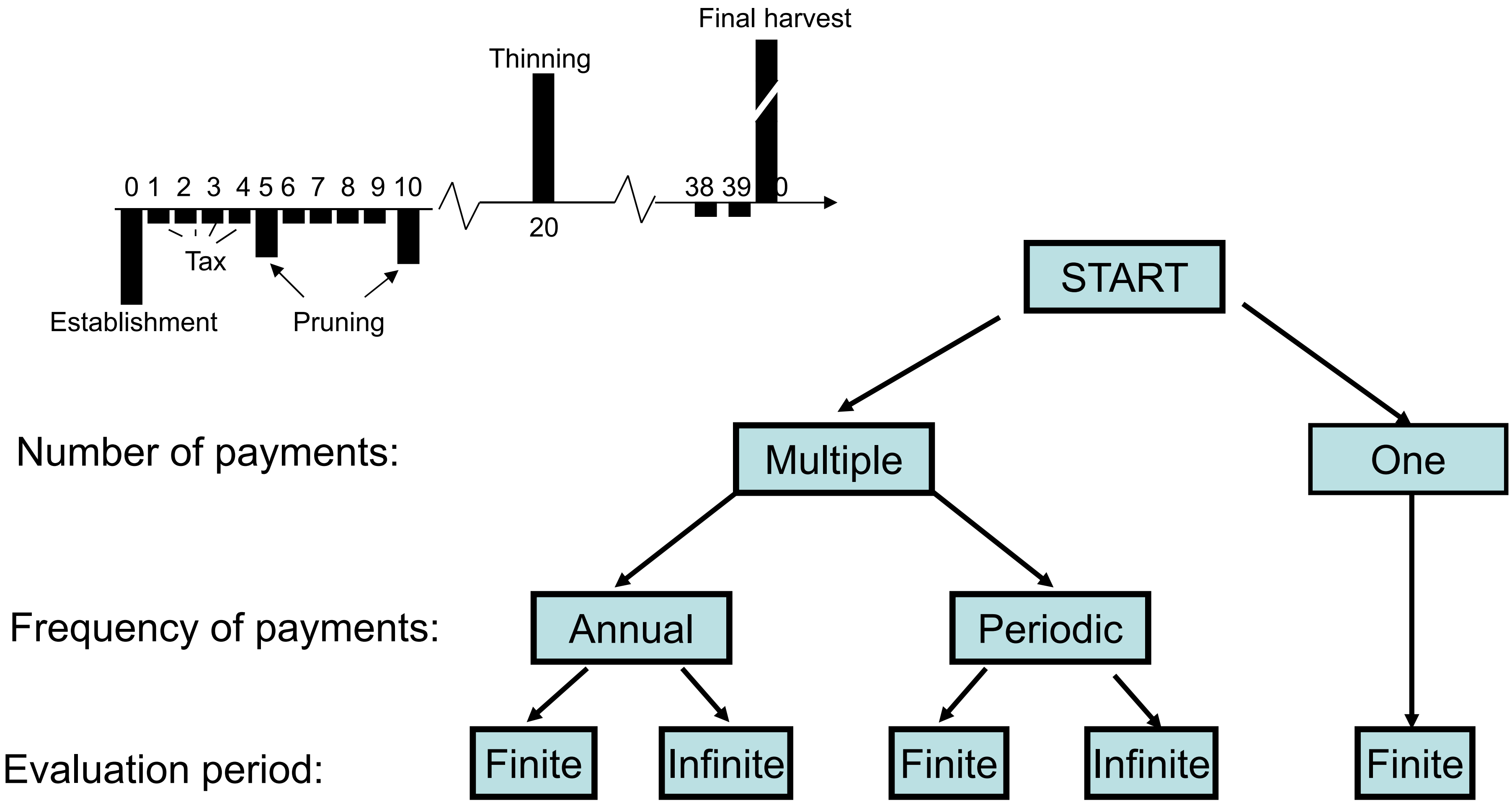
Overview of Discounting

Lecture 3 (04/6/2016)

Decision tree for selecting the formulas



Finite Annual Payments

Present Value: $$V_0 = \frac{R[(1+i)^n - 1]}{i(1+i)^n}$$

Future Value: $$V_n = \frac{R[(1+i)^n - 1]}{i}$$

Infinite Annual Payments

Present Value: $$V_0 = \frac{R}{i}$$

Future Value: Infinity

Finite Periodic Payments

Present Value: $$V_0 = \frac{R[(1+i)^n - 1]}{[(1+i)^t - 1](1+i)^n}$$

Future Value: $$V_n = \frac{R[(1+i)^n - 1]}{(1+i)^t - 1}$$

Infinite Periodic Payments

Present Value: $$V_0 = \frac{R}{(1+i)^t - 1}$$

Future Value: Infinity

Single Value Formulas

Present Value: $$V_0 = V_n(1+i)^{-n} = \frac{V_n}{(1+i)^n}$$

Future Value: $$V_n = V_0(1+i)^n$$

Financial Analysis with Inflation I.

Definition

- Inflation: an increase in average price level, reducing the purchasing power of a unit currency (deflation is the reverse process)
- Inflation rate: average annual rate of increase in the price of goods

Measuring Inflation

- Consumer Price Index (CPI)*: measures the average increase in the cost of a standard collection of consumer goods (market basket)
- Producer Price Index (PPI): measures the average increase in the cost of a standard collection of production inputs

*CPI: *the Consumer Price Index for All Urban Consumers (CPI-U) for the U.S. City Average for All Items, 1982-84=100.*

The Average Annual Inflation Rate

$$k = \sqrt[(t_2 - t_1)]{\frac{CPI_{t_2}}{CPI_{t_1}}} - 1$$

- Example: Calculate the average annual inflation rate for the last 30 years (1985-2015)
- Solution: Use the website at http://stats.bls.gov to get CPIs:

$$k = \sqrt[(2015-1985)]{\frac{CPI_{2015}}{CPI_{1985}}} - 1 = \sqrt[30]{\frac{207.8}{105.5}} - 1 = 0.02285 = \underline{\underline{2.285\%}}$$

Components of the Interest Rate

- The nominal rate: includes both the cost of capital and inflation;
- The real rate: is the rate earned on an investment after accounting for inflation. This is the real return for investing one's money.

the nominal rate ≈ the inflation rate + the real rate

$$\underline{i \approx k + r}$$

Components of the Real Rate

- Real rate ≈ the pure rate
  + risk premium
  + illiquidity premium
  + tax premium
  + transaction cost premium
  + time period premium

Combining Interest Rates

- Let *i* = the nominal rate;
- *r* = the real rate; and
- *k* = the inflation rate.

$$i = \frac{\left[R(1+r)\right](1+k) - R}{R} = (1+r)(1+k) - 1 =$$

$$= \underline{\underline{r + k + rk}} \approx r + k$$

$$\boxed{i = r + k + rk; \qquad r = \frac{(1+i)}{(1+k)} - 1; \qquad k = \frac{(1+i)}{(1+r)} - 1}$$

Combining Interest Rates

- <u>Example:</u> You bought a house in 1985 for $120,000. In 2015 it was appraised at $450,000. How much was your real rate of return on this house if the average annual inflation rate between 1985 and 2015 was 3%?
- <u>Solution:</u>
  - What do we know? V_{1985}=$120K; V_{2015}=$450K; k = 0.02285.
  - What do we need to know? r = ?

- <u>Solution:</u>
  - Which formula to use? $$r = \frac{(1+i)}{(1+k)} - 1$$
  - How do we calculate *i* ?

$$i = \sqrt[(2015-1985)]{\frac{V_{2015}}{V_{1985}}} - 1 = \sqrt[30]{\frac{\$450,000}{\$120,000}} - 1 = \underline{0.045044}$$

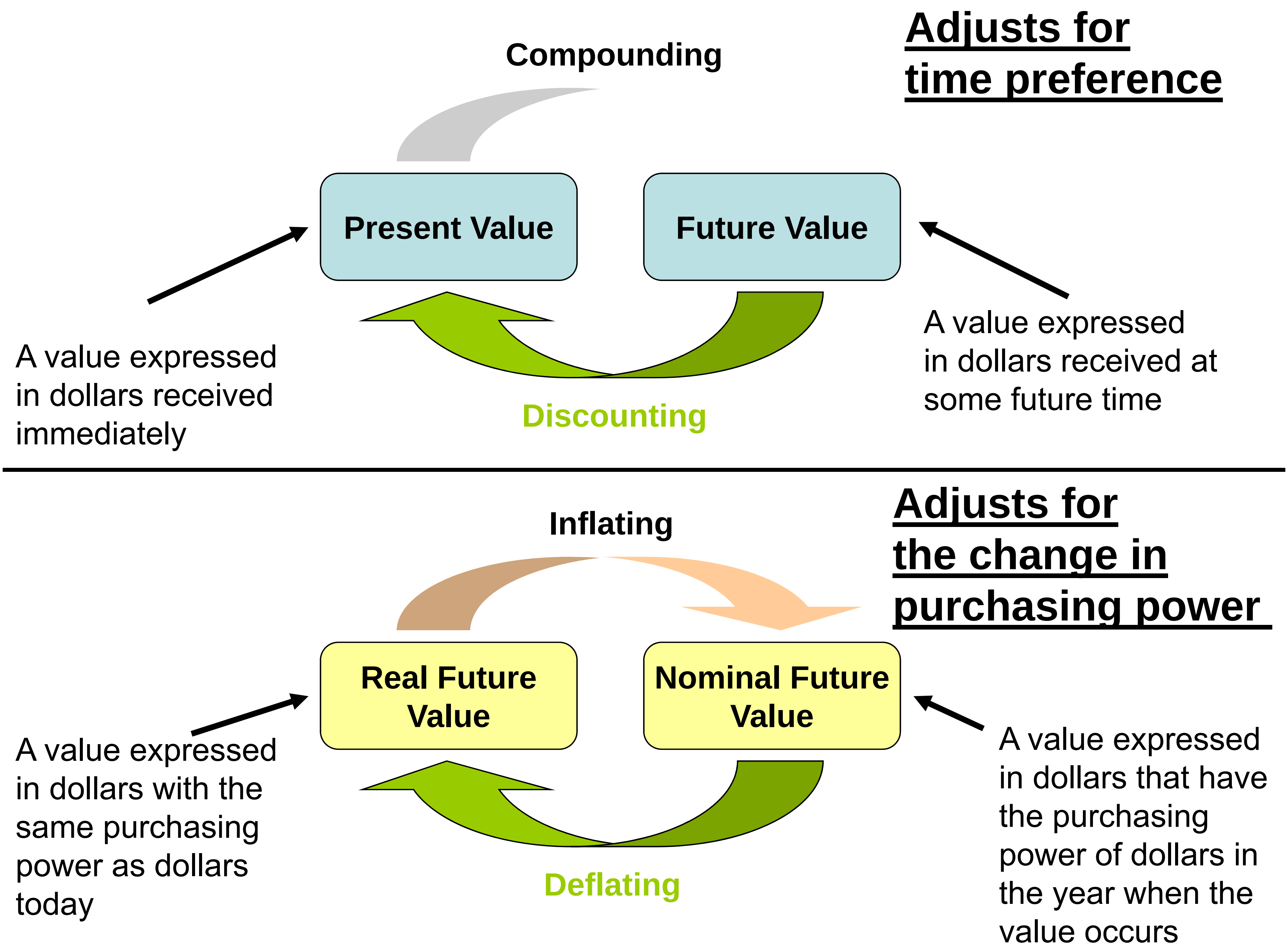
  - Calculate *r*:

$$r = \frac{1+0.045044}{1+0.02285} - 1 = 0.0217 \approx \underline{\underline{2.2\%}}$$

Deflating and Inflating

- Deflating: The process of converting a value expressed in the currency of a given point in time into a value expressed in the currency of an earlier time with the same purchasing power ;
- Inflating: is the reverse process.

Note: Historical inflation rates are available to inflate past values to the present.


Adjusts for time preference
Compounding
Present Value
Future Value
Discounting
A value expressed in dollars received immediately
A value expressed in dollars received at some future time
Adjusts for the change in purchasing power
Inflating
Real Future Value
Nominal Future Value
Deflating
A value expressed in dollars with the same purchasing power as dollars today
A value expressed in dollars that have the purchasing power of dollars in the year when the value occurs

Deflating and Inflating

$V_n^* =$ nominal value occuring in year n,

$V_n =$ real value occuring in year n, and

$\mathrm{n}_0 =$ reference year.

$$\underline{\text{Real value:}}\;\; V_n = (1+k)^{-(n-n_0)} V_n^*$$

$$\underline{\text{Nominal value:}}\;\; V_n^* = (1+k)^{(n-n_0)} V_n$$

Note: Deflating/inflating is mathematically same as discounting/compounding but conceptually very different.

Example 1

- How much would a salary of \$100,000 in 2020 be worth in current (2015) dollars if the forecasted average annual inflation rate is 3%?
- <u>Solution:</u>

1. What do we know? $V^*_{2020} = \$100,000,\ \mathrm{n}_0 = 2015$
2. What do we need to know? $V_{2015} = ?$
3. Which formula to use?

$$V_{2015} = (1+0.03)^{-(2020-2015)} V^*_{2020} =$$

$$= 1.03^{-5} \cdot \$100,000 = \underline{\underline{\$86,260.88}}$$

Example 2

- How much would a salary of $15,000 in 1976 be worth in current dollars (2015)?
- <u>Solution:</u>

1. What do we know? $V_{1976}^{*} = \$15,000,\ \mathrm{n}_0 = 2015$

2. What do we need to know? $V_{1976} = ?$

3. Which formula to use?

$$V_{1976} = (1+k)^{-(1976-2015)} V_{1976}^{*},$$

$$where\ \ k = \sqrt[2015-1976]{\frac{CPI_{2015}}{CPI_{1976}}} - 1$$

Example 2

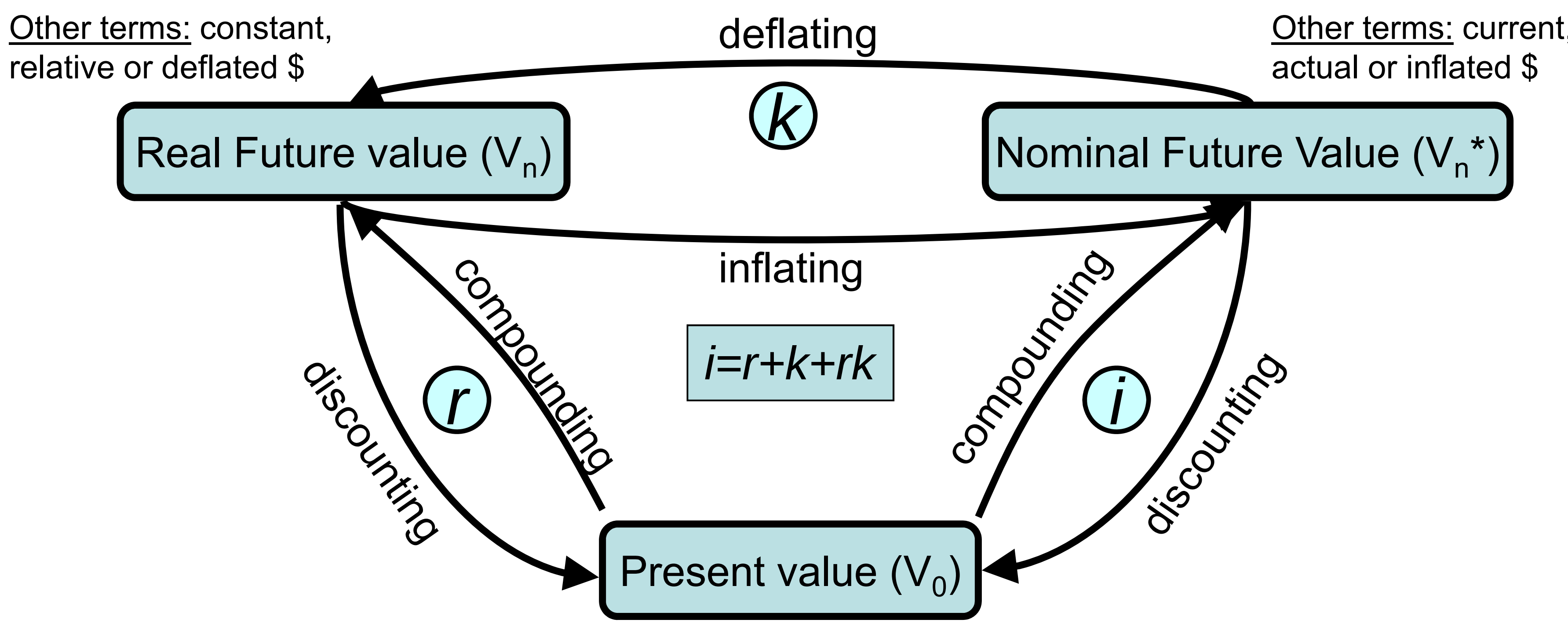
$$V_{1976} = \left(1 + \sqrt[39]{\frac{CPI_{2015}}{CPI_{1976}}} - 1\right)^{39} V^*_{1976} = \left(\sqrt[39]{\frac{CPI_{2015}}{CPI_{1976}}}\right)^{39} V^*_{1976} =$$

$$= \frac{CPI_{2015}}{CPI_{1976}} \cdot V^*_{1976} = \frac{207.8}{56.9} \cdot \$15,000 = \underline{\underline{\$54,780.32}}$$

Rules of discounting with inflation

- Discount nominal future values with a nominal rate and discount real future values with a real rate;
- When a present value is compounded by a real rate, then the result is a real future value;
- When a present value is compounded by a nominal rate, then the result is a nominal future value.

Discounting with inflation

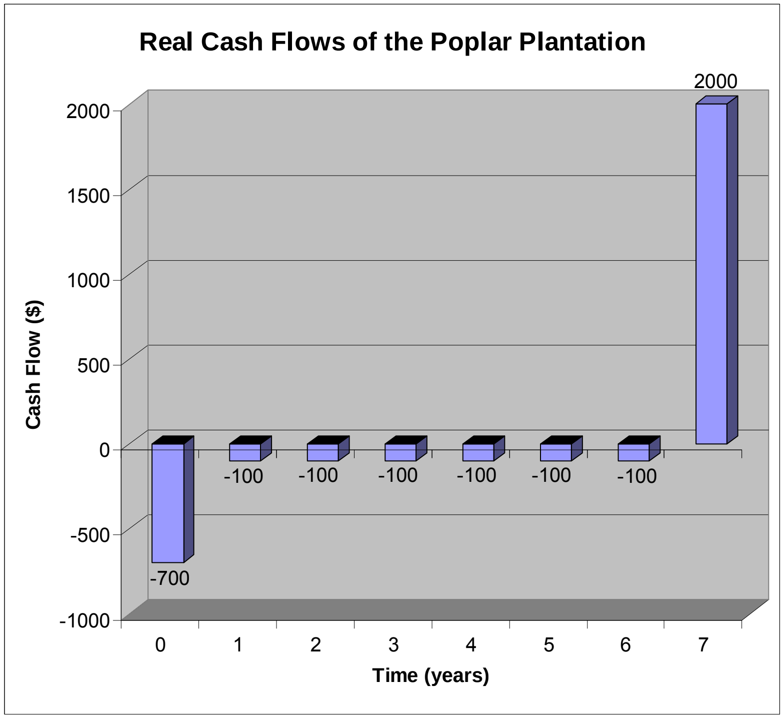


Note: It is often hard to tell if a future value is real or nominal

A hybrid poplar plantation

- The plantation can be established for $600/ac on a land that can be rented for $100/ac/year. You expect the land rent to go up at about the same rate as the inflation rate (=4%/year). After 7 years, the plantation will produce 20 tons of chips per acre. The current price for chips is $100/ton and you expect this price to go up at the rate of the inflation. What is the present value of the poplar project at an 8% real interest rate?

- **The cash flows**



Year	Future Values		Present Values ($)
	Real (constan values)	Nominal (current values)	
0	-700		
1	-100		
2	-100		
3	-100		
4	-100		
5	-100		
6	-100		
7	2000		
NPV	N/A	N/A	

$\text{Real} \xrightarrow{k} \text{Nominal} \xrightarrow{i} \text{Present Values}$, $\quad \text{Real} \xrightarrow{r} \text{Present Values}$

k = 4%
r = 8%

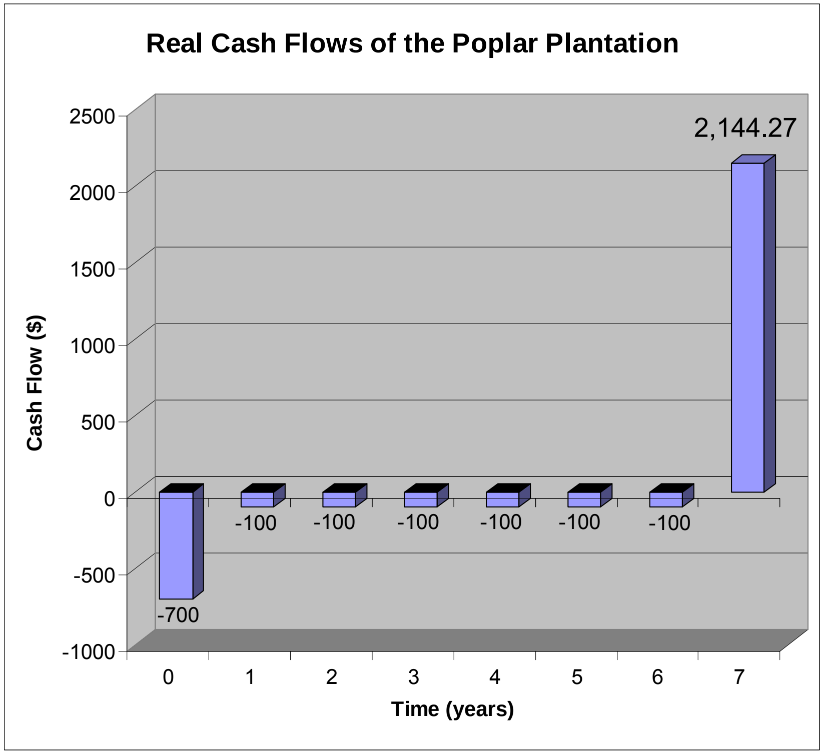
$$i = r + k + rk = 0.08 + 0.04 + 0.08 \cdot 0.04 =$$

$$= 0.1232 = \underline{12.32\%}$$

The poplar plantation example with changing prices

- Everything is the same as in the poplar project except that the price per ton of chips will now rise by 1% above inflation (i.e., there is a 1% real annual increase in the price of chips). What is the present value of the poplar project at an 8% real interest rate?

- **The cash flows**



Year	Future Values		Present Values ($)
	Real (constant values)	Nominal (current values)	
0	-700		
1	-100		
2	-100		
3	-100		
4	-100		
5	-100		
6	-100		
7			
NPV	N/A	N/A	

$k = 4\%$

$r = 8\%$

$i = 12.32\%$

$P_{chips,7}=100*(1+0.01)^7=\$107.21/ton$

Calculating real Softwood Sawtimber Stumpage Price Changes in Louisiana

- In the first quarter of 1986, softwood sawtimber stumpage in Louisiana sold for an average of $101.68/mbf. In the first quarter of 2006, the average sawtimber stumpage price in the state was $348.68. The PPI for the first quarter of 1986 was 95.03, and in the first quarter of 2006 it was 165.17. What were the real and nominal rates of change in the price of LA softwood sawtimber over this 20-year period?

1. What do we know?

   PPI$_{1986}$=95.03;   PPI$_{2006}$= 165.17

   P*$_{1986}$=$101.68;   P*$_{2006}$ = P$_{2006}$ =$ 348.68

2. Nominal price change (annual average):

$$i_{sst\_price} = \sqrt[20]{\frac{\$348.68}{\$101.68}} - 1 = \underline{\underline{6.3554\%}}$$

3. Real price change (annual average):

   1. Inflate the nominal, 1986 price to current (2006) dollars:

$$P_{1986} = P^{*}_{1986} \cdot \frac{PPI_{2006}}{PPI_{1986}} = \$101.68 \cdot \frac{165.17}{95.03} = \underline{\$176.73}$$

2. Calculate average annual real price change:

$$r_{sst\_price} = \sqrt[20]{\frac{\$348.68}{\$176.73}} - 1 = \underline{\underline{3.456\%}}$$

4. <u>An alternative way to calculate the real rate of price change:</u>
   1. First, calculate the inflation rate from the PPIs:

$$k = \sqrt[20]{\frac{165.17}{95.03}} - 1 = \underline{2.8025\%}$$

   2. Calculate the real rate using the nominal rate and the inflation rate:

$$r_{sst\_price} = \frac{1 + i_{sst\_price}}{1 + k} - 1 = \frac{1.063554}{1.028025} - 1 = \underline{\underline{3.456\%}}$$

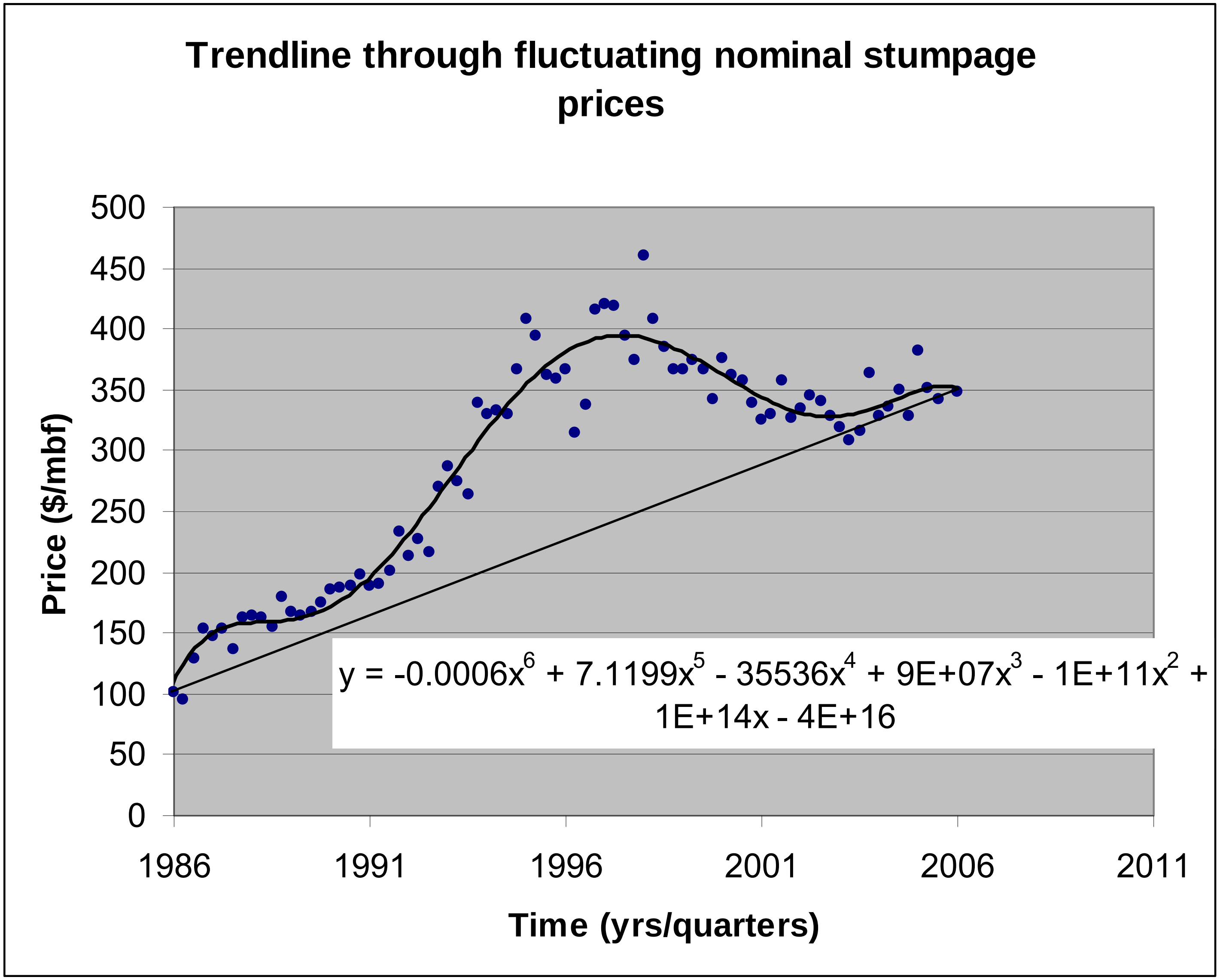
Projection of LA softwood sawtimber stumpage prices

- What will the nominal and real softwood sawtimber stumpage price be in LA in 2010 if the average annual inflation rate as well as the nominal and real rates of price increases are assumed to remain the same as in the last 20 years?

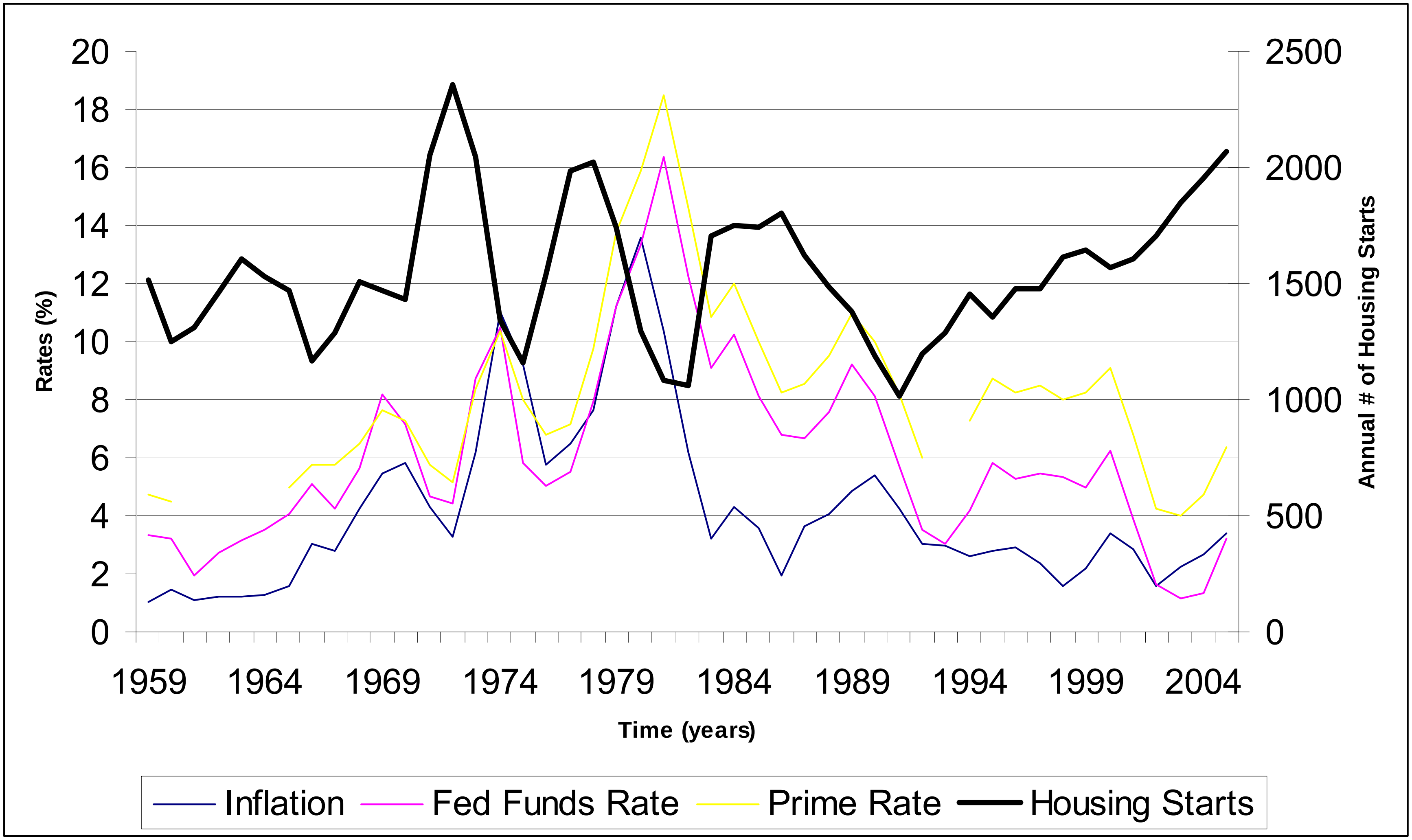
$$P_{2010}^{*} = P_{2006} \cdot (1 + i_{sst\_price})^{4} = \$348.69 \cdot (1 + 0.063554)^{4} = \underline{\underline{\$446.15}}$$

$$P_{2010} = P_{2006} \cdot (1 + r_{sst\_price})^{4} = \$348.69 \cdot (1 + 0.03456)^{4} = \underline{\underline{\$399.45}}$$

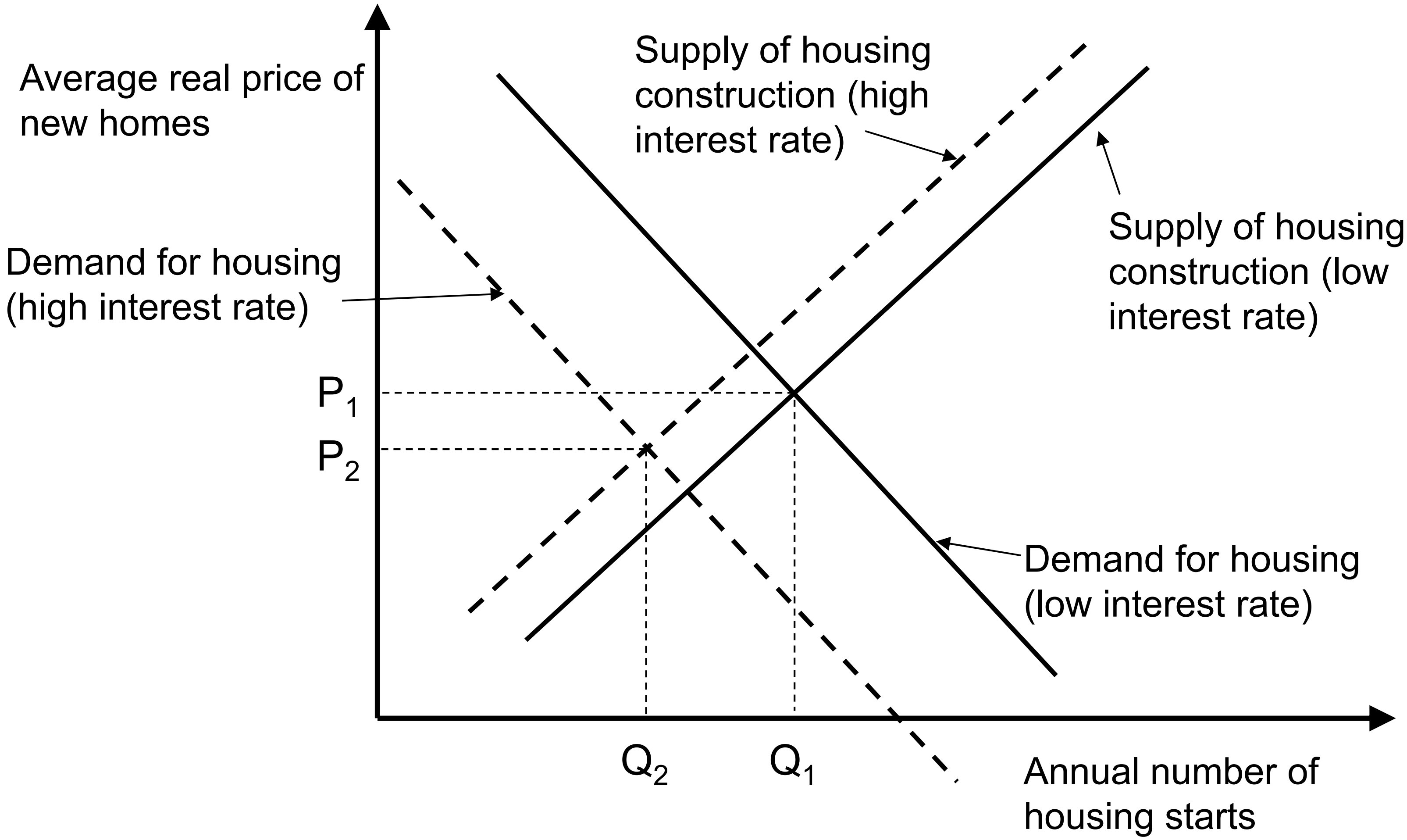
Projecting prices using trendlines



Housing Starts and Inflation



Supply and Demand for Housing Starts



Key interest rates

- The Federal Funds Rate: is the overnight rate that banks charge other banks for borrowing money. This rate is the Federal Reserve's main tool for influencing economic growth and stability.
- "The Prime Rate is the lending interest rate banks charge their most steady, credit-worthy customers [usually large, conservatively financed businesses]. The prime rate is the same for almost all major banks." (source: http://beginnersinvest.about.com/od/primerate/) This rate can be used to approximate mortgage rates.