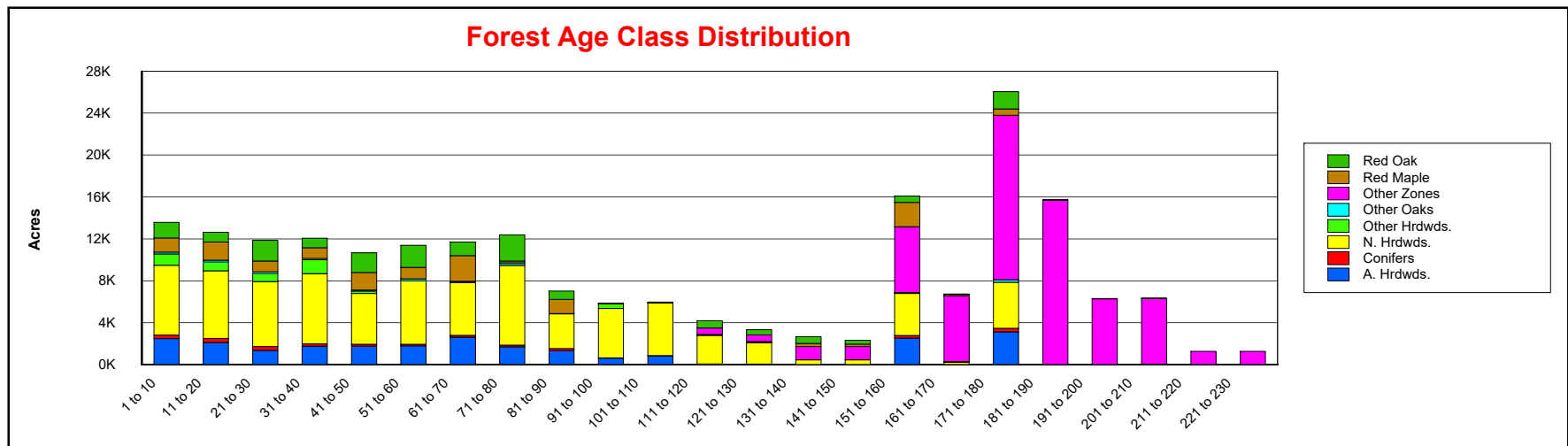
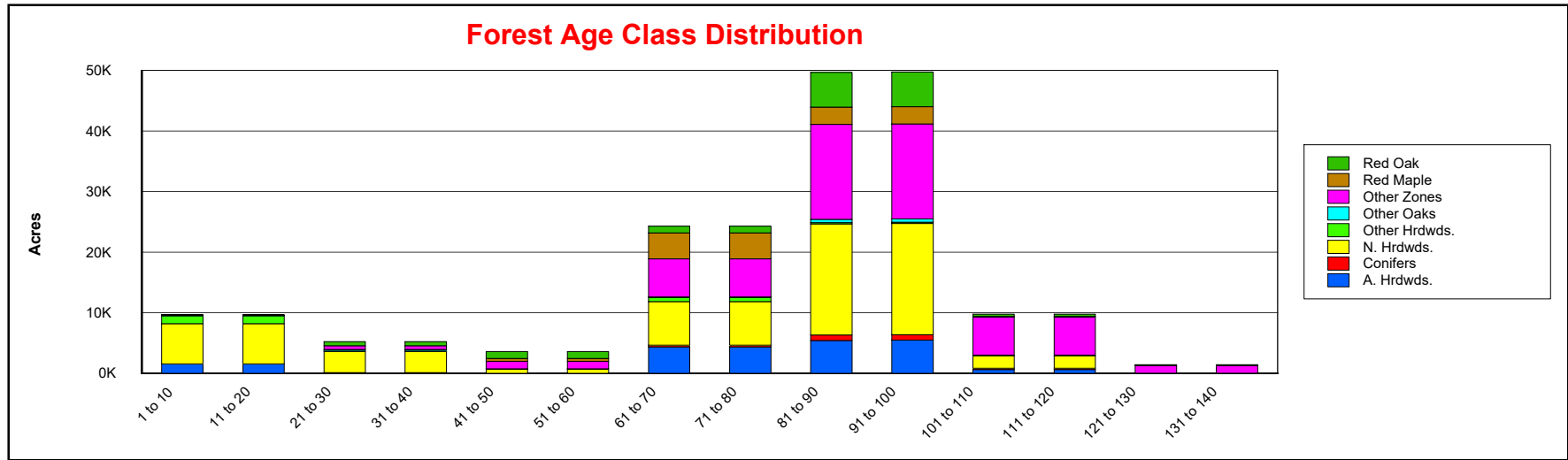


Modeling Non-Timber Objectives in Harvest Scheduling with Linear Programming

Lecture 14 (5/18/2017)

- Extended rotations: a minimum of 1,500ac will be required from each Site I and II to be older than 40 and 30 yrs, respectively;
- Wildlife openings: set aside 500ac of wildlife openings – clearcut these in pd. 1 and maintain them for browse and forage;
- Stream-side management zones: no harvests allowed in 8% of the forest (4% in each site class)

Balancing the Age-class Distribution



Harvest scheduling prescriptions with up to two possible harvests within a 40-year planning horizon and a minimum rotation of 20 yrs

Prescription	Planning Period				
	1	2	3	4	
1	Harvest				X_{sa10}
2	Harvest		Harvest		X_{sa13}
3	Harvest			Harvest	X_{sa14}
4		Harvest			X_{sa20}
5		Harvest		Harvest	X_{sa24}
6			Harvest		X_{sa30}
7				Harvest	X_{sa40}
8	No Harvest				X_{sa00}

$X_{sap_1p_2}$ = the number of acres from site class s , initial age-class a assigned to be harvested first in period p_1 and again in period p_2 .

Extended Rotations

Initial age-class distribution

Period	Variable							
	X_{sa10}	X_{sa13}	X_{sa14}	X_{sa20}	X_{sa24}	X_{sa30}	X_{sa40}	X_{sa00}
1	0	0	0	a	a	a	a	a
2	1	1	1	0	0	a+1	a+1	a+1
3	2	0	2	1	1	0	a+2	a+2
4	3	1	0	2	0	1	0	a+3

Age Classes	Acres by site class	
	Site I	Site II
0-10	3,000	8,000
11-20	6,000	4,000
21-30	9,000	7,000
Total	18,000	19,000

$$X_{2330} + X_{2340} + X_{2300} \geq 1,500 \quad (\text{ext. rot. constraint for site class II, pd.2})$$

$$X_{2240} + X_{2200} + X_{2340} + X_{2300} \geq 1,500 \quad (\text{ext. rot. constraint for site class II, pd.3})$$

$$X_{1340} + X_{1300} \geq 1,500 \quad (\text{ext. rot. constraint for site class I, pd.3})$$

$$X_{2100} + X_{2200} + X_{2300} \geq 1,500 \quad (\text{ext. rot. constraint for site class II, pd.4})$$

$$X_{1200} + X_{1300} \geq 1,500 \quad (\text{ext. rot. constraint for site class I, pd.4})$$

Modifying the Average Ending Age Constraint

$$\sum_{a=1}^3 \left[\text{Age}_{sa00}^{40} \cdot X_{sa00} + \sum_{p_1=1}^4 \left(\text{Age}_{sap_1 0}^{40} \cdot X_{sap_1 0} + \sum_{p_2=p_1+2}^4 \text{Age}_{sap_1 p_2}^{40} \cdot X_{sap_1 p_2} \right) \right] \geq \overline{\text{Age}}^{40} \cdot \text{TotalArea}_s$$

$$\text{TotalArea}_s = \sum_{a=1}^3 \left[X_{sa00} + \sum_{p_1=1}^4 \left(X_{sap_1 0} + \sum_{p_2=p_1+2}^4 X_{sap_1 p_2} \right) \right]$$

$$\overline{\text{Age}}^{40} \times \text{TotalArea}_s = \overline{\text{Age}}^{40} \times \sum_{a=1}^3 \left[X_{sa00} + \sum_{p_1=1}^4 \left(X_{sap_1 0} + \sum_{p_2=p_1+2}^4 X_{sap_1 p_2} \right) \right] =$$

$$\sum_{a=1}^3 \left[\overline{\text{Age}}^{40} X_{sa00} + \sum_{p_1=1}^4 \left(\overline{\text{Age}}^{40} X_{sap_1 0} + \sum_{p_2=p_1+2}^4 \overline{\text{Age}}^{40} X_{sap_1 p_2} \right) \right]$$

Modifying the Average Ending Age Constraint (continued)

$$\sum_{a=1}^3 \left[\text{AgeDif}_{sa00}^{40} \cdot X_{sa00} + \sum_{p_1=1}^4 \left(\text{AgeDif}_{sap_1 0}^{40} \cdot X_{sap_1 0} + \sum_{p_2=p_1+1}^4 \text{AgeDif}_{sap_1 p_2}^{40} \cdot X_{sap_1 p_2} \right) \right] \geq 0$$

$$\text{AgeDif}_{1100}^{40} \cdot X_{1100} + \sum_{a=1}^3 \sum_{p_1=1}^4 \left(\text{AgeDif}_{1ap_1 0}^{40} \cdot X_{1ap_1 0} + \sum_{p_2=p_1+2}^4 \text{AgeDif}_{1ap_1 p_2}^{40} \cdot X_{1ap_1 p_2} \right) \geq 0$$

$$\sum_{a=1}^3 \sum_{p_1=1}^4 \left(\text{AgeDif}_{2ap_1 0}^{40} \cdot X_{2ap_1 0} + \sum_{p_2=p_1+2}^4 \text{AgeDif}_{2ap_1 p_2}^{40} \cdot X_{2ap_1 p_2} \right) \geq 0$$

Implementing Wildlife Openings

- Forest management activity scheduling model;
- Creating 500 acres of wildlife browse habitat;
- The openings are cleared in Pd. 1 and will be maintained over time by planting browse species at a cost of \$10/ac/yr

1. Defining the wildlife opening variables:

W_{sa} = the number of acres from site class s , initial age class a assigned to be cleared in pd. 1 and maintained as wildlife openings for the remainder of the planning horizon

2. Modifying the Area Constraints

$$X_{sa00} + \sum_{p=1}^4 \left[X_{sap_1 0} + \sum_{p_2=p_1+2}^4 X_{sap_1 p_2} \right] + W_{sa} \leq A_{sa} \quad \text{for } s=1,2 \text{ and } a=1,2,3$$

Note: wildlife openings as the ninth prescription

3. Specifying the target area for wildlife openings

$$\sum_{s=1}^2 \sum_{a=1}^3 W_{sa} \geq 500$$

or,

$$\sum_{a=1}^3 W_{sa} \geq 500 \quad s=1,2$$

4. Modifying the objective function (accounting for the costs and revenues that are associated with the new activities)

$$\text{Max } Z = \sum_{s=1}^2 \sum_{a=1}^3 \left[\sum_{p_1=1}^4 \left(c_{sap_1 0}^p \cdot X_{sap_1 0} + \sum_{p_2=p_1+2}^4 c_{sap_1 p_2}^p \cdot X_{sap_1 p_2} \right) + c_{sa}^{wp} \cdot W_{sa} \right]$$

where

$c_{sap_1 p_2}^p$ = the discounted net revenue (profit) from assigning one acre from site class s , initial age class a to be harvested in periods p_1 and p_2 , and

c_{sa}^{wp} = the discounted net revenue from assigning an acre from site class s , initial age class a to be managed as a wildlife opening.

5. The objective function coefficients

$$c_{sa}^{wp} = \frac{(P - s_v)v_{sa10}^1 - s_f}{(1+r)^5} - \frac{c_w[(1+r)^{35} - 1]}{r(1+r)^{35}(1+r)^5}$$

where

P = the wood price,

s_v = the variable (per cord) timber sale administration cost,

s_f = the fixed (per acre) timber sale administration cost,

v_{sa10}^1 = the volume of wood that will be harvested in period 1 for each acre assigned to the variable W_{sa} .

c_w = the annual, per-acre cost of maintaining the wildlife openings, and

r = the real interest rate.

Example Objective Function Coefficient Calculation

Economic data

Item	Symbol	Amount
Wood Price	P	\$25/cd
Planting Cost	E	\$100.00/ac
Timber Sales Cost -per acre	s_f	\$15.00/ac
-per cord	s_v	\$0.20/cd
Interest Rate	r	4%

Yield

Harvest Age	Acres by site class	
	Site I	Site II
10	2	5
20	10	14
30	20	27
40	31	38
50	37	47
60	42	54
70	46	60

$$C_{23}^{WP} = \frac{(25 - 0.2) \cdot 27 - 15}{(1.04)^5} - \frac{10 \left[(1.04)^{35} - 1 \right]}{0.04(1.04)^{40}} = \underline{\underline{\$384.62}}$$

6. Modifying the harvest accounting constraints

$$\sum_{s=1}^2 \sum_{a=1}^3 \left[v_{sa10}^1 X_{sa10} + v_{sa13}^1 X_{sa13} + v_{sa14}^1 X_{sa14} + v_{sa10}^1 W_{sa} \right] - H_1 = 0$$

7. Average ending age constraints

The wildlife opening variables should be excluded from these constraints

Implementing Stream-side Management Zones (SMZs)

- It is assumed that the area assigned to SMZs has already been calculated, and
- 8% of each analysis area will be reserved for SMZs.

$$A_{sa}^* = A_{sa} \times (1 - smz_{sa})$$

$$X_{sa00} + \sum_{p=1}^4 \left[X_{sap_1 0} + \sum_{p_2=p_1+2}^4 X_{sap_1 p_2} \right] \leq A_{sa}^* \quad \text{for } s=1,2 \text{ and } a=1,2,3$$