+r)^{10 \cdot 1-5}}$$

$$= \frac{(\$25/cd - \$.2/cd) \cdot 27cd/ac - \$100/ac - \$15/ac}{(1.04)^5} = \$455.84/ac$$

The specific objective function for the profit maximization formulation of the example problem is:

$$\begin{aligned} \text{Max } Z = & -53.75\, X_{111} + 73.85\, X_{112} + 109.32\, X_{121} + 211.56\, X_{122} \\ & + 313.15\, X_{131} + 363.03\, X_{132} + 7.40\, X_{211} + 128.93\, X_{212} \\ & + 190.85\, X_{221} + 307.95\, X_{222} + 455.84\, X_{231} + 459.43\, X_{232} \end{aligned}$$

As always, you should verify several of these coefficients for yourself.

2. Constraints for the Profit-Maximization Model

The profit-maximization model could be formulated with exactly the same constraints as the cost-minimization model. The area constraints are necessary, and there is no reason to change them. A variety of ending constraints could be used, but the ending constraint from the cost-minimization model is just as appropriate for this model. As discussed earlier, the same harvest constraints used in the cost-minimization model could be used here, but other types of harvest constraints will generally make more sense for a profit-maximization model. We will use harvest fluctuation constraints in our profit-maximization model. These constraints limit the amount the harvest can go up or down from one period to the next and help ensure a fairly even flow of timber from the forest.

Area Constraints

The area constraints for the profit maximization problem are the same as those used in the profit minimization problem. In general, they are:

$$\sum_{p=0}^{2} X_{sap} \le A_{sa} \quad s = 1,2 \quad a = 1,2,3$$

The specific area constraints for this problem are:

$X_{110} + X_{111} + X_{112}$ # 3,000
$X_{120} + X_{121} + X_{122}$ # 6,000
$X_{130} + X_{131} + X_{132}$ # 9,000
$X_{210} + X_{211} + X_{212}$ # 8,000
$X_{220} + X_{221} + X_{222}$ # 4,000
$X_{230} + X_{231} + X_{232}$ # 7,000

Harvest Fluctuation Constraints

With the cost-minimization problem, the harvest constraints were absolutely necessary. Without them, the cost-minimizing solution would be to harvest nothing. With a profit-maximization objective, we could exclude the harvest constraints could be excluded completely and the model would determine the harvest levels that produce the maximum profit. If a certain minimum amount of wood is needed in each period and the harvest is below this minimum, the remainder could be purchased on the open market. If the model indicates a harvest that is more than is needed, the excess could be sold on the open market. Or, as mentioned earlier, the same constraints that were used in the cost minimization problem, requiring that a minimum amount should be produced in each period, could be used here.

One has a lot more flexibility in specifying harvest constraints with the profit maximization formulation. Generally, however, the model is given too much latitude in setting harvest levels, the harvest level may fluctuate wildly from one period to the next. Therefore, a new set of harvest constraints that do not limit the level of the harvest will be introduced in this example. These constraints will limit the amount that the harvest level will be allowed to fluctuate from one period to the next.

Let's say that we want to ensure that the harvest does not fluctuate from one period to the next by more than 15%. That is, we want to make sure that the harvest in period 2 is not less than 15% below the harvest in period 1 and not more than 15% above the harvest in period 1. This can be accomplished with some constraints that look like this:

$H_2 \geq .85\, H_1$ and $H_2 \leq 1.15\, H_1$.

The first of these constraints requires the harvest in period 2 to be at least 85% of the harvest in period 1. The second constraint says the harvest in period 2 cannot be more than 15% greater than the harvest in period 1. Constraints such as these would serve to limit the amount the harvest can go up or down between period 1 and period 2 to not more than 15%.

In order to use harvest fluctuation constraints like those above, it will be necessary to introduce the variables H_1 and H_2, which should equal the total harvest in their respective periods. To do this, a set of *harvest accounting constraints* must be created. A harvest accounting constraint sums up the harvest for a period and puts this sum into a variable such as H_1 or H_2. To see how such a constraint might be formulated, consider the harvest target constraints that were used in the cost minimization formulation:

$$\sum_{s=1}^{2}\sum_{a=1}^{3} v_{sap} \cdot X_{sap} \geq H_p \qquad p = 1,2$$

These constraints require the total harvest to be greater than some minimum harvest target for the period — H_p. The left-hand side of the constraints adds up the total harvest for period p and the right-hand side of the constraint specifies the minimum harvest target. In these constraints, the H_p's are parameters — i.e., fixed values. These constraints can be converted to harvest accounting constraints by treating the H_p's as variables and converting the inequality to an equality. Since the H_p's are now variables, they should appear on the left-hand side. This is done by subtracting them from both sides of the equation. This results in the following harvest accounting constraints:

$$\sum_{s=1}^{2}\sum_{a=1}^{3} v_{sap} \cdot X_{sap} - H_p = 0 \qquad p = 1,2$$

Note the similarity – and the differences – between these constraints and the harvest target constraints. The coefficients on the X_{sap} variables are the same in these constraints as in the harvest target constraints. However, these constraints are equality constraints, and the quantity H_p is a variable in these constraints, rather than a coefficient.

These constraints do not actually constrain the solution of the problem in any way. Rather, these constraints simply sum up the harvest for the given period and plug that value into the harvest variable for the period. They do not put any limits on what the harvest can or should be. This is why these constraints are called "accounting" constraints. Accounting constraints simply sum up some quantity that can be expressed as a linear function of the variables and define a new variable to store the sum. Revenue and cost accounting constraints could also be formulated so that the solution would automatically report the revenue and cost for each period.

The constraints introduced earlier to tie together the harvests from one period to the next can now be expressed in terms of these new variables — the H_p's. They need to be rearranged slightly, since the variables must always appear on the left-hand side of the constraint. The harvest fluctuation constraints for this problem are:

$$.85\,H_1 - H_2 \# 0$$
$$-1.15\,H_1 + H_2 \# 0$$

The specific harvest accounting constraints for this problem are:

$$2\,X_{111} + 10\,X_{121} + 20\,X_{131} + 5\,X_{211} + 14\,X_{221} + 27\,X_{231} - H_1 = 0$$
$$10\,X_{112} + 20\,X_{122} + 31\,X_{132} + 14\,X_{212} + 27\,X_{222} + 38\,X_{232} - H_2 = 0$$

Average Ending Age Constraint

The average ending age constraint is the same for both the cost minimization formulation and the profit maximizing formulation. It's general form is:

$$\sum_{s=1}^{2}\sum_{a=1}^{3}\sum_{p=0}^{2} Age_{sap}^{20} \times X_{sap} \geq \overline{Age}^{20} \times TotalArea$$

The specific form of the average ending age constraint for this problem is:

$$25\,X_{110} + 15\,X_{111} + 5\,X_{112} + 35\,X_{120} + 15\,X_{121} + 5\,X_{122}$$
$$+ 45\,X_{130} + 15\,X_{131} + 5\,X_{132} + 25\,X_{210} + 15\,X_{211} + 5\,X_{212}$$
$$+ 35\,X_{220} + 15\,X_{221} + 5\,X_{222} + 45\,X_{230} + 15\,X_{231} + 5\,X_{232}\ \$629{,}000$$

Non-negativity Constraints

As always, it is necessary to include the non-negativity constraints. They are:

X_{sap} \$ 0 $\quad s = 1, 2 \quad a = 1, 2, 3 \quad p = 0, 1, 2$

and H_p \$ 0 $\quad p = 1, 2$

3. <u>The Complete Profit-Maximization Problem Formulation</u>

This section summarizes the complete profit-maximization problem formulation, first, in its general form, and then with the specific coefficient values for this example.

$$Max\ Z = \sum_{s=1}^{2}\sum_{a=1}^{3}\sum_{p=0}^{2} c_{sap}^{p} \cdot X_{sap} \qquad \text{(Objective function)}$$

Subject to:

$$\sum_{p=0}^{2} X_{sap} \le A_{sa} \quad s = 1, 2 \quad a = 1, 2, 3 \qquad \text{(Area constraints)}$$

$$\sum_{s=1}^{2}\sum_{a=1}^{3} v_{sap} \cdot X_{sap} - H_p = 0 \quad p = 1, 2 \qquad \text{(Harvest constraints)}$$

$.85\ H_1 - H_2$ # 0

$-1.15\ H_1 + H_2$ # 0

$$\sum_{s=1}^{2}\sum_{a=1}^{3}\sum_{p=0}^{2} Age_{sap}^{20} \times X_{sap} \ge \overline{Age}^{20} \times TotalArea \qquad \text{(Ending age constraints)}$$

X_{sap} $ 0 s = 1, 2 a = 1, 2, 3 p = 0, 1, 2   (Non-negativity constraints)
and H_p $ 0 p = 1, 2

where X_{sap} = the number of acres cut from site class s (where s = 1 or 2) and initial age class a (where a = 1, 2, or 3) in period p, (where p = 0, 1, or 2 and where p = 0 means no harvest during the planning horizon);
c_{sap}^{p} = the present value of the net revenue of assigning one acre to the variable X_{sap}.
A_{sa} = the total number of acres in site class s, initial age class a;
v_{sap} = the harvest volume for each acre assigned to the variable X_{sap};
H_p = the volume harvested in decade p (in cords);
Age_{sap}^{20} = the age in year 20 of acres assigned to the variable X_{sap};
$\overline{Age}^{20}$ = the target (minimum) average age of the forest in year 20; and
$TotalArea$ = the total area of the forest.

The specific formulation of the profit-maximization linear program for the example forest is:

Max Z = -53.75 X_{111} + 73.85 X_{112} + 109.32 X_{121} + 211.56 X_{122}
+ 313.15 X_{131} + 363.03 X_{132} + 7.40 X_{211} + 128.93 X_{212}
+ 190.85 X_{221} + 307.95 X_{222} + 455.84 X_{231} + 459.43 X_{232}

Subject to:

$X_{110} + X_{111} + X_{112}$ # 3,000
$X_{120} + X_{121} + X_{122}$ # 6,000
$X_{130} + X_{131} + X_{132}$ # 9,000
$X_{210} + X_{211} + X_{212}$ # 8,000
$X_{220} + X_{221} + X_{222}$ # 4,000

$X_{230} + X_{231} + X_{232} \# 7{,}000$
$2\, X_{111} + 10\, X_{121} + 20\, X_{131} + 5\, X_{211} + 14\, X_{221} + 27\, X_{231} - H_1 = 0$
$10\, X_{112} + 20\, X_{122} + 31\, X_{132} + 14\, X_{212} + 27\, X_{222} + 38\, X_{232} - H_2 = 0$
$.85\, H_1 - H_2 \# 0$
$-1.15\, H_1 + H_2 \# 0$
$25\, X_{110} + 15\, X_{111} + 5\, X_{112} + 35\, X_{120} + 15\, X_{121} + 5\, X_{122} + 45\, X_{130} + 15\, X_{131} + 5\, X_{132}$
$25\, X_{210} + 15\, X_{211} + 5\, X_{212} + 35\, X_{220} + 15\, X_{221} + 5\, X_{222} + 45\, X_{230} + 15\, X_{231} + 5\, X_{232}$
$\$629{,}000$
$X_{110} \$ 0; X_{111} \$ 0; X_{112} \$ 0; X_{120} \$ 0; X_{121} \$ 0; X_{122} \$ 0; X_{130} \$ 0; X_{131} \$ 0; X_{132} \$ 0;$
$X_{210} \$ 0; X_{211} \$ 0; X_{212} \$ 0; X_{220} \$ 0; X_{221} \$ 0; X_{222} \$ 0; X_{230} \$ 0; X_{231} \$ 0; X_{232} \$ 0;$
$H_1 \$ 0; H_2 \$ 0$

4. Interpreting the Solution to the Profit-Maximization Problem

The LINDO solution for the profit-maximization formulation of the example problem is shown in Figure 13.1. The optimal objective function value for this problem is 7,995,010. This means that the maximum present value of the net revenues over the next 20 years for this forest under the specified constraints is close to $8 million.

Interpreting the Optimal Variable Values

Tables 13.1 through 13.5 interpret the results for the optimal solution to the profit-maximization formulation of the example problem. Table 13.1 shows the acres harvested by period and analysis area (site class and initial age class). Table 13.2 shows the harvest schedule by period, site class and age at harvest. Table 13.2 also summarizes total acres harvested by period and site class and acres not harvested by site class. Table 13.3 summarizes the acres and volume harvested and costs and revenues by period. You should note that the solution to the profit maximization problem is quite different from the solution to the cost minimization problem. The most important difference is that the solution to the profit maximization formulation involves harvesting more in the first decade than in the second decade. You should not feel comfortable with this result. The initial forest is understocked relative to the state the forest would be in if it was regulated using the optimal economic rotations for each site class. It would make sense to follow the type of harvesting pattern was prescribed for the cost minimization problem — harvesting less than the LTSY for a couple of periods to allow the inventory to build up. While it is not obvious why the model would suggest harvesting more in the first period, you should suspect the most glaring problem with the formulation — the short planning horizon. The next chapter will extend the planning horizon for this problem to 40 years, and the solution will change significantly. You may recall that one of the tests of whether a planning horizon is long enough is whether the solution changes significantly when a longer planning horizon is used.

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LP OPTIMUM FOUND AT STEP     22

       OBJECTIVE FUNCTION VALUE

       1)      7995010.

 VARIABLE        VALUE          REDUCED COST
     X111           .000000       128.450800
     X112           .000000        68.851610
     X121           .000000        42.623970
     X122       2831.855000          .000000
     X131       5895.564000          .000000
     X132       3104.436000          .000000
     X211           .000000        68.572790
     X212           .000000        11.771880
     X221           .000000        62.676190
     X222       4000.000000          .000000
     X231       7000.000000          .000000
     X232           .000000        39.825640
     X110       3000.000000          .000000
     X120       3168.145000          .000000
     X130           .000000        83.121450
     X210       8000.000000          .000000
     X220           .000000        99.889540
     X230           .000000       222.836700
       H1     306911.300000          .000000
       H2     260874.600000          .000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)           .000000       184.619400
       3)           .000000       258.467200
       4)           .000000       415.436500
       5)           .000000       184.619400
       6)           .000000       358.356800
       7)           .000000       555.151700
       8)           .000000          .424470
       9)           .000000        -.499377
      10)           .000000          .499377
      11)      92073.380000          .000000
      12)           .000000        -7.384778

NO. ITERATIONS=      22
```

Figure 13.1. LINDO solution to the example profit-maximization harvest scheduling problem.

The overall harvest for the two periods is lower with the profit-maximization model than with the cost-minimization model. This should not be surprising, since the dual prices associated with the harvest constraints in the cost-minimization model were quite a bit higher than the market price. That was an indication that it would be cheaper to buy some wood on the open market than to try to meet the harvest constraints in the cost-minimization model with wood grown on the example forest. Since the profit-maximization model is not required to produce any particular amount of wood, less wood is harvested. Note that the profit maximization solution cuts less heavily in site class II than does the cost-minimization model. In order to meet the higher harvests produced by the cost-minimization model, site class II had to be harvested more heavily.

Table 13.1. Acres assigned to each prescription, by site class and initial age class.

Site Class	Initial Age Class	Harvest in Pd 1	Harvest in Pd 2	No Harvest
I	0-10 yr	0	0	3,000
	11-20 yr	0	2,831.9	3,168.1
	21-30 yr	5,895.6	3,104.4	0
II	0-10 yr	0	0	8,000
	11-20 yr	0	4,000	0
	21-30 yr	7,000	0	0
Total		12,895.6	9,936.3	14,168.1

Table 13.2. Acres harvested by period, by site class, and by age at harvest.

Planning Period	Age at Harvest	Site I	Site II	Total
1	30	5,895.6	7,000	12,895.6
2	30	2,831.9	4,000	6,831.9
	40	3,104.4	0	3,104.4
	Total	5,936.3	4,000	9,936.3
Total acres harvested		11,831.9	11,000	22,831.9
Acres not harvested		6,168.1	8,000	14,168.1

Table 13.3. Acres and volume harvested, and revenues and costs by period for the example forest.

Quantity	Period 1	Period 2
Acres harvested	12,895.6	9,936.3
Volume harvested (cords)	306,911.3	260,874.6
Gross Revenues	$7,672,783	$6,521,865
Costs	$1,544,376	$1,194,849
Net Revenues	$6,128,406	$5,327,016

Tables 13.4 and 13.5 show the projected age-class distribution at the end of periods 1 and 2, respectively, under the profit-maximization plan. The age-class distribution at the end of the planning horizon is less balanced than the age-class distribution produced by the cost-minimization model. However, it does meet the minimum average-ending-age requirement.

Table 13.4. Age-class distribution of the example forest after period 1.

Age Classes	Acres by site class	
	Site I	Site II
0 to 10	5,895.6	7,000
11 to 20	3,000	8,000
21 to 30	6,000	4,000
31 to 40	3,104.4	0
Total	18,000	19,000

Table 13.5. Age-class distribution of the example forest after period 2.

Age Classes	Acres by site class	
	Site I	Site II
0 to 10	5,936.3	4,000
11 to 20	5,895.6	7,000
21 to 30	3,000	8,000
31 to 40	3,168.1	0
Total	18,000	19,000

Interpreting the Reduced Cost Coefficients

As with the cost-minimization model, the reduced cost coefficients indicate how much the objective function coefficient corresponding to each variable whose optimal value is zero would have to be improved before that variable will take on a positive value in the optimal solution. In the context of the profit-maximization model, the reduced cost coefficients indicate how much the discounted net revenue per acre from the corresponding prescription would have to be increased before it would be optimal to apply that prescription on any acres from the corresponding analysis area.

The variable with the lowest reduced cost value is X_{212}. The reduced cost for this variable is \$11.77. This means that it would become profitable to harvest in period 2 some of the acres from site class 1, initial age class 2, if the discounted profit from that prescription could be increased by \$11.78 per acre.

Interpreting the Slack/Surplus Coefficients and the Dual Prices

One of the slack or surplus coefficients in this example is positive. Recall that each row in this output block corresponds to a problem constraint. The constraint with the positive slack value is the one that says that the harvest cannot increase by more than 15% between periods 1 and 2. Both harvest fluctuation constraints — the constraint that limits the amount that the harvest can decrease and the constraint that limits the amount of increase — cannot be binding at the same time, so you will always have at least one non-binding constraint (with a positive slack) with the profit-maximization formulation. All of the other constraints are binding.

Each binding constraint has a non-zero dual price. As with the cost-minimization model, the dual prices for the less-than-or-equal constraints are positive and the dual prices for the greater-than-or-equal constraints are negative. Recall that the dual prices indicate how much the objective function would be improved if the right-hand side of the corresponding constraint was increased by 1. Since most of the constraints in the profit-maximization formulation are the same as their counterparts in the cost-minimization formulation, their interpretation is about the same. The dual prices corresponding to the area constraints indicate the increase in the discounted net revenues that would be possible if one more acre was available in that analysis area. The dual price corresponding to the average ending-age constraint indicates the loss in profits if one acre was required to be one year older at the end of the planning horizon.

The dual prices on the harvest accounting constraints are related to the dual prices for the harvest fluctuation constraints. We will not be too concerned about their values since the harvest accounting constraints do not relate to any limiting resource. The dual prices associated with binding harvest fluctuation constraints indicate how much additional profit could be earned if one more cord could be shifted from one period to the other. In this case, more wood is scheduled to be produced in period 1 than in period 2. Therefore, the binding constraint is the one that says that the harvest cannot decline by more than 15%. The dual price on this constraint says that fifty cents more profit could be made if one more cord could be produced in period 1 instead of period 2.

5. Study Questions for the Profit-Maximization Harvest Scheduling Model

1. What are the advantages and disadvantages of formulating a harvest scheduling model as profit-maximization problem versus formulating it as a cost-minimization problem?

2. In the objective function of the example problem, the coefficient on the variable X_{221} is 190.85. What does this coefficient represent? Show how it was calculated.

3. What is a harvest accounting constraint? How is it different from a harvest target constraint? How is it similar? Why is it called an "accounting" constraint? What other types of accounting constraints might be useful?

4. Do the harvest accounting constraints limit the solution to the problem? If so, how? If not, why not?

5. What is the purpose of the harvest fluctuation constraints? Why would you want to limit the amount of variation in the harvest levels over time? How should you decide how much to allow the harvest level to fluctuate from one period to the next?

6. What does a positive slack or surplus value in the LINDO solution tell you?

7. How should the dual prices on the harvest fluctuation constraints be interpreted?

6. Exercises

1. It is your job to develop a management plan for a 5,800 acre forest. The age class distribution by site class is given in Table 13.6. Table 13.6 gives the expected yield per acre by age and site class.

Table 13.6. Initial forest acreage by site and age class.

Age Classes	Acres by site class	
	Site I	Site II
0-10	700	800
11-20	800	1,300
21-30	1,000	1,200

Table 13.7. Expected yield by site and age.

Age	Cords/acre by site class	
	Site I	Site II
10	0	5
20	9	15
30	16	24
40	21	32
50	25	39

a. Formulate (Do not solve!) this management problem with the following assumptions:

i) You want to maximize the discounted net revenue from the forest over a 20-year planning horizon using an interest rate of 4%.

ii) Real stumpage prices are $20 per cord. It costs $10 per acre plus $1.20 per cord to prepare a timber sale. It costs $50 per acre to regenerate stands that are harvested.

iii) Harvest levels in any decade should not be more than 15% larger than or less than 15% smaller than the harvest level in any other decade.

iv) You want the average age of your ending inventory to be at least 13 years old.

b. Note the similarity between this problem and problem 1 in Chapter 12. Compare the formulations for the two problems.

2. The following data are for a 5,800 acre forest. The age class distribution by site class is given in Table 13.8. Table 13.9 gives the expected yield per acre by age and site class.

Table 13.8. Initial forest acreage by site and age class.

Age Classes	Acres by site class	
	Site I	Site II
0-10	480	950
11-20	910	1,080
21-30	1,060	1,320

Table 13.9. Expected yield by site and age.

Age	Cords/acre by site class	
	Site I	Site II
10	0	2
20	14	18
30	22	31
40	34	43
50	40	53

a. Formulate, and solve using Excel, a harvest scheduling model for this forest using the following assumptions:

i) The objective is to maximize the discounted net revenue from the forest over a 20-year planning horizon using an interest rate of 4%.

ii) Real stumpage prices are $30 per cord. It costs $15 per acre plus $0.30 per cord to prepare a timber sale. It costs $150 per acre to re-plant stands that are harvested.

iii) Harvest levels in any decade should not be more than 10% larger than or less than 10% smaller than the harvest level in any other decade.

iv) The average age of the forest at the end of the 20-year planning horizon should be at least 17 years old.

b. Use this information from the Excel Answer Report to complete Tables 13.10 through 13.12 on the next pages.

Table 13.10. Acres assigned to each prescription, by site class and initial age class.

Site Class	Initial Age Class	Harvest in Pd 1	Harvest in Pd 2	No Harvest
I	0-10 yr			
	11-20 yr			
	21-30 yr			
II	0-10 yr			
	11-20 yr			
	21-30 yr			
Total				

Table 13.11. Acres harvested by period, by site class, and by age at harvest.

Planning Period	Age at Harvest	Site I	Site II	Total
1	20			
	30			
	Total			
2	20			
	30			
	40			
	Total			
Total acres harvested				
Acres not harvested				

Table 13.12. Acres and volume harvested, and revenues and costs by period for the example forest.

Quantity	Period 1	Period 2
Acres harvested		
Volume harvested (cords)		
Gross Revenues		
Costs		
Net Revenues		

3. It is your job to develop a management plan for a 1.2 million acre forest. The age class distribution by site class is given in Table 13.13. Table 13.14 gives the expected yield per acre by age and site class.

   a. Formulate this management problem as a linear program. Be sure to clearly define all of your variables. Use the following assumptions:

      i) You want to maximize the discounted net revenue from the forest over a 30 year planning horizon using an interest rate of 4%.

      ii) Real stumpage prices are $30 per cord. It costs $50 per acre plus $0.10 per cord to prepare a timber sale. It costs $100 per acre to re-plant stands that are harvested.

      iii) Harvest levels should not fluctuate by more than 15% each decade.

      iv) You want the average age of your ending inventory to be at least 20 years old.

Table 13.13. Initial forest acreage by site and age class.

Age Classes	Acres by site class		
	Site I	Site II	Site III
0-10	120,000	70,000	90,000
11-20	80,000	130,000	130,000
21-30	90,000	170,000	100,000
31-40	140,000	40,000	40,000

Table 13.14. Expected yield by site and age.

Age	Cords per acre by site class		
	Site I	Site II	Site III
10	2	5	7
20	10	14	17
30	17	22	26
40	21	26	30
50	22	28	33
60	23	29	36
70	24	30	38

b. Solve the problem you formulated in part a using Excel or LINDO. Use the solution to the problem to create a harvest schedule table and a summary table similar to Tables 13.11 and 13.12.