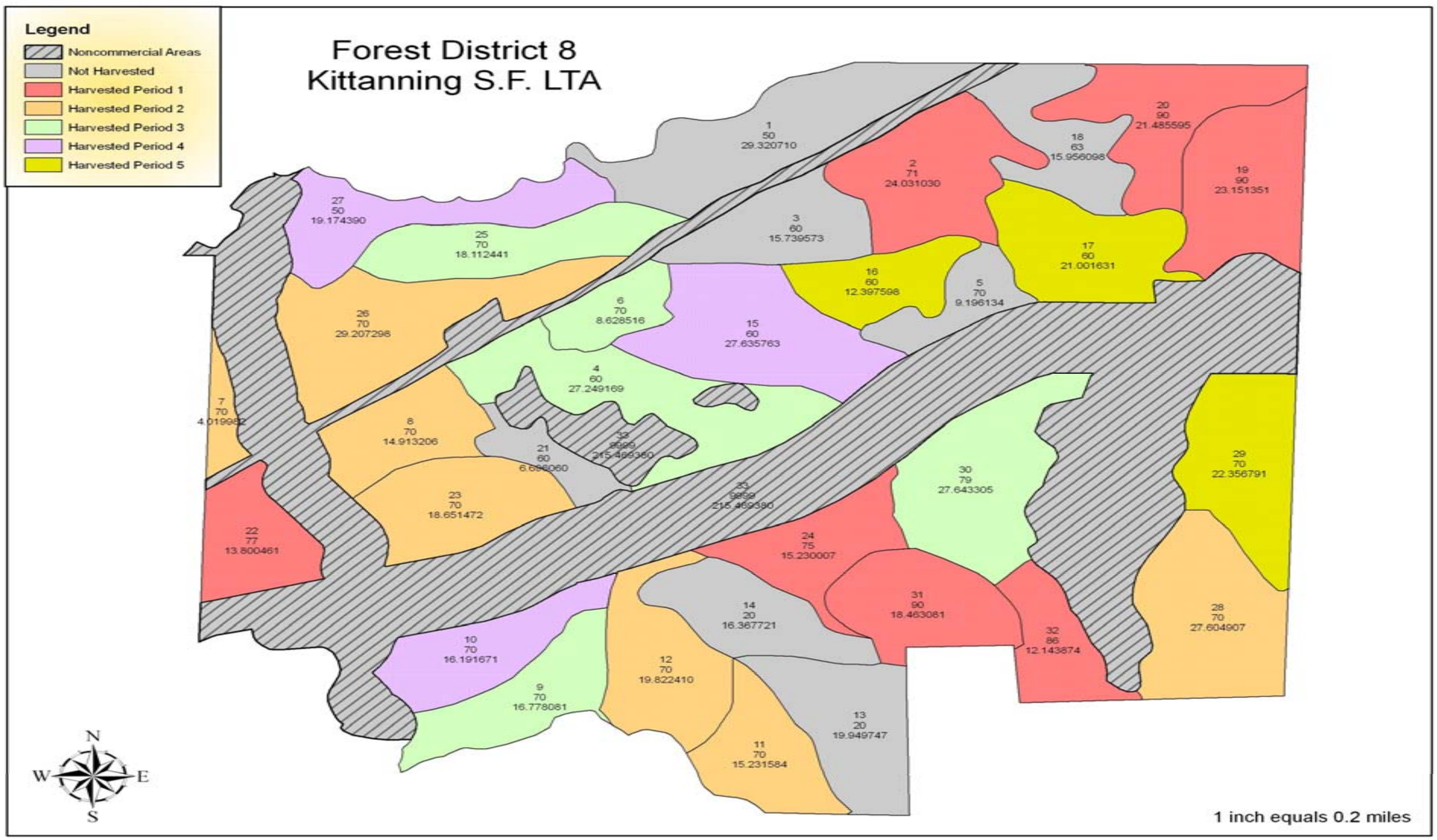
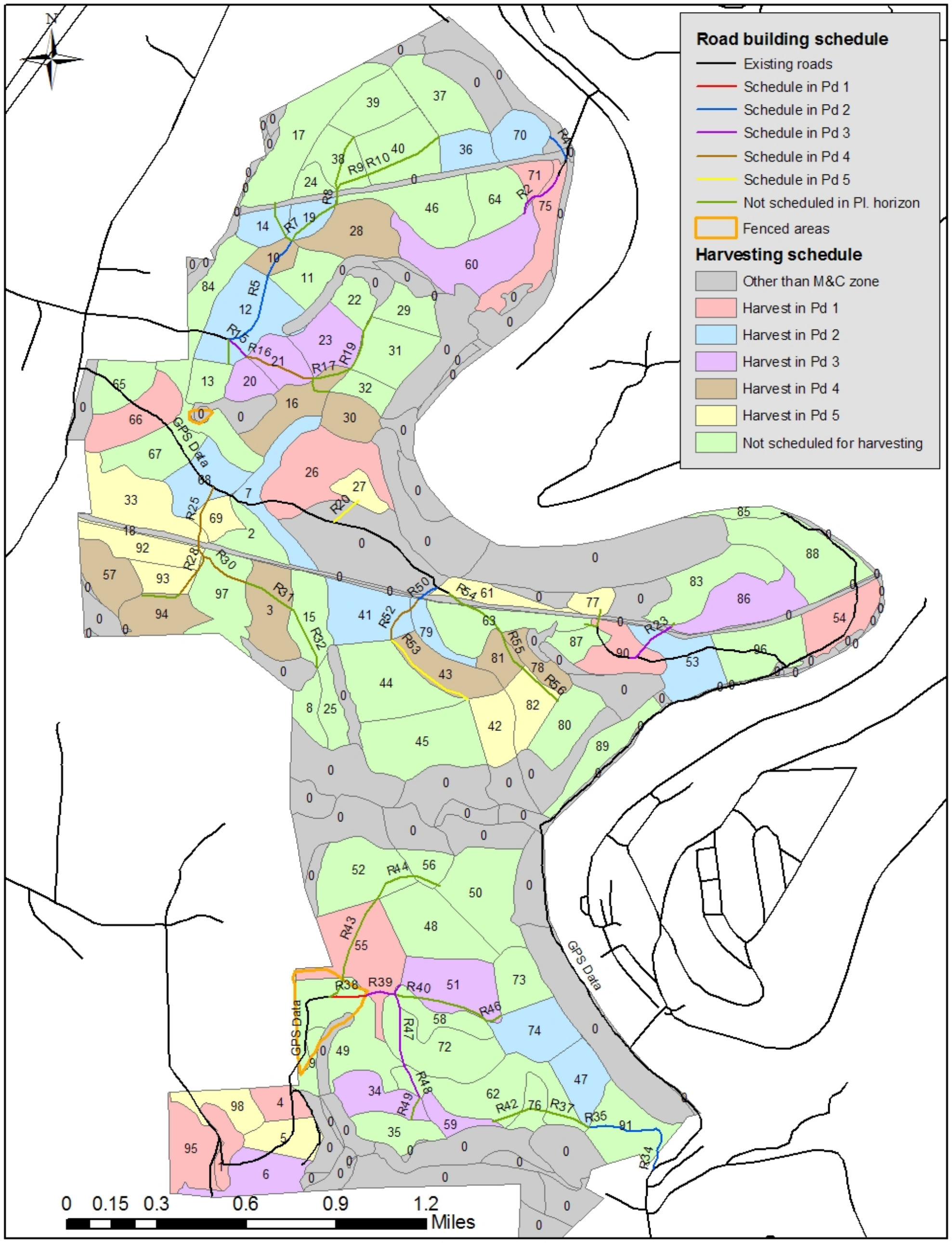
# Spatial Forest Planning with Integer Programming

Lecture 10 (5/8/2017)

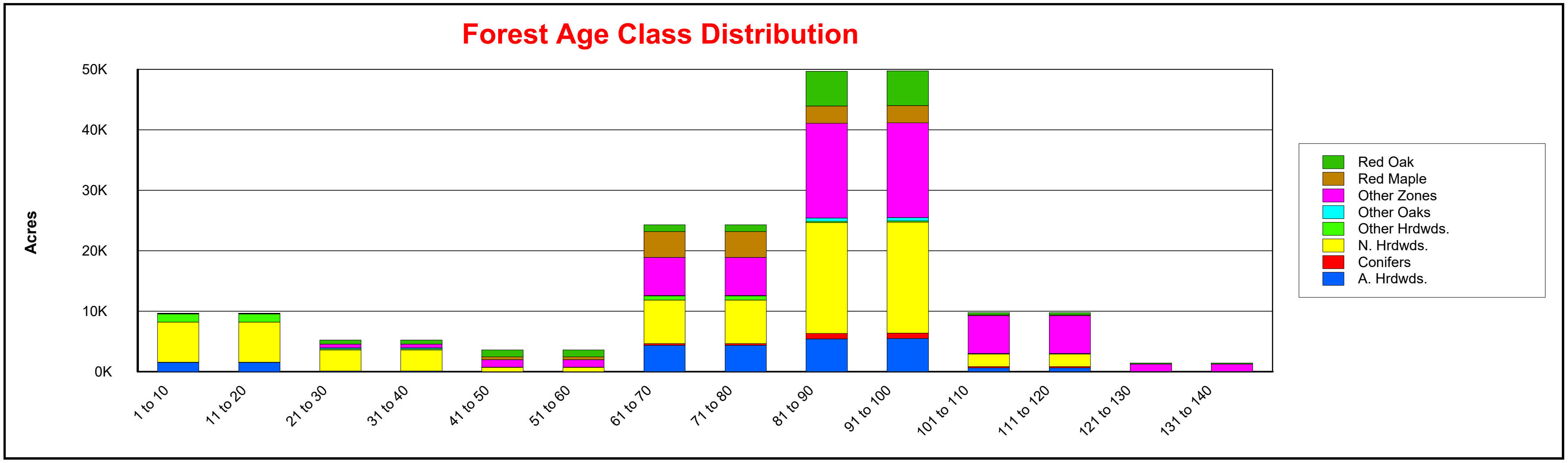
# Spatial Forest Planning

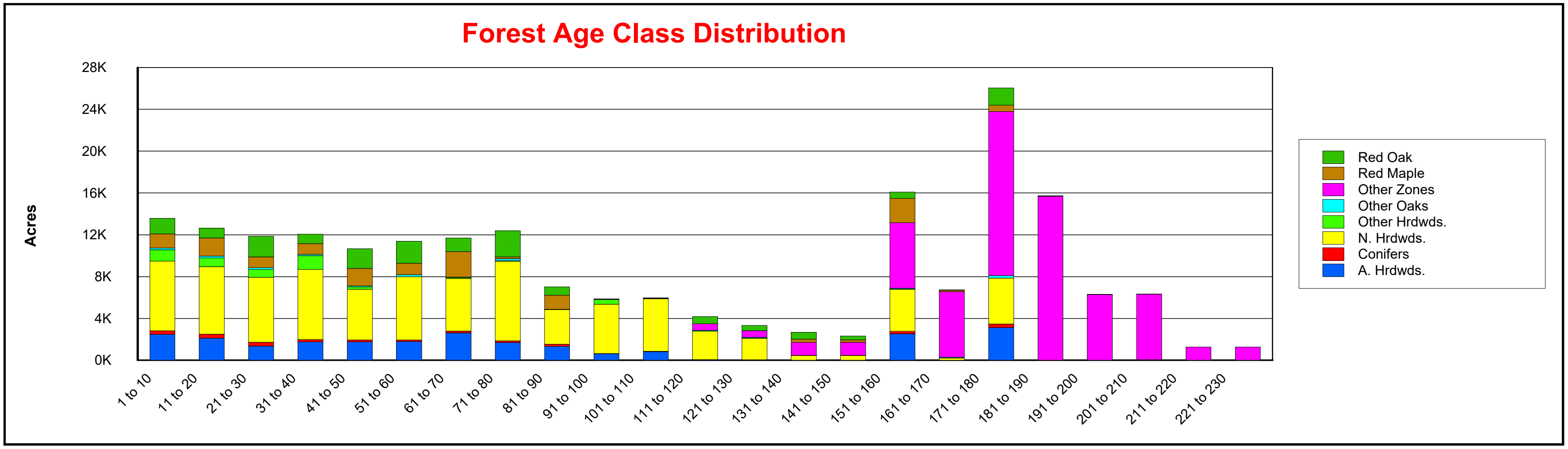


# Spatial Forest Planning with Roads



# Balancing the Age-class Distribution



# Spatially-explicit Harvest Scheduling Models

- Set of management units
- T planning periods
- Decision: whether and when to harvest management units
  - Modeled with 0-1 variables
  - $x_{mt}$ = 1 if unit m is harvested in period t, 0 otherwise

# Spatially-explicit Harvest Scheduling Models (continued)

- Constraints
  - Logical: can only harvest a unit once, at most
  - Harvest volume, area and revenue flow control
  - Ending conditions
    - Minimum average ending age
    - Extended rotations
    - Target ending inventories
  - Maximum harvest area (green-up)
  - Others spatial concerns:
    - Roads, mature patches, etc…

# Integer Programming Model for Spatial Forest Planning

$$MaxZ = \sum_{m=1}^{M} A_m [c_{m0} x_{m0} + \sum_{t=h_m}^{T} c_{mt} x_{mt}]$$

subject to:

$$x_{m0} + \sum_{t=h_m}^{T} x_{mt} \le 1$$ one constraint for each $m$=1,2,…,$M$

$$\sum_{m \in M_{ht}} v_{mt} \cdot A_m \cdot x_{mt} - H_t = 0$$ one constraint for each $t$=1,2,…,$T$

$$b_{l,t} H_t - H_{t+1} \le 0$$ one constraint for each $t$=1,2,…,$T$-1

$$-b_{h,t} H_t - H_{t+1} \le 0$$ one constraint for each $t$=1,2,…,$T$-1

$$\sum_{j \in C} x_{jt} \le |C| - 1$$ one constraint for each $\forall C \in \Omega$ and for each $t$=$h_i$,…,$T$

$$\sum_{m=1}^{M} A_m [(Age_{m0}^{T} - \overline{Age}^{T}) x_{m0} + \sum_{t=h_m}^{T} (Age_{mt}^{T} - \overline{Age}^{T}) x_{mt}] \ge 0$$

$$x_{mt} \in \{0,1\}$$ for each $m$=1,2,…,$M$ and for each $t$=1,2,…,$T$

# Notation

where

$h_m$ =the first period in which management unit *m* is old enough to be harvested,

$x_{mt}$ = a binary variable whose value is 1 if management unit *m* is to be harvested in period *t* for $t = h_m, \ldots T$; when $t = 0$, the value of the binary variable is 1 if management unit *m* is not harvested at all during the planning horizon (i.e., *xm0* represents the "do-nothing" alternative for management unit *m*),

$M$ =the number of management units in the forest,

$T$ = the number of periods in the planning horizon,

$c_{mt}$ = the net discounted net revenue per hectare plus the discounted expected forest value at the end of the planning horizon if management unit *m* is harvested in period *t*,

$M_{ht}$ = the set of management units that are old enough to be harvested in period *t*,

$A_m$ = the area of management unit *m* in hectares,

$v_{mt}$ = the volume of sawtimber in $m^3$/ha harvested from management unit *m* if it is harvested in period *t*,

$H_t$ = the total volume of sawtimber in $m^3$ harvested in period *t*,

# Notation cont.

and

$b_{l,t}$ =a lower bound on decreases in the harvest level between periods $t$ and $t+1$ (where, for example, $bl,t$ = 1 would require non-declining harvests and $bl,t$ = 0.9 would allow a decrease of up to 10%),

$b_{h,t}$ = an upper bound on increases in the harvest level between periods $t$ and $t+1$ (where $bh,t$ = 1 would allow no increase in the harvest level and $bh,t$ = 1.1 would allow an increase of up to 10%),

$C$ = the set of indexes corresponding to the management units in cover $C$,

$\Omega$ = the set of covers that arise from the problem,

$h_i$ = the first period in which the youngest management unit in cover $i$ is old enough to be harvested,

$Age_{mt}^{T}$ = the age of management unit $m$ at the end of the planning horizon if it is harvested in period $t$; and

$\overline{Age}^{T}$ = the minimum average age of the forest at the end of the planning horizon.

# The Challenge of Solving Spatially-Explicit Forest Management Models

- Formulations involve many binary (0-1) decision variables
  - Feasible region is not convex, or even continuous
  - In fact, it is a potentially immense set of points in n-dimensional space
- Solution times could increase more than exponentially with problem size

1.) Unit Restriction Model (URM):

*adjacent units cannot be cut simultaneously*

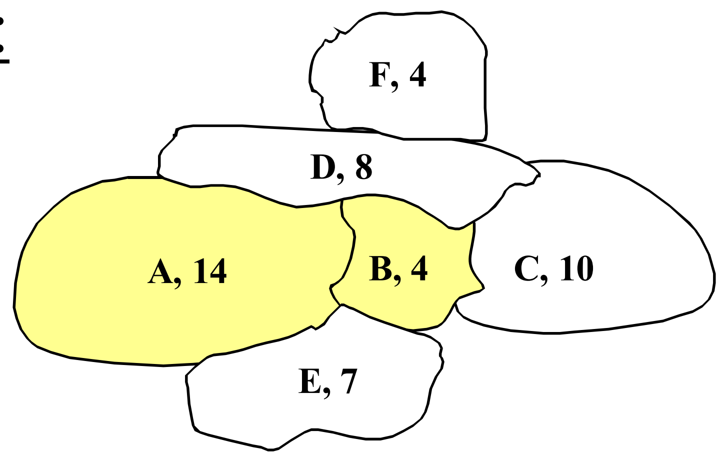
2.) Area Restriction Model (ARM):

*adjacent units can be cut simultaneously as long as their combined area doesn't exceed the maximum harvest opening size*

# Pair-wise Constraints for URM

<u>Adjacency list:</u>

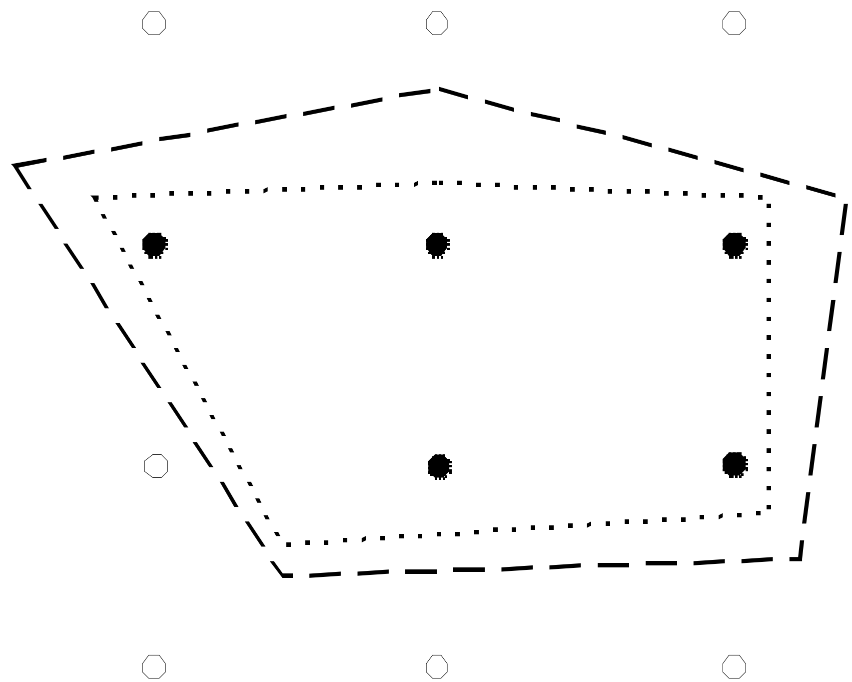
AB
AD
AE
BC
BD
BE
CD
DF




Pair-wise adjacency constraint for AB is

$$x_{At} + x_{Bt} \leq 1$$

# What is a “better” formulation?



# Maximal Clique Constraints for URM

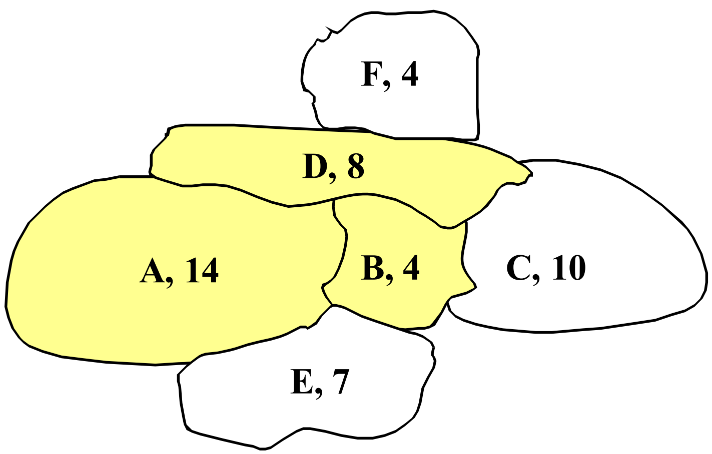
Maximal clique list:

ABD

ABE

BCD

DF



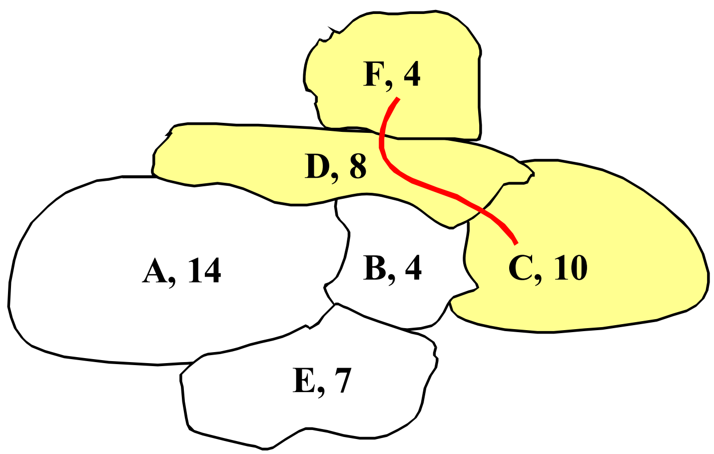

Clique adjacency constraint for ABD is

$$x_{At} + x_{Bt} + x_{Dt} \leq 1$$

# Cover Constraints for ARM

Cover list:

ABC
AD
BCD
AE
BCE
BDEF
CDF




$$A_{\max} = 20ha$$

Cover constraint for FDC is:

$$x_{Ct} + x_{Dt} + x_{Ft} \leq 2$$

In general:

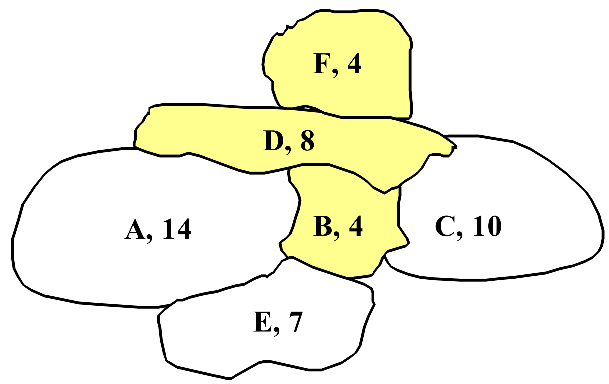
$$\sum_{m \in C} x_{mt} \leq |C| - 1$$

McDill et al. 2002

# GMU Constraints for ARM

<u>GMU list:</u>

A-F
AB
BC
BD
BDE
BDF
BE
CD
CF




$$A_{\max} = 20ha$$

GMU variable for BDF is: $X_{BDF\,t}$

The constraint in general form: $\sum_{n \in K_{jt}} x_{nt} \leq 1 \qquad \forall j \in J \;\; and \;\; t = h_j, \; \ldots, \; T$

Where $K_{jt}$ is the set of GMUs that contain at least one stand in max clique j;
$J$ is the set of max cliques; and
$h_j$ is the first period in which the youngest stand in clique j is old enough to be cut.

McDill et al. 2002 and Goycoolea et al. 2005

# The "Bucket" Formulation of ARM

- Introduce a class of "clearcuts": $\mathrm{K}$ , and
- Introduce variables $x_m^{it}$ that take the value of 1 if management unit *m* is assigned to clearcut *i* in period *t*.
- Objective function:

$$MaxZ = \sum_{m=1}^{M} \sum_{i \in \mathrm{K}} a_m [c_{m0} x_m^{i0} + \sum_{t=h_m}^{T} c_{mt} x_m^{it}]$$

Constantino et al. 2008

# The "Bucket" Formulation of ARM (cont.)

- Constraints:

$$\sum_{t=0,t=h_m}^{T} \sum_{i \in K} x_m^{it} \le 1$$ for $m$ = 1, 2, ..., $M$

$$\sum_{m=1}^{M} a_m x_m^{it} \le A_{\max}$$ for $i \in K$ and $t = h_m, \ldots T$

$$x_m^{it} \le w_Q^{it}$$ for $Q \in M,\ m \in Q,\ i \le m$ and $t = h_m, \ldots T$

$$\sum_{i \in K} w_Q^{it} \le 1$$ for $Q \in M$ and $t = h_m, \ldots T$

# The “Bucket” Formulation of ARM (cont.)

- Constraints:

$$\sum_{m \in M_{ht}, i \in \mathrm{K}} v_{mt} \cdot a_m \cdot x_m^{it} - H_t = 0$$

for *t* = 1, 2, . . . *T*

$$b_{l,t} H_t - H_{t+1} \leq 0$$

for *t* = 1, 2, . . . *T*-1

$$-b_{h,t} H_t - H_{t+1} \leq 0$$

$$\sum_{i \in \mathrm{K}} \sum_{m=1}^{M} a_m [(Age_{m0}^T - \overline{Age}^T) x_m^{i0} + \sum_{t=h_m}^{T} (Age_{mt}^T - \overline{Age}^T) x_m^{it}] \geq 0$$

# The "Bucket" Formulation of ARM (cont.)

- $w_Q^{it}$ takes the value of one whenever a unit in maximal clique $Q$ is assigned to clearcut *i* in period *t*;
- $Q \in \mathrm{M}$ is a maximal clique in the set of maximal cliques.

# Strengthening and Lifting Covers

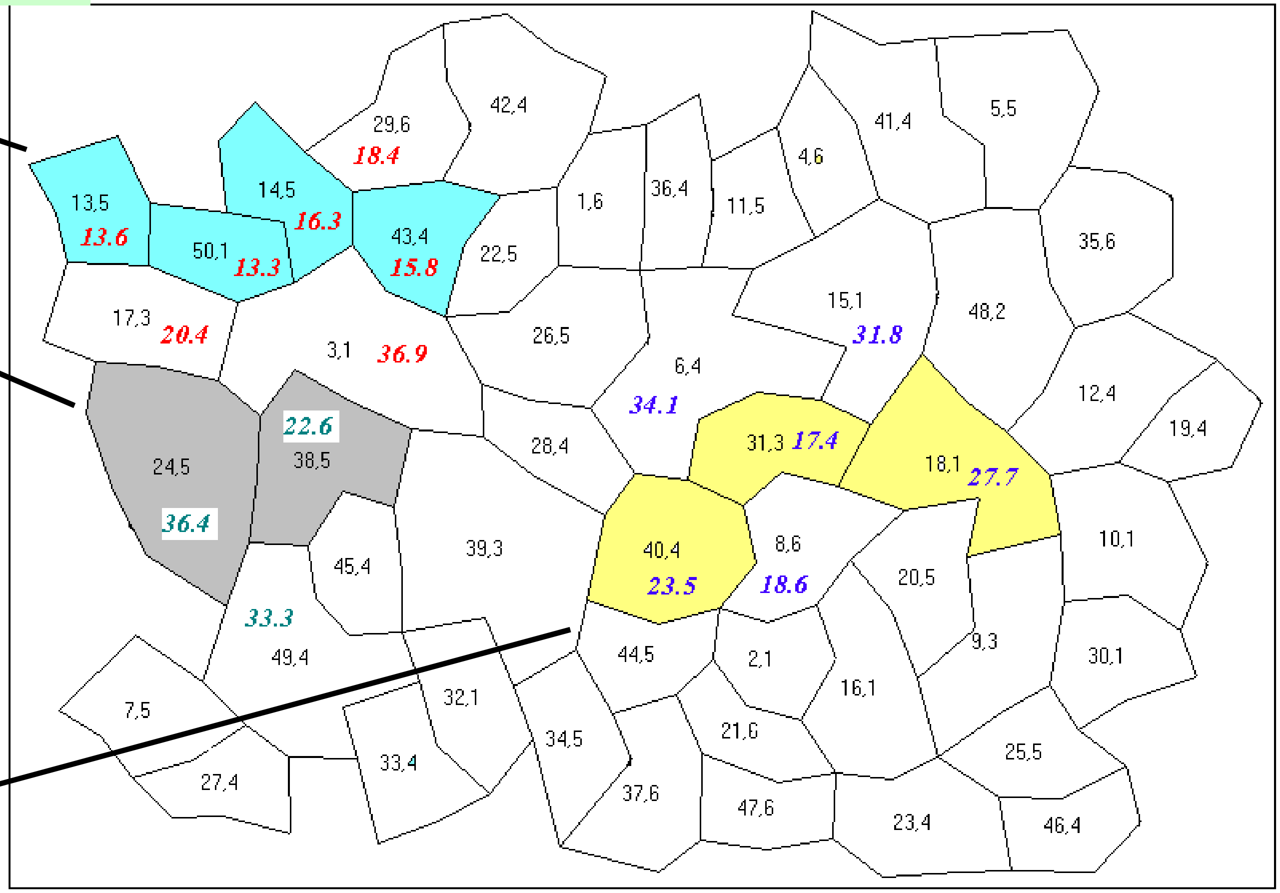
$$x_{13} + x_{14} + x_{43} + x_{50} + 2x_3 \leq 3$$

$$x_{13} + x_{14} + x_{43} + x_{50} \leq 3$$

$$x_{24} + x_{38} \leq 1$$

$$x_{24} + x_{38} + x_3 \leq 1$$

$$x_{24} + x_{38} + x_{49} \leq 1$$




$$x_{18} + x_{31} + x_{40} \leq 2$$

$$x_{18} + x_{31} + x_{40} + x_6 + x_{15} \leq 2$$

$$x_{18} + x_{31} + x_{40} + x_8 \leq 2$$

$$A_{\max} = 48ha$$

# Formalization

The minimal cover constraint has the general form of:

$$\sum_{j \in C} x_j \leq |C| - 1$$

where C is a set of management units (nodes) that form a connected sub-graph of the underlying adjacency graph, and for which $\sum_{j \in C} a_j > A_{\max}$ holds, but $\sum_{j \in C \setminus \{l\}} a_j \leq A_{\max}$ for any $l \in C$.

# Strengthening the Minimal Covers

**<u>Notation</u>:** Let $\Omega$ denote the set of all possible minimal covers that arise from a certain forest planning problem (or adjacency graph).

<u>Define</u>: $P = \{x \in \{0,1\}^n : \sum_{i \in C} x_i \leq |C| - 1, \; \forall C \in \Omega\}.$

For every set of management units A, let $N(A)$ represent the set of all management units adjacent, but not belonging, to A.

<u>Define</u>: $\pi(s, C) = \max\{\sum_{j \in N(s) \cap C} x_j \, : \, x \in P \text{ and } x_s = 1\}$

# Strengthening the Minimal Covers Cont.

$\pi(s,C) = \max\{\sum_{j \in N(s) \cap C} x_j \ : \ x \in P \text{ and } x_s = 1\}$

**Proposition:** Consider a minimal cover C and $s \in N(C)$.

Define: $\alpha^* = \left(\left|N(s) \cap C\right| - \pi(s,C) - 1\right)$.

Then, for all $\alpha \leq \alpha^*$:

$$\sum_{j \in C} x_j + \alpha x_s \leq \left|C\right| - 1 \quad \text{is valid for P.}$$

# Strengthening the Minimal Covers Cont.

**Proof:** Consider $x \in P$. If $x_s = 0$, then the inequality holds by the definition of minimal cover C.

If $x_s = 1$, then:

$$\sum_{j \in C} x_j + \alpha x_s = \sum_{j \in C \setminus N(s)} x_j + \sum_{j \in N(s) \cap C} x_j + \alpha$$

$$\leq |C \setminus N(s)| + \sum_{j \in N(s) \cap C} x_j + \alpha^*$$

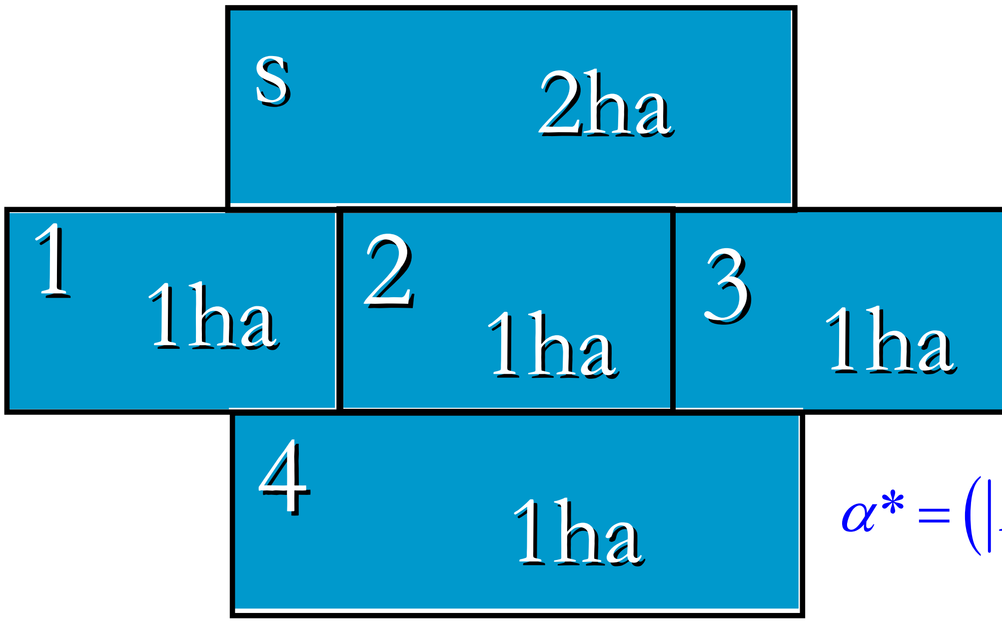
$$\leq |C \setminus N(s)| + \pi(s, C) + \alpha^*$$

$$= |C| - 1$$

$$\alpha^* = \left(|N(s) \cap C| - \pi(s, C) - 1\right)$$

# How strong are these inequalities?

$$A_{\max} = 3ha$$



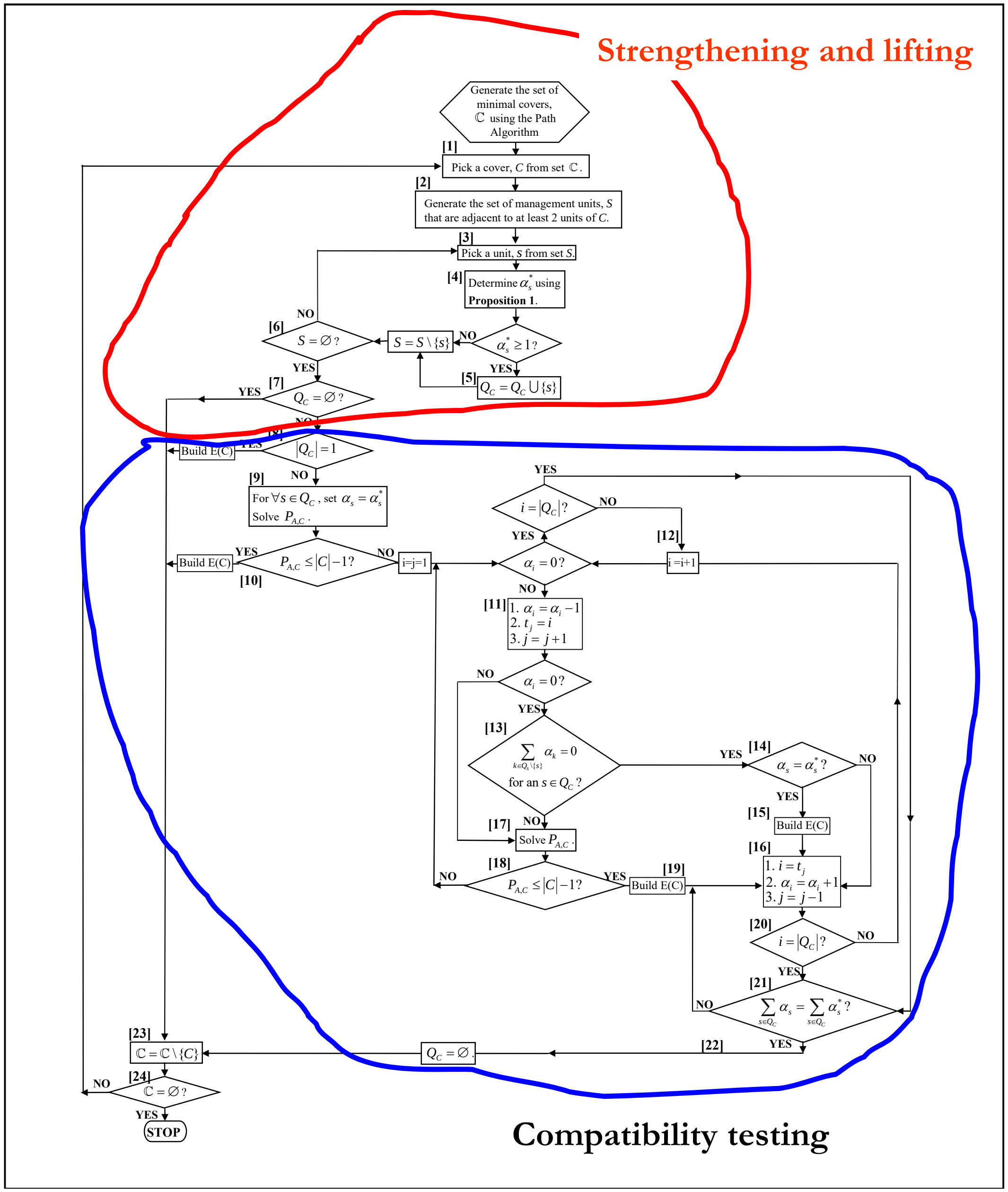

$$\alpha^* = \left(|N(s) \cap C| - \pi(s,C) - 1\right)$$

$$x_1 + x_2 + x_3 + x_4 \leq 3$$

$$x_1 + x_2 + x_3 + x_4 + 1x_s \leq 3$$

$$\boxed{x_1 + x_2 + x_3 + x_4 + 2x_s \leq 3}$$

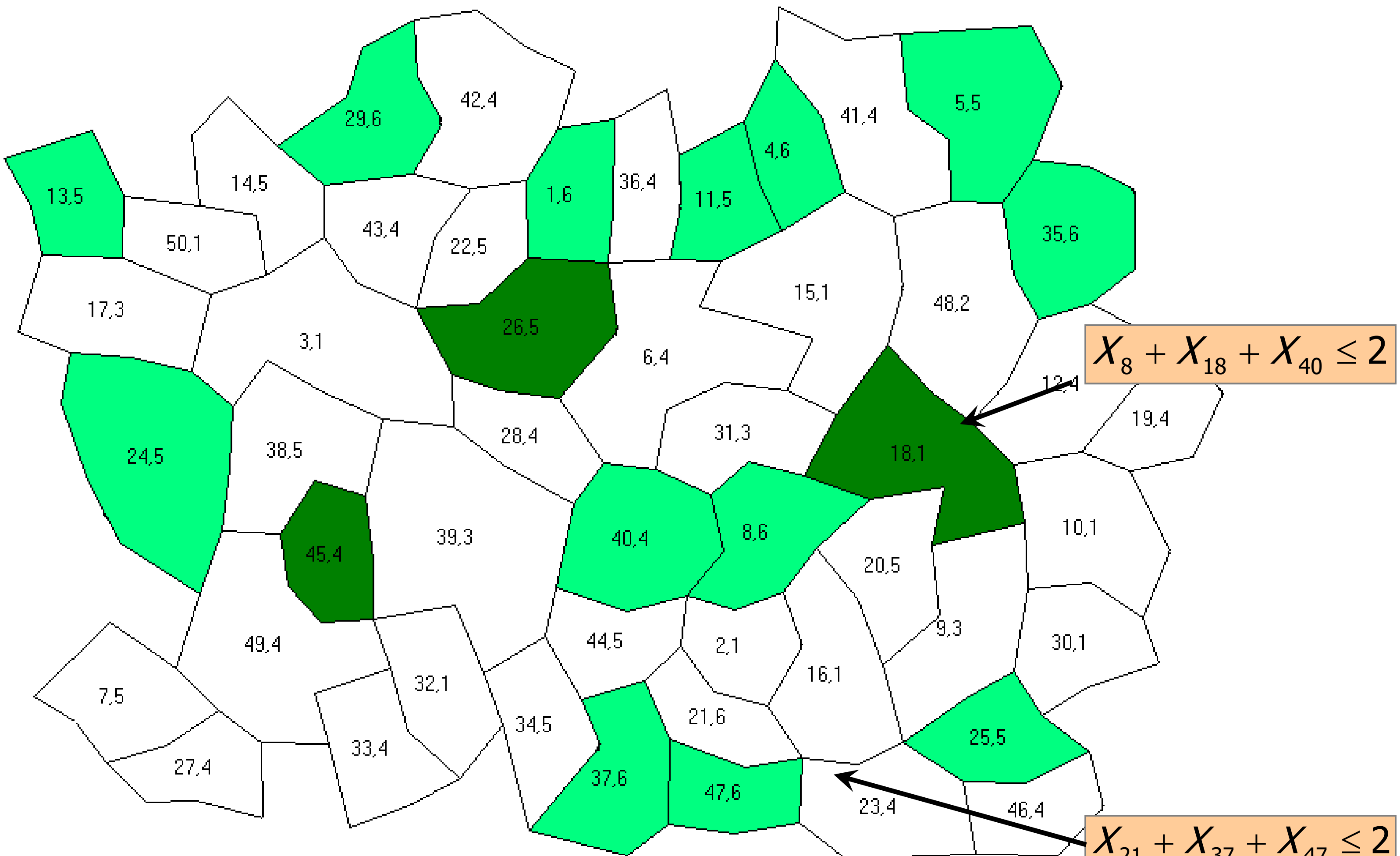
# Sequential Lifting Algorithm



# Open Questions:

1) Can one define a rule for multiple lifting?
2) Can one find even stronger inequalities?
3) Is there a better formulation?

# A “Cutting Plane” Algorithm for the Cover Formulation




**Stands to harvest after adding the first round of ARM cuts**