# LEV and the Forest Value 

Lecture 7 (04/18/2016)

## The Financially Optimal Rotation Age



## LEV and MAI



## Marginal Analysis of the Rotation Decision

- The marginal benefits:

$$
M B_{a}=P \cdot \Delta Y_{\text {Price of wood }} P Y_{a+1} \cdot(1-{\underset{\underbrace{}}{\text { Income tax rate }}}_{\left.t_{i n c}\right)}
$$

Growth of wood $\mathrm{b} / \mathrm{w}$ age a and $\mathrm{a}+1$

- The marginal costs:

$$
\begin{aligned}
M C_{a}= & \underbrace{A\left(1-t_{\text {inc }}\right)}_{\text {annual management costs }} \underbrace{+t_{\text {prop }}\left(1-t_{\text {inc }}\right)}_{\text {property taxes }} \underbrace{+r \cdot L E V^{*}}_{\text {land rent }}+ \\
& \underbrace{+r \cdot P \cdot Y_{a}\left(1-t_{\text {inc }}\right)}_{\text {inventory cost }}
\end{aligned}
$$

## Marginal Costs and Revenues and the LEV



## An increase in the Interest Rate

| Establishment Cost (E ) | 200 |
| :--- | ---: |
| Price of Wood(P) | 600 |
| Property Tax ( $\mathrm{t}_{\text {inc }}$ ) | 2 |
| Annual Management Costs (A) | 1 |
| Income Tax $\left(\mathrm{t}_{\text {inc }}\right)$ | 0.22 |
| Interest Rate r | 0.03 |



## An increase in Stumpage Price

| Establishment Cost (E ) | 200 |
| :--- | ---: |
| Price of Wood(P) | 600 |
| Property Tax ( $\mathrm{t}_{\text {inc }}$ ) | 2 |
| Annual Management Costs (A) | 1 |
| Income Tax ( $\mathrm{t}_{\text {inc }}$ ) | 0.22 |
| Interest Rate $r$ | 0.04 |



| Establishment Cost (E ) | 200 |
| :--- | ---: |
| Price of Wood(P) | 800 |
| Property Tax ( $\mathrm{t}_{\text {inc }}$ ) | 2 |
| Annual Management Costs (A) | 1 |
| Income Tax $\left(\mathrm{t}_{\text {inc }}\right)$ | 0.22 |
| Interest Rate r | 0.04 |



- In increase in establishment costs?
- In increase in annual management costs and property taxes?
- In increase in severance tax?
- In increase in income tax?


## The impact of changes in economic variables on the financially optimal rotation age and LEV

|  | An increase in... | Financially Optimal <br> Rotation Age $\left(R^{*}\right)$ |  | LEV |
| :--- | :--- | :--- | :---: | :--- |
| $r$ | Real rate | Negative |  | Negative |
| $P$ | Price of wood | $E=0$ <br> 0 |  | $E>0$ <br> Neg. |
| $E$ | Establishment Cost | Positive |  | Positive |
| $A$ | Annual man. costs | 0 |  | Negative |
| $t_{\text {prop }}$ | Property tax | 0 |  | Negative |
| $t_{\text {inc }}$ | Income tax | 0 |  | Negative |
| $t_{\text {sever }}$ | Severance tax | $E=0$ <br> 0 | $E>0$ <br> Pos. | Negative |

$$
L E V=\frac{-E(1+r)^{R}+\sum_{t=1}^{R-1} I_{t}(1+r)^{(R-t)}+\sum_{p=1}^{n} P_{p} \cdot Y_{p, R}-C_{h}}{(1+r)^{R}-1}+\frac{A}{r}
$$

## Forest Value

- Land Expectation Value: present value of costs and revenues from an infinite series of identical even-aged forest rotations starting from bare land;
- Forest Value (a generalization of LEV): the present value of a property with an existing stand of trees + the present value of a LEV for all future rotations of timber that will be grown on the property after harvesting the current stand.


## The Forest Value allows us:

- To determine when a given stand should be cut;
- To separate the management of the current stand from that of future stands;
- To account for price changes that might occur during the life of the current stand;

Note: We will still assume that the rotations and prices associated with the future stands (i.e., the stands that are established after the current stand is cut) will be the same.


## When to cut the stand?

- Cut it now:
\$5,948/ac
- Forest Value $=$ Current Timber Value + LEV

$$
L E V=\frac{-E(1+r)^{R}+\sum_{t=1}^{R-1} I_{t}(1+r)^{(R-t)}+\sum_{p=1}^{n} P_{p} \cdot Y_{p, R}-C_{h}}{(1+r)^{R}-1}+\frac{A}{r}=
$$

$$
=\frac{\$ 84(1.05)^{(60-30)}+\$ 4,400}{(1.05)^{60}-1}-\frac{\$ 5}{0.05}=
$$

$$
=\frac{\$ 363.04316+\$ 4,400}{17.67919}-\$ 100=\underline{\$ 169.42 / a c}
$$

$$
F V_{0}=\$ 5,948 / a c+\$ 169.42 / a c=\underline{\underline{\$ 6,117.42 / a c}}
$$

## When to cut the stand?

- Cut it 10 years from now:
- Forest Value $=$ Present Value of Costs and Revenues for first 10 years + Present Value of LEV

$$
\begin{aligned}
& P V_{L E V}=\frac{L E V}{(1+0.05)^{10}}=\frac{\$ 169.42}{1.62889}=\underline{\$ 104.01 / a c} \\
& P V_{\text {CurrentRotation }}=\frac{\$ 7,884}{(1.05)^{10}}-\frac{\$ 5\left(1.05^{10}-1\right)}{0.05(1.05)^{10}}= \\
& \quad=\$ 4,840.09-\$ 38.61=\$ 4,801.48 / a c
\end{aligned}
$$

$$
F V_{10}=P V_{\text {CurrentRotation }}+P V_{L E V}=\underline{\underline{\$ 4,905.49 / a c}}
$$

## Forest Value

- Assumptions:

1. The current stand will be harvested;
2. A new stand will be established;
3. All future rotations of the new stand will be identical.

- Definition:
- The Forest Value is the present value of the projected costs and revenues from an existing forest tract, plus the present value of an infinite series of identical future forest rotations that starts after the current tract is harvested.


## Calculating the Forest Value

## - New notation:

$T_{0}=$ the time when the currect stand is to be cut;
$\mathrm{Y}_{\mathrm{p}, \mathrm{T}_{0}}^{C}=$ the expected yield of product p from the current stand at time $\mathrm{T}_{0}$; and
$C_{h}^{C}=$ the cost of selling the current stand of timber.

- Forest Value formula:


## Land value and timber value

- Forest Value = Land Value + Timber Value
- Land Value = LEV
- Timber Value = Forest Value - LEV

Timber Value $=\frac{\sum_{p=1}^{n} P_{p} \cdot Y_{p, T_{0}}^{C}-C_{h}^{C}}{(1+r)^{T_{0}}}-\frac{\stackrel{\text { Annual Land Cost }}{(r \cdot L E V-A)} \cdot\left[(1+r)^{T_{0}}-1\right]}{r(1+r)^{T_{0}}}$

## What if real prices change?

- Assumption: the price changes will end by the end of the current rotation

Timber Value $=\frac{\sum_{p=1}^{n} P_{p, T_{0}} \cdot Y_{p, T_{0}}^{C}-C_{h}^{C}}{(1+r)^{T_{0}}}-\frac{\stackrel{\text { Annual Land Cost }}{(r \cdot L E V-A)} \cdot\left[(1+r)^{T_{0}}-1\right]}{r(1+r)^{T_{0}}}$

When calculating the LEV, use the new, steady state price: $P_{p, \infty}$

## An example

| Item | Amount |
| :--- | :--- |
| Assumptions for the Current and Future Stands |  |
| Current sawtimber volume | $18 \mathrm{mbf} / \mathrm{ac}$ |
| Current pulpwood volume | $14 \mathrm{cords} / \mathrm{ac}$ |
| Current sawtimber price | $\$ 325 / \mathrm{mbf}$ |
| Current pulpwood price | $\$ 7 / \mathrm{cord}$ |
| Expected sawtimber volume in 10yrs | $24 \mathrm{mbf} / \mathrm{ac}$ |
| Expected pulpwood volume in 10yrs | $12 \mathrm{cords} / \mathrm{ac}$ |
| Expected real sawtimber price in 10yrs | $\$ 450 / \mathrm{mbf}$ |
| Expected real pulpwood price in 10yrs | $\$ 15 / \mathrm{cord}$ |
| Property tax | $\$ 5$ |
| Real alternate rate of return | $5 \%$ |
| Assumptions for the Current and Future Stands |  |
| Timber stand improvement cut (age 30 yrs) <br> pulpwood harvest | 12 cords/ac |
| Final (age 60) sawtimber harvest | $13 \mathrm{mbf} / \mathrm{ac}$ |
| Final (age 60) pulpwood harvest | 25 cords/ac |

## Cut now:

Timber value $=\sum_{p=1}^{2} P_{p, 0} \cdot Y_{p, 0}^{c}=\$ 325 / m b f \cdot 18 m b f+\$ 7 / c d \cdot 14 c d=\$ 5,948$
$F V^{\prime}{ }_{R 1}=12 \cdot \$ 15 \cdot(1.05)^{30}+13 \cdot \$ 450+25 \cdot \$ 15=\$ 7,002.95$
$L E V=\frac{F V^{\prime}{ }_{R 1}}{(1+r)^{R}-1}-\frac{\operatorname{tax}}{r}=\frac{\$ 7,002.95}{(1.05)^{60}-1}-\frac{\$ 5}{0.05}=\$ 296.11$
ForestValue $_{\text {cutnow }}=\$ 5,948+\$ 296.11=\underline{\underline{\$ 6,244.13}}$

Cut in 10 yrs:
Timber value $=\sum_{p=1}^{2} P_{p, 10} \cdot Y_{p, 10}^{c}=\$ 450 / m b f \cdot 24 m b f+\$ 15 / c d \cdot 12 c d=\$ 10,980$ $P V_{\text {timber }}=\frac{\$ 10,980}{(1.05)^{10}}-\$ 38.61=\$ 6,740.77-\$ 38.61=\$ 6,702.16$
$P V_{L E V}=\frac{\$ 296.11}{(1.05)^{10}}=\$ 181.79$

ForestValue $_{\text {Cuttl10 } y \text { rs }}=\$ 6,702.16+\$ 181.79=\underline{\underline{\$ 6,883.95}}$

