Introduction to Discounting

Lecture 2 (03/30/2017)

Outline

- What is financial analysis?
- Why is it important in forestry?
- Basic discounting concepts
  - Present and future values
  - The interest rate
  - The single value formula
- Examples

Financial Analysis

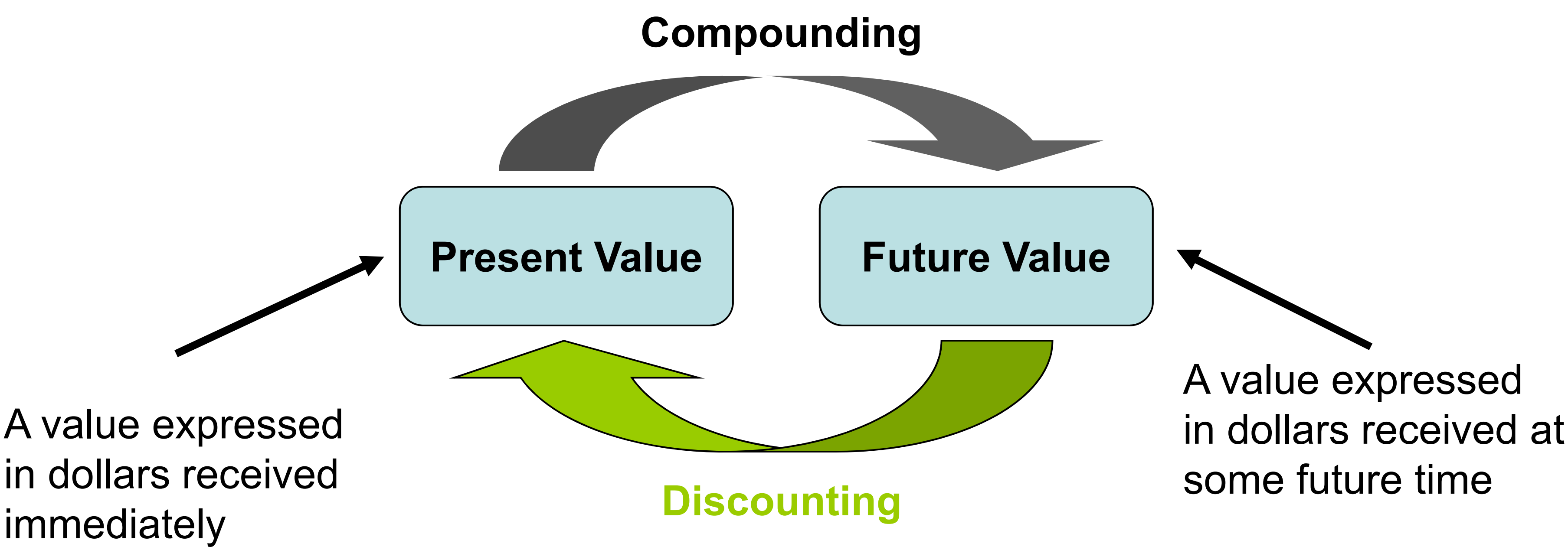
- Evaluates forest management alternatives based on profitability;
- Evaluates the opportunity costs of alternatives;
- Cash flows of costs and revenues;
- The timing of payments is important. Why?

Why is financial analysis important in forestry?

- Forests are valuable financial assets: if foresters do not understand financial analysis, forests will be managed by those who do;
- Financial analyses allow foresters to make financially sound forest management decisions;
- <u>Remember, however:</u> the financial criterion is only one of the many decision factors in forestry!

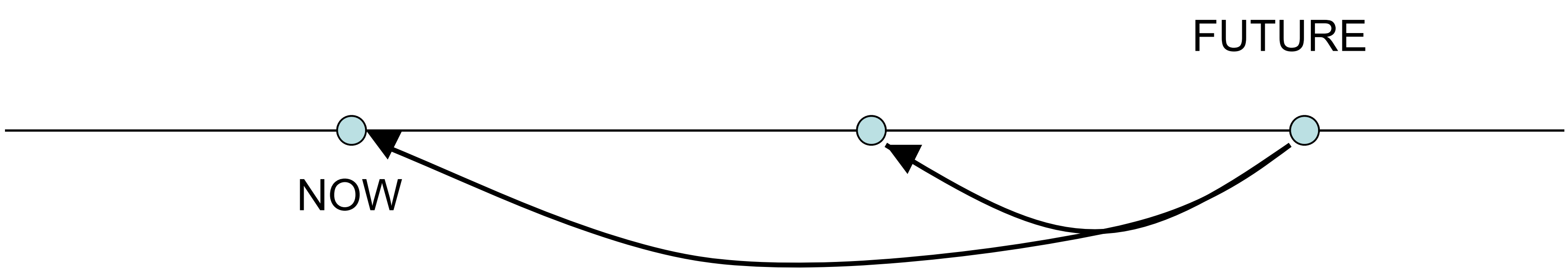
What is discounting?

- A process that accounts for time preferences
- Converts *future values* to *present values*



Definition of Discounting

- *The process of converting values expressed in dollars received at one point in time to an equivalent value expressed in dollars received at an earlier point in time*
- *Compounding is the reverse process)*

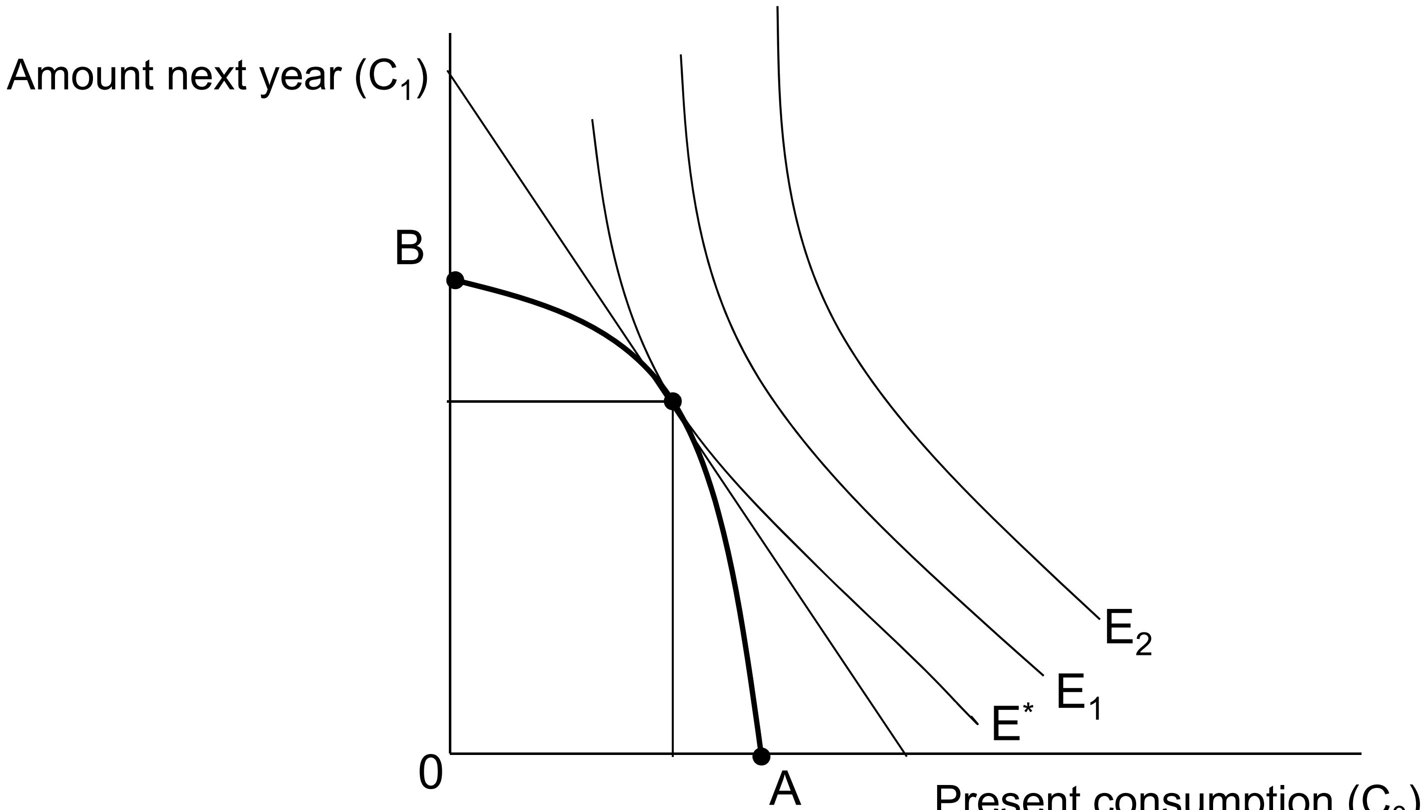


The interest rate

- Time preference: = human nature + potential investments
- Money can make money over time
- Corollary: using money costs money
- *The interest rate determines the relationship between present and future values*

Interest rate as a trade-off

(the economy of Robinson Crusoe, Buongiorno & Gilles 2003)



Source: Buongiorno and Gilles 2003, p. 374

The interest rate

- Also: *the interest rate is the percentage of the amount invested or borrowed that is paid in interest after one unit of time*

$$V_1 = V_0 + iV_0 = principal + interest$$

$$\boxed{V_1 = V_0(1+i)}$$

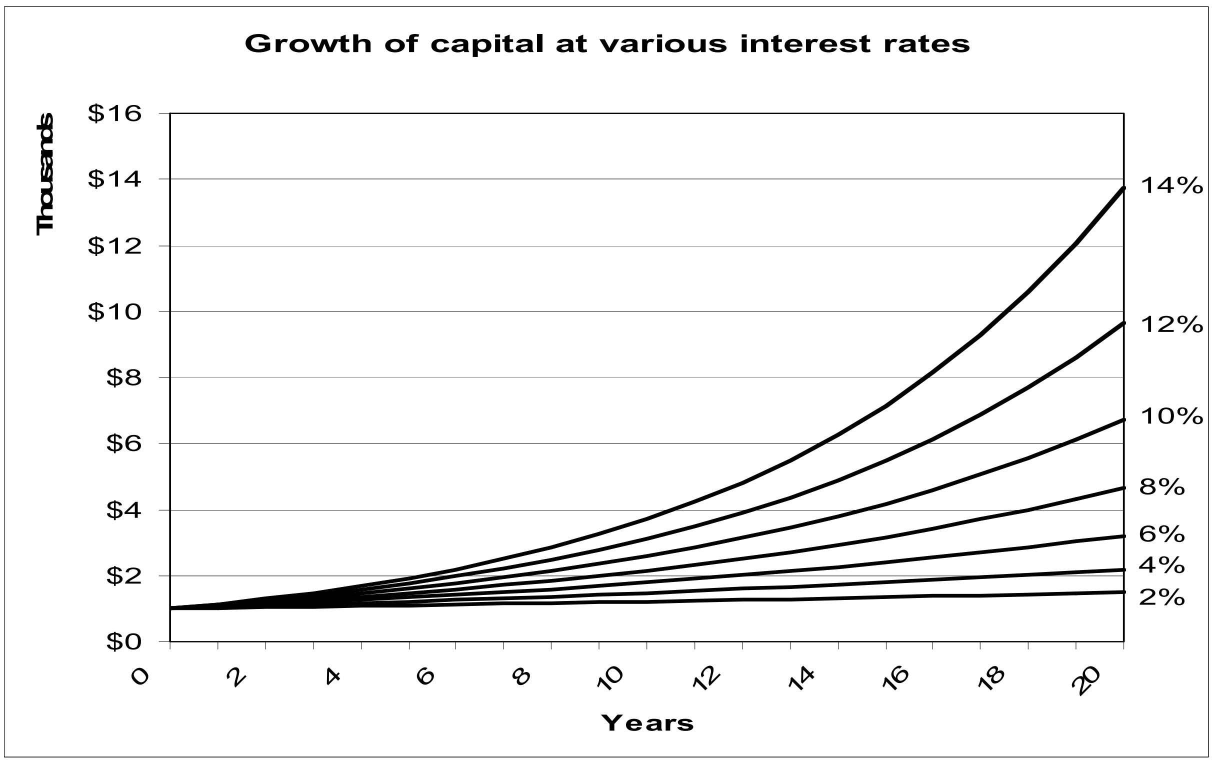
The future value at a compound interest:

$$V_2 = \left(V_0(1+i)\right)i + V_0(1+i) = V_0(1+i)(1+i) = \boxed{\underline{V_0(1+i)^2}}$$

$$V_3 = \left(V_0(1+i)^2\right)i + V_0(1+i)^2 = V_0(1+i)^2(1+i) = \boxed{\underline{V_0(1+i)^3}}$$

The Single Value Formula

$$Future\ Value:\ V_n = V_0(1+i)^n$$



Example:

- If you put $100 into a bank account today at a 6.5% interest, how much will you have after 12 years?

- <u>Solution:</u>

$$V_n = V_0(1+i)^n \rightarrow V_{12} = \$100 \cdot (1+0.065)^{12} = \underline{\underline{\$212.91}}$$

$$Present\ Value:\ V_0 = V_n(1+i)^{-n} = \frac{V_n}{(1+i)^n}$$

Example: How much money do you need to invest today at an interest rate of 5% to get $5,000 in future dollars 10 years from now?

$$Solution:$$

$$V_0 = \$5,000 * (1+0.05)^{-10} = \underline{\underline{\$3,069.57}}$$

Solving for the interest rate:

$$i = \left[\sqrt[n]{\frac{V_{t+n}}{V_n}} \right] - 1 = \left[\frac{V_{t+n}}{V_n} \right]^{1/n} - 1$$

Example: If a farmer converts his land to a pine plantation next year, costing him \$5,000, and expects to make \$23,000 from harvesting it 26 years later, what would his rate of return be on this forestry investment?

Solution: $$i = \sqrt[(27-1)]{\frac{\$23,000}{\$5,000}} - 1 = \underline{\underline{6.045\%}}$$

Solving financial analysis problems

1. What do I know?
2. What do I need to know?
3. How do I get to what I need to know from what I know? (select or derive the appropriate formula or formulas)
4. Plug the known values into the formula and calculate the solution
5. Interpret the solution

Notes

- Forest is a capital, planting trees is an investment for the future
- Forestry is a capital-intensive enterprise
- The reason for investments (a sacrifice of current consumption) is their productivity

Discounting when multiple payments/costs are involved

Overview

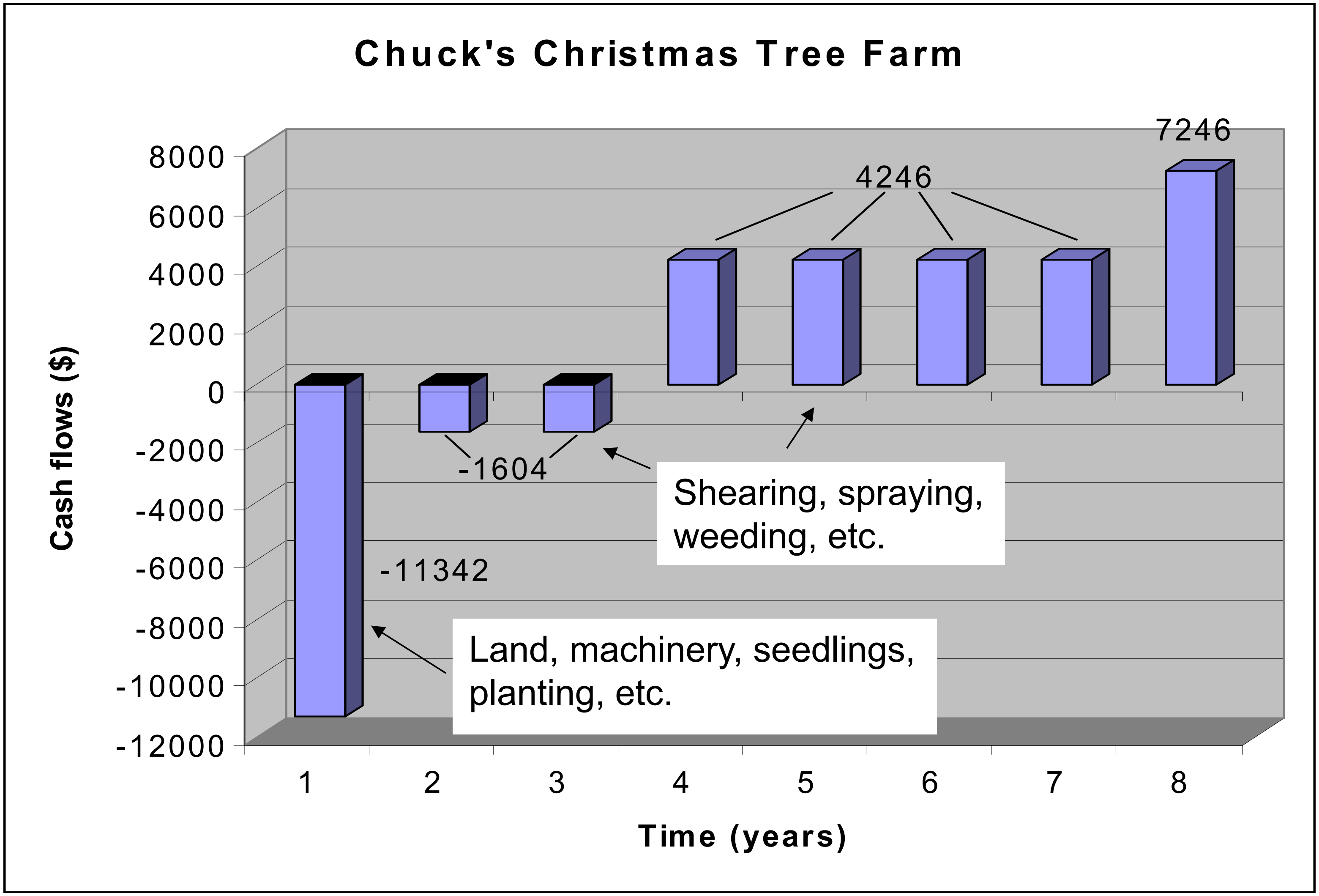
- The Net Present Value
- Cash flows of costs and revenues
- Formulas:
  - Infinite annual payments
  - Infinite periodic payment
  - Finite annual payments
  - Finite periodic payments

The Net Present Value (NPV)

- The NPV is the present value of revenues minus the present value of costs:

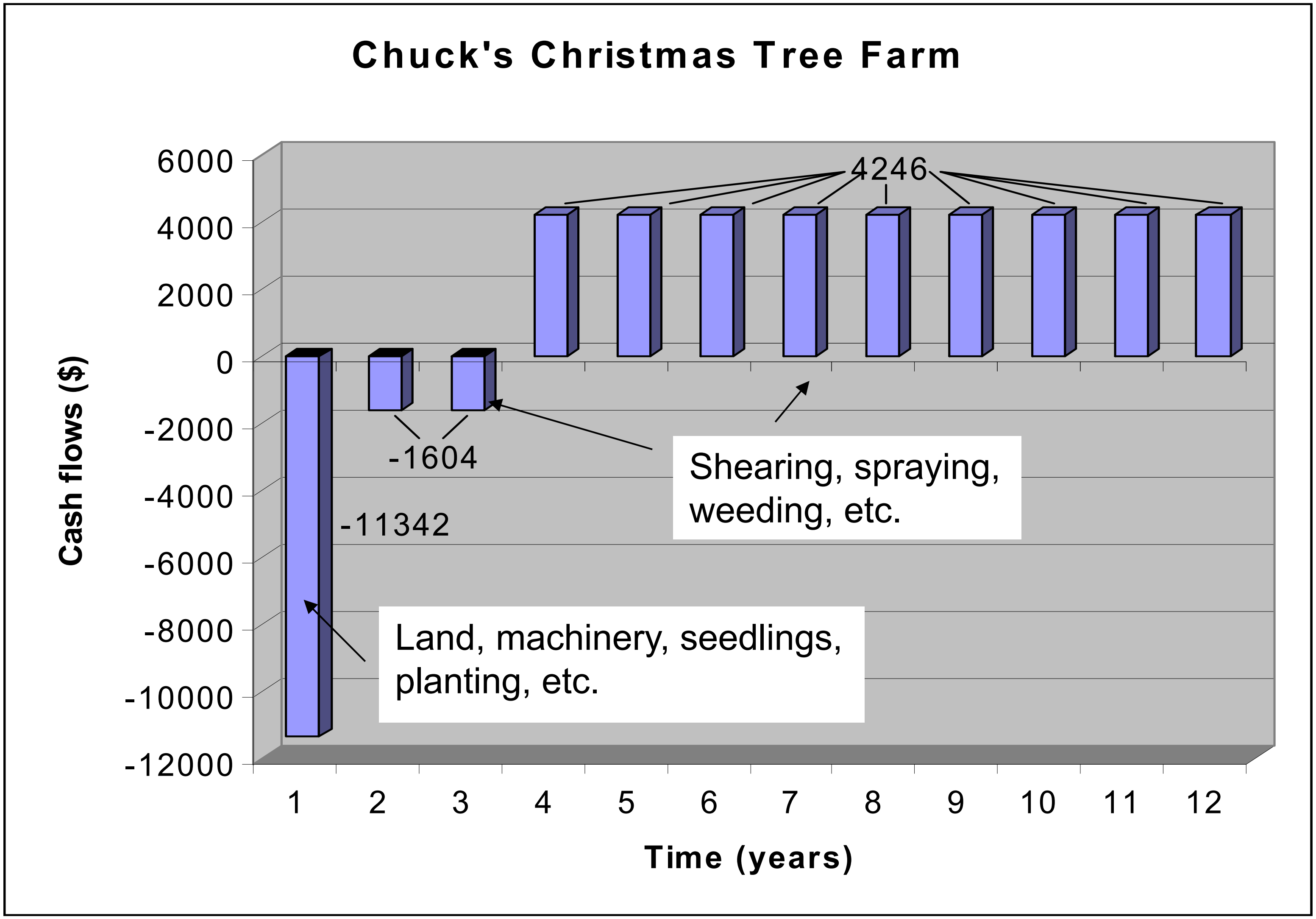
$$NPV = \frac{R_1}{(1+i)^1} + \frac{R_2}{(1+i)^2} + \ldots + \frac{R_n}{(1+i)^n} - \frac{C_1}{(1+i)^1} - \frac{C_2}{(1+i)^2} - \ldots - \frac{R_n}{(1+i)^n}$$

Cash flows



Chuck's Christmas Tree Farm
8000
6000
4000
2000
0
-2000
-4000
-6000
-8000
-10000
-12000
Cash flows ($)
7246
4246
-1604
-11342
Shearing, spraying, weeding, etc.
Land, machinery, seedlings, planting, etc.
1
2
3
4
5
6
7
8
Time (years)

Infinite annual payments



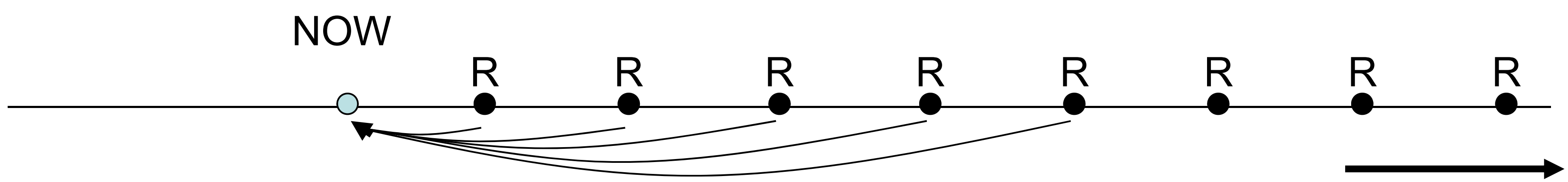
Derivation of the infinite annual series formula

$$V_0 = \frac{R}{1+i} + \frac{R}{(1+i)^2} + \frac{R}{(1+i)^3} + ...$$

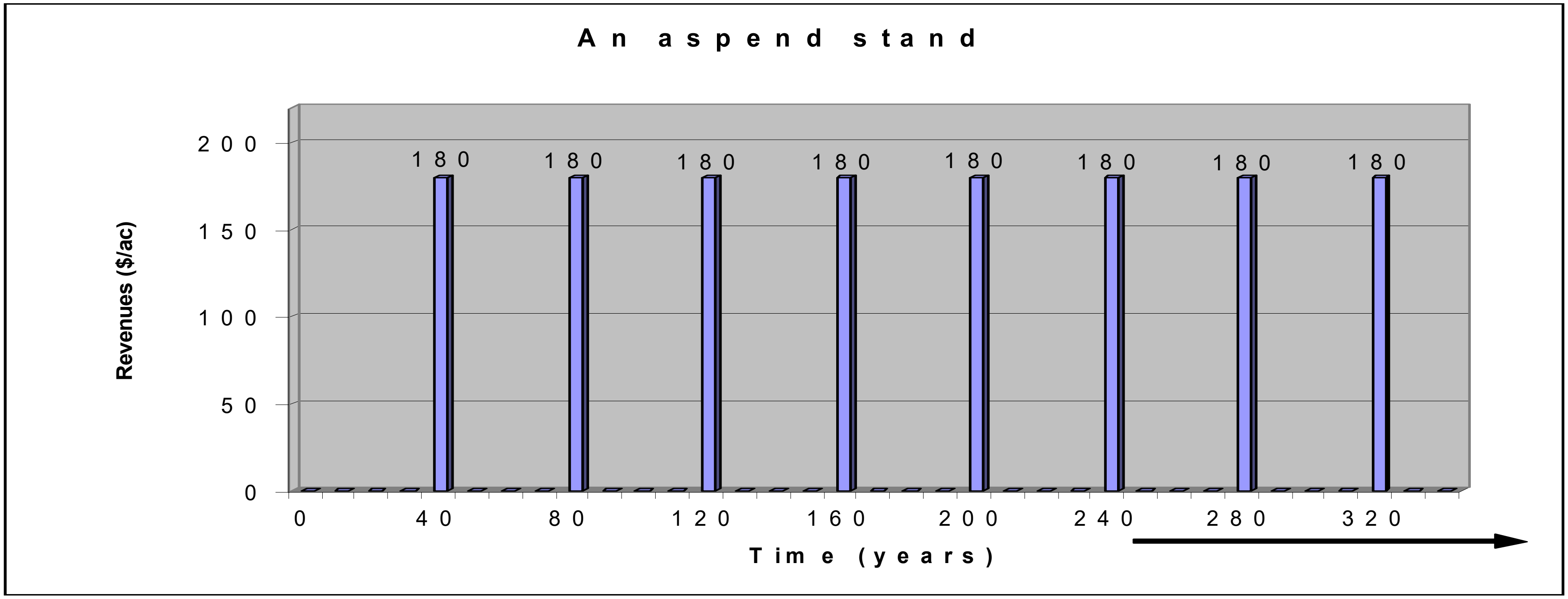
1. Leave $100 in a bank account forever at an interest rate of 5%. How much can you withdraw each year?
2. Answer: $100*0.05=$5/yr
3. In other words: $V_0 i = R$ $\boxed{\rightarrow V_0 = R / i}$

Infinite annual series

- Present value: $$V_0 = \frac{R}{i}$$
- The payment: $$R = iV_0$$
- The interest: $$i = \frac{R}{V_0}$$



Infinite series of periodic payments



Let's use the infinite annual payment formula, but substitute the annual interest rate with a 40 year compound interest rate:

$$V_0 = \frac{R}{i_{40}} = \frac{R}{\frac{R(1+i)^{40} - R}{R}} = \frac{R}{(1+i)^{40} - 1}$$

Infinite periodic series

- Present value: $$V_0 = \frac{R}{(1+i)^t - 1}$$
- The payment: $$R = V_0 \left[(1+i)^t - 1 \right]$$
- The interest: $$i = \sqrt[t]{R / V_0 + 1} - 1$$

Infinite series of periodic payments (the aspen example)

- So, how much is the present value of the revenues generated by the aspen stand at a 6% interest rate?

- <u>Solution:</u>

$$V_0 = \frac{R}{(1+i)^t - 1} = \frac{\$180}{(1+0.06)^{40} - 1} = \underline{\underline{\$19.38}}$$

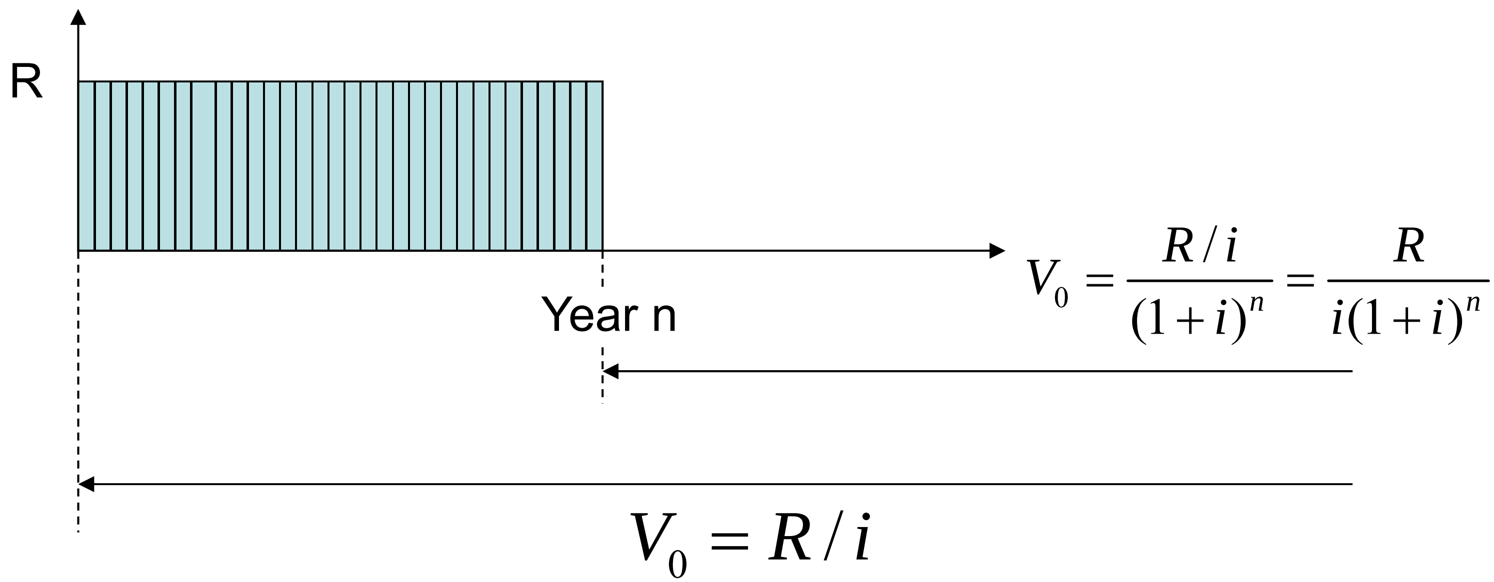
Finite series of annual payments

- Examples:
  - Calculating regular, annual payments on a loan for a fix period of time;
  - Calculating annual rent/tax payments or management costs for a fix period of time;
  - Or, calculating monthly payments.
- Calculating monthly interest rates:

$$i_m = [\sqrt[12]{i+1}] - 1 = (i+1)^{1/12} - 1 \approx \underline{i/12}$$

Finite series of annual payments

- Derivation of the formula:



$$V_0 = \frac{R}{i} - \frac{R}{i(1+i)^n} = \frac{R[(1+i)^n - 1]}{i(1+i)^n}$$

Finite series of annual payments

Present Value: $V_0 = \dfrac{R[(1+i)^n - 1]}{i(1+i)^n}$

Future Value (in year n): $V_n = \dfrac{R[(1+i)^n - 1]}{i}$

Payment to achieve a given Present and Future Value:

$$R = \frac{V_0 i(1+i)^n}{(1+i)^n - 1} = \frac{V_n i}{(1+i)^n - 1}$$

Example

- You want to buy a house in Seattle for $450,000. You don’t have any money to put down, so you get a loan with a 3.5% interest. How much would you have to pay each month to pay the loan off within 15 years?

Solution procedure

1. What do we know?
   - V_0=$450,000
   - i=0.035
   - n=15yrs
2. What do we want to know?
   - R_m=?
3. What formula(s) to use?

$$R = \frac{V_0 i (1+i)^n}{(1+i)^n - 1} \quad \rightarrow \quad R_m = \frac{V_0 i_m (1+i_m)^n}{(1+i_m)^n - 1}$$

Solution procedure cont.

3. Convert the annual interest rate of 3.5% to a monthly interest rate

$$i_m = [\sqrt[12]{i+1}] - 1 = [\sqrt[12]{0.035+1}] - 1 =$$

$$= 0.002871 = \underline{0.2871\%}$$

4. Plug the monthly interest rate into the payment formula:

$$R_m = \frac{V_0 i_m (1+i_m)^n}{(1+i_m)^n - 1} = \frac{\$450,000 \cdot 0.002871 \cdot (1+0.002871)^{180}}{(1+0.002871)^{180} - 1} =$$

$$= \frac{\$2,164.391}{0.675349} = \underline{\underline{\$3,204.85}}$$

Finite series of periodic payments

- There is a fixed amount (*R*) that you receive or pay every *t* years for *n* years (where *n* is an integer multiple of *t*);
- <u>Example:</u> An intensively managed black locust stand (*Robinia pseudoacacia*) is coppiced three times at 20-year intervals. After the third coppice (at age 60), the stand has to be replanted. At ages 20, 40 and 60 yrs the stand produces $1,000 per acre. Using a 5% discount rate, what would the present value of these harvests be?

Solution procedure

- What do we know?
  1. R_{20}=\$1,000
  2. n=60 yrs, t=20 yrs
  3. i=5%=0.05
- What do we need to know?
  - Present Value (V_0)
- What formula to use?
  - Use the finite annual payment formula with a 20-year compound interest rate.

Solution procedure cont.

- First let's calculate the 20 year compound interest rate:

$$i_{20} = (1+0.05)^{20} - 1 = \underline{165.3298\%}$$

- Plug in the 20-yr interest rate into the finite annual series formula:

$$V_0 = \frac{R[(1+i)^n - 1]}{i(1+i)^n} = \frac{\$1,000[(1+1.6533)^3 - 1]}{1.6533(1+1.6533)^3} =$$

$$= \frac{\$17,679.23436}{30.88238} = \underline{\underline{\$572.47}}$$

Finite periodic payments formula

- In general:

$$V_0 = \frac{R[(1+i)^n - 1]}{[(1+i)^t - 1](1+i)^n}$$

- The payment to achieve a given present value:

$$R = \frac{V_0[(1+i)^t - 1](1+i)^n}{[(1+i)^n - 1]}$$