Integer Programming and Spatial Landscape Planning

(an example from reserve design)

Lecture 15 (6/1/2017)

Spatial Optimization to Aid Reserve Design

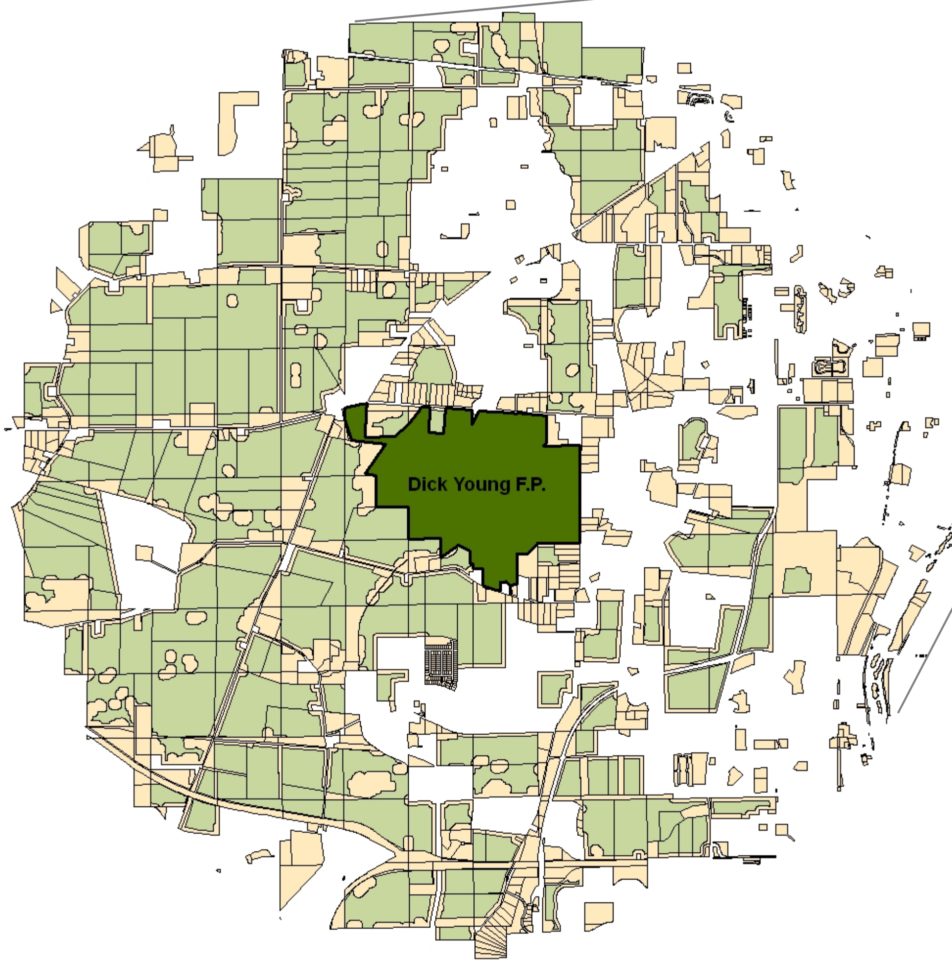
- Constructing reserve networks with spatial structures that are conducive to the health and integrity of the ecosystem that we wish to preserve

Spatial Attributes

- Size
- Connectivity
- Perimeter-area ratio
- Proximity
- Contiguity

Protecting Grassland Birds in Illinois

The **Henslow's Sparrow**
Photo by Merilee Janusz

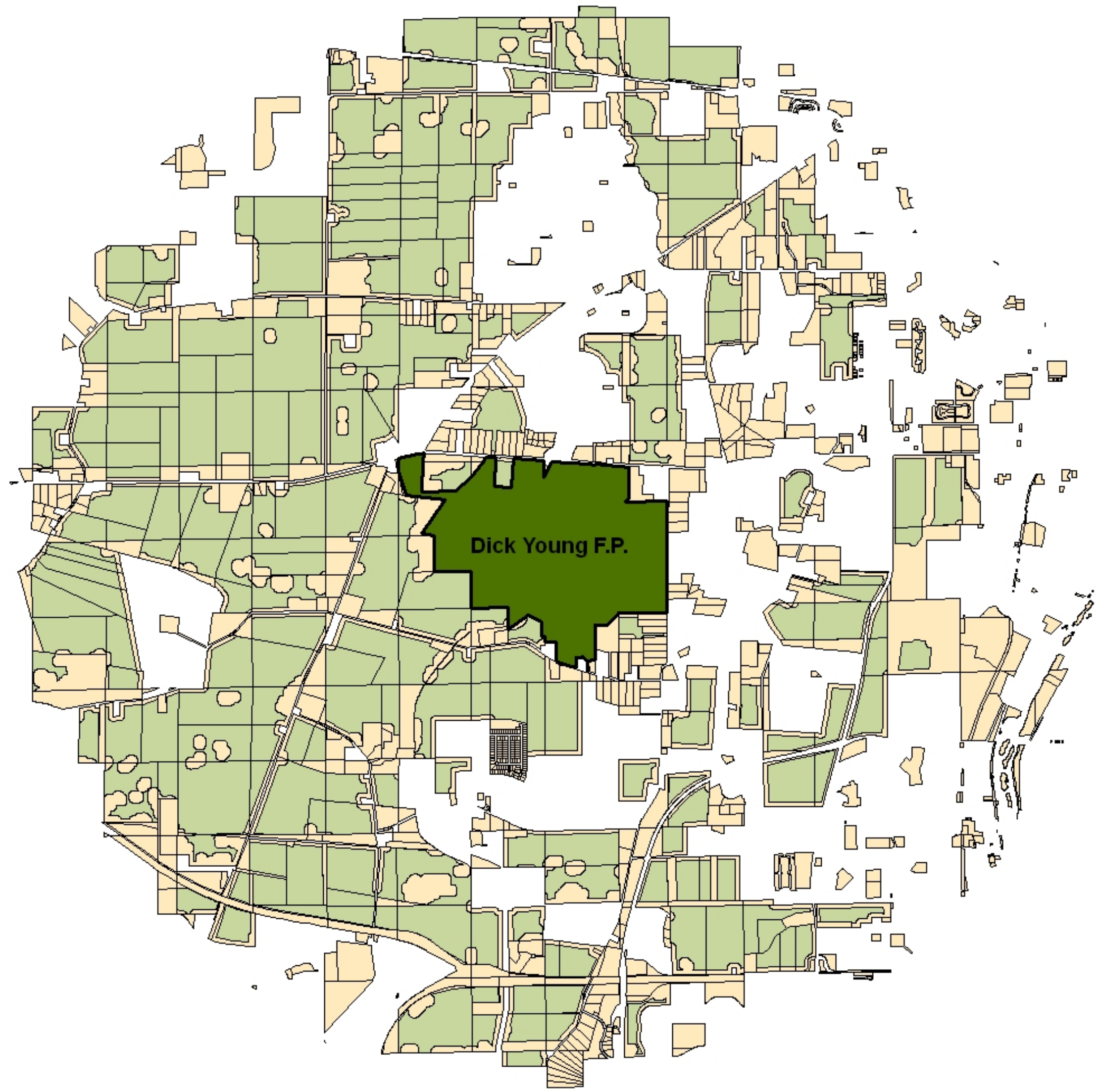


Upland Sandpiper
Photo by Dave Spleha

Eastern Meadowlark
Photo by Gene Oleynik

Protecting Grassland Birds in Illinois

- Which parcels should be selected for purchase to maximize the total area protected without exceeding the budget?

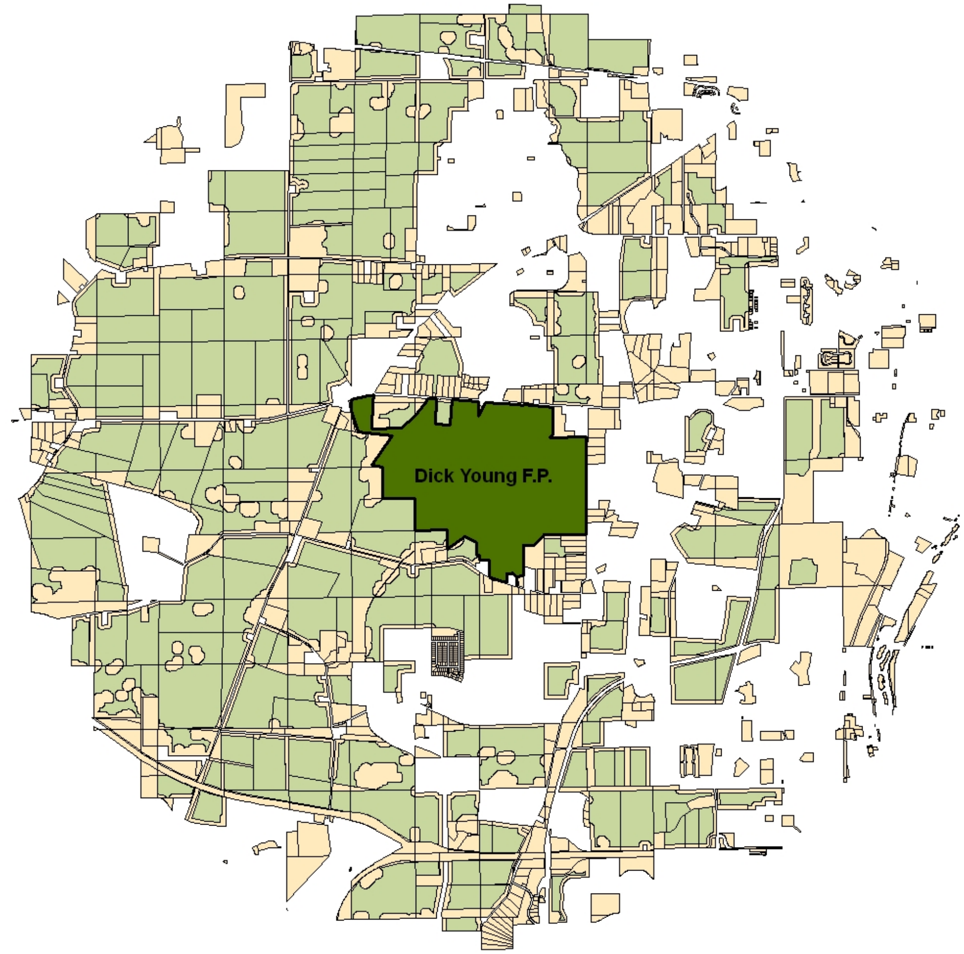


- Budget = B;
- Purchase price of parcel i is c_i;
- Area of parcel i is a_i

- Budget = B;
- Purchase price of parcel i is c_i;
- Area of parcel i is a_i

Mathematical model formulation:

- Define the decision variables:
  - x_i is one if parcel i is to be purchased, and 0 otherwise
- Define objective function (what do we want?):
  - $Max \sum_i a_i x_i$
- Define constraints:
  - $\sum_i c_i x_i \le B$
  - $x_i \in \{0,1\}$



0-1 mathematical program

Species Representation

➢Suppose you want to represent each species at least once in your reserve network:

- *Preserve species 'a' in at least one location:*

$$x_1 + x_7 + x_9 \geq 1$$

- *Preserve species 'b' in at least one location:*

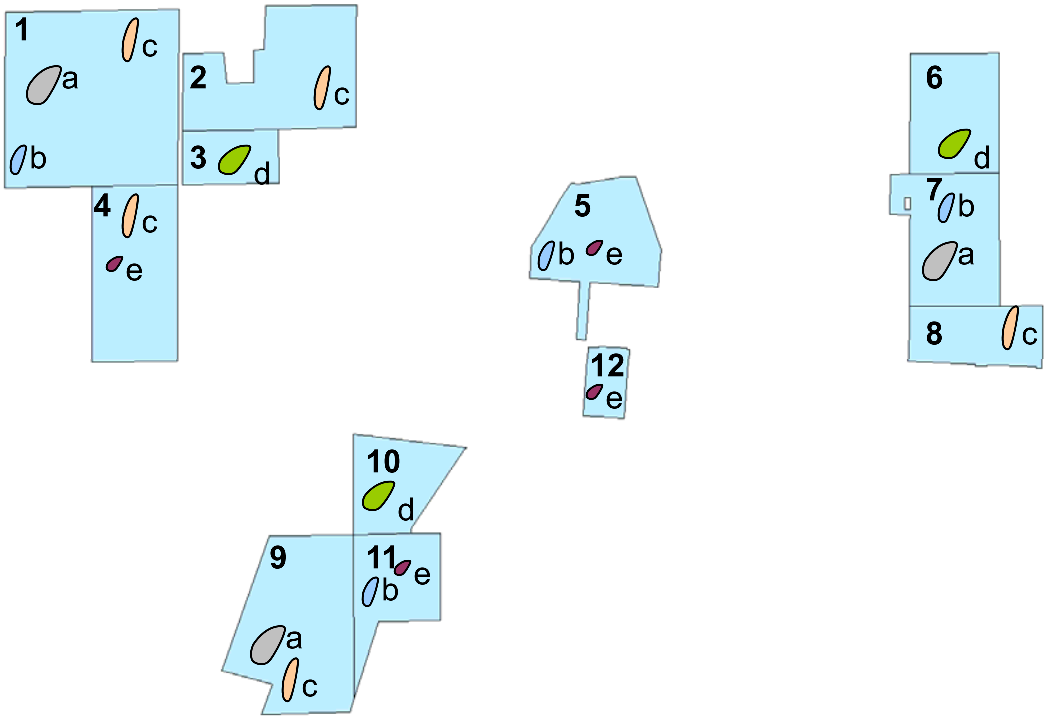
$$x_1 + x_5 + x_7 + x_{11} \geq 1$$

- *Preserve species 'c', 'd', 'e' in at least one location:*

$$x_1 + x_2 + x_4 + x_8 + x_9 \geq 1$$

$$x_3 + x_6 + x_{10} \geq 1$$

$$x_4 + x_5 + x_{11} + x_{12} \geq 1$$



Species Representation cont.

- *…and of course we still have a budget and want to maximize the total protected area:*

$$Max \sum_{i=1}^{12} a_i x_i$$

$$s.t.$$

$$\sum_{i=1}^{12} c_i x_i \le B$$

$$x_1 + x_7 + x_9 \ge 1$$

$$x_1 + x_5 + x_7 + x_{11} \ge 1$$

$$x_1 + x_2 + x_4 + x_8 + x_9 \ge 1$$

$$x_3 + x_6 + x_{10} \ge 1$$

$$x_4 + x_5 + x_{11} + x_{12} \ge 1$$

$$x_i \in \{0,1\}$$

Species Representation cont.

- In a general form:

$$
\begin{aligned}
&Max \sum_{i \in I} a_i x_i \\
&s.t. \\
&\sum_{i \in I} c_i x_i \leq B \\
&\sum_{i \in S_j} x_i \geq 1 \qquad \forall j \in J \\
&x_i \in \{0,1\}
\end{aligned}
$$

where: i indexes the sites, j the species. I is the complete set of sites available for conservation purchase, J is the set of species to preserve and S_j is the set of sites that contain a population of species j.

Maximal Species Representation

- Let's introduce a binary indicator variable y_j that turns on (takes the value of one) if species *j* is protected in at least one viable population.
- We need a trigger mechanism that drives the value of y_j.
- Let's add a constraint:

$$\sum_{i \in S_j} x_i \geq y_j \qquad \forall j \in J$$

- Our objective function will become:

$$Max \sum_{j \in J} y_j$$

Maximal Species Representation cont.

- Our mathematical program becomes:

$$Max \sum_{j \in J} y_j$$

$$s.t.:$$

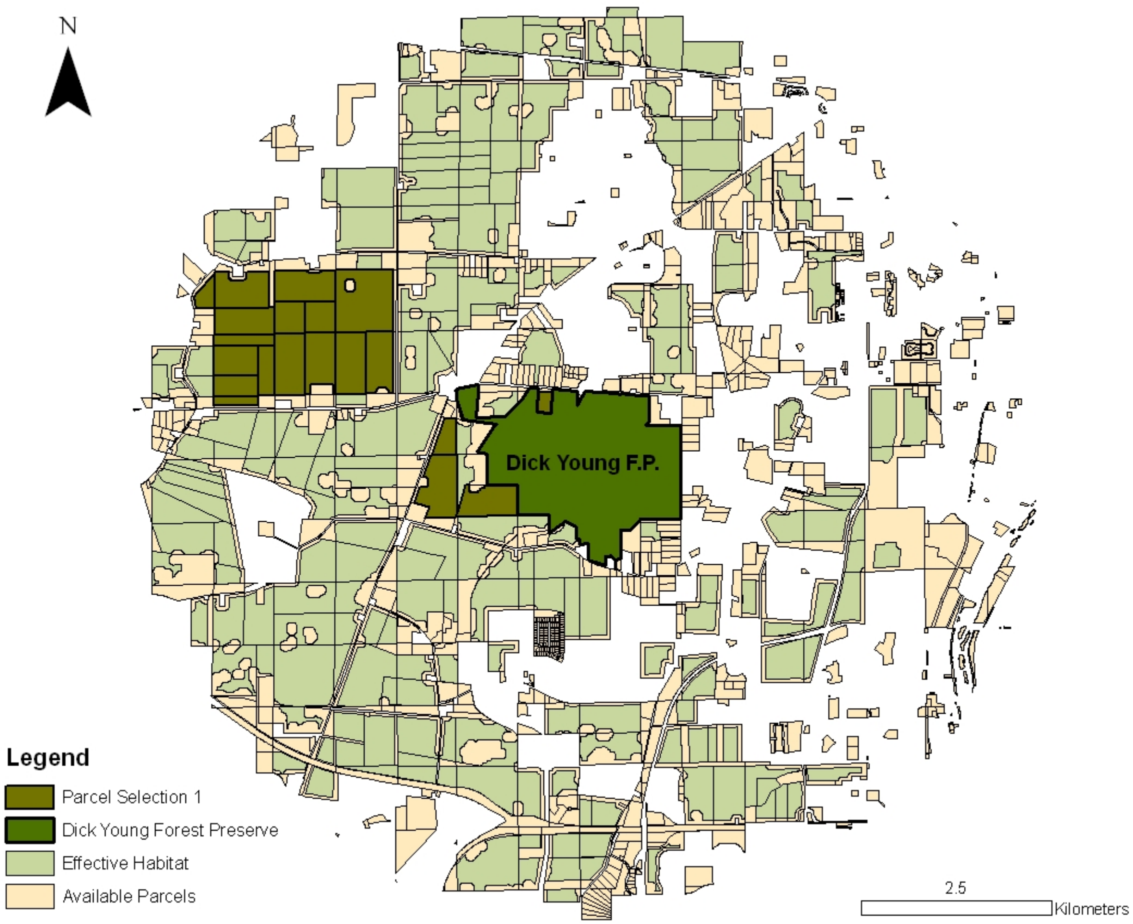
$$\sum_{i \in I} c_i x_i \leq B$$

$$\sum_{i \in S_j} x_i \geq y_j \qquad \forall j \in J$$

$$x_i, y_j \in \{0,1\}$$

Spatially Explicit Reserve Selection Subject to Minimum Contiguous Habitat Size Requirements

- Threatened grassland birds in the analysis area require at least 100 ha of habitat in contiguous patches

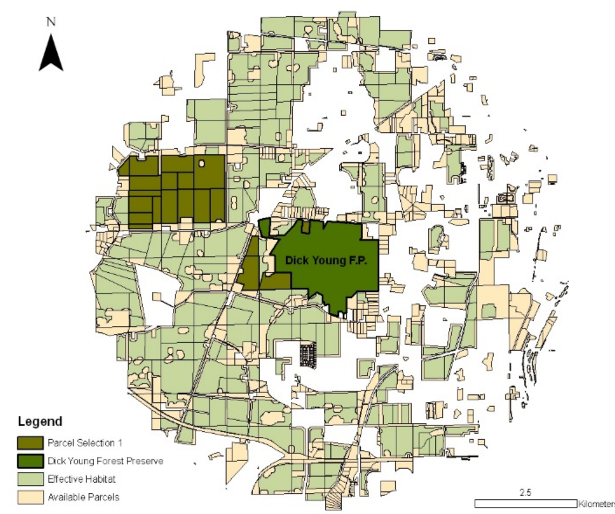


Minimum Contiguous Habitat Size Requirements

- <u>Step 1:</u> Enumerate all feasible contiguous clusters of parcels. Let *C* denote this potentially enormous set.
- <u>Step 2:</u> Enforce the logical condition that a parcel can only be protected if it is part of at least one feasible cluster that is protected. To enforce this condition, introduce indicator variable y_j that turns on if cluster *j* is protected.

$$\sum_{j \in P_i} y_j \geq x_i \qquad \forall i \in I$$

where P_i is the set of feasible clusters that contain parcel *i* .



Minimum Contiguous Habitat Size Requirements cont.

- <u>Step 3:</u> We also need to ensure that a cluster is declared to be protected only if each parcel that comprise the cluster is protected:

$$\sum_{i \in C_j} x_i \geq |C_j| y_j \qquad \forall C_j \in C$$

Here y_j may turn on if all x_is in C_j are on. Is that enough?

We also need to make sure that: y_j must turn on if all x_is in C_j are on. Why and how can we do that?

$$\sum_{i \in C_j} x_i - y_j \leq |C_j| - 1 \qquad \forall C_j \in C$$

Minimum Contiguous Habitat Size Requirements cont.

$$Max \sum_i a_i x_i$$

Protected area

$$subject\ to:$$

$$\sum_i c_i x_i \leq B$$

Budget

A parcel can only be protected if it is part of at least one feasible cluster that is protected

$$\sum_{j \in P_i} y_j \geq x_i \qquad \forall\ i \in I$$

A cluster <u>may be</u> protected if each parcel that makes up the cluster is protected

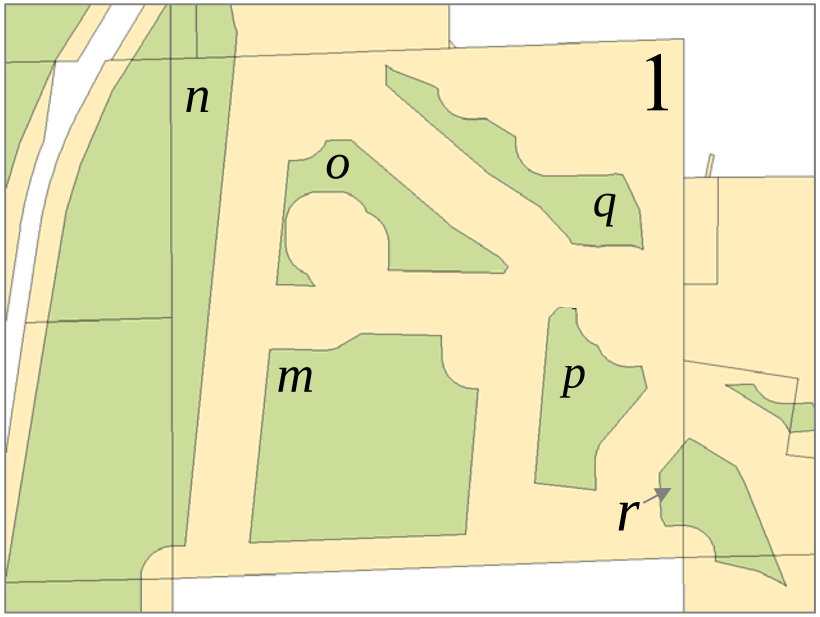
$$\sum_{i \in C_j} x_i \geq |C_j| y_j \qquad \forall\ C_j \in C$$

A cluster <u>must be</u> protected if each parcel that makes up the cluster is protected

$$\sum_{i \in C_j} x_i - y_j \leq |C_j| - 1 \qquad \forall\ C_j \in C$$

$$x_i, y_j \in \{0,1\}$$

Programming Disjoint Habitat Patches



Option 1:

$$x_n = x_m,\ x_n = x_o,\ x_n = x_p,\ x_n = x_q,\ x_n = x_r,$$

$$x_m = x_o,\ x_m = x_p,\ x_m = x_q,\ x_m = x_r,$$

$$x_o = x_p,\ x_o = x_q,\ x_o = x_r,$$

$$x_p = x_q,\ x_p = x_r,\ and$$

$$x_q = x_r$$

Option 2:

$$x_n + x_m + x_o + x_p + x_q + x_r = 6z_1$$

(the credit goes to Liam Stacey, CFR grad student for conceiving this construct)

A Stronger Formulation for the Minimum Contiguous Habitat Size Problem

$$Min \sum_{i} c_i x_i$$

$$subject\ to:$$

$$\sum_{i} a_i x_i \geq A$$

$$\sum_{j \in P_i} y_j \geq x_i \qquad \forall\ i \in I$$

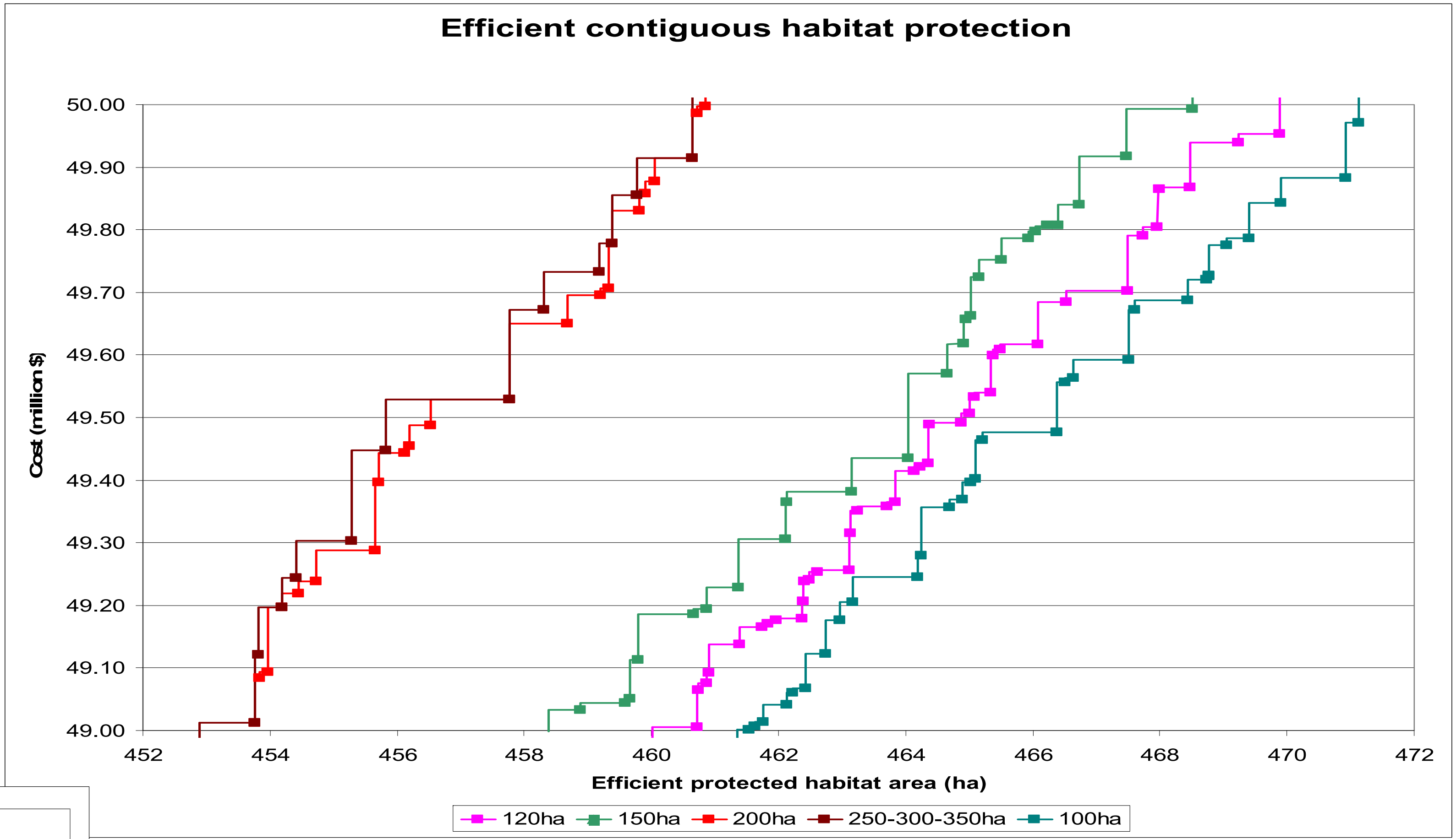
$$\sum_{i \in C_j} x_i \geq |C_j| y_j \qquad \forall\ C_j \in C$$

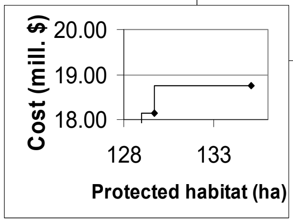
$$x_i \geq y_j \qquad \forall i \in C_j \text{ and } \forall C_j \in C$$

$$\sum_{i \in C_j} x_i - y_j \leq |C_j| - 1 \qquad \forall\ C_j \in C$$

$$x_i, y_j \in \{0,1\}$$

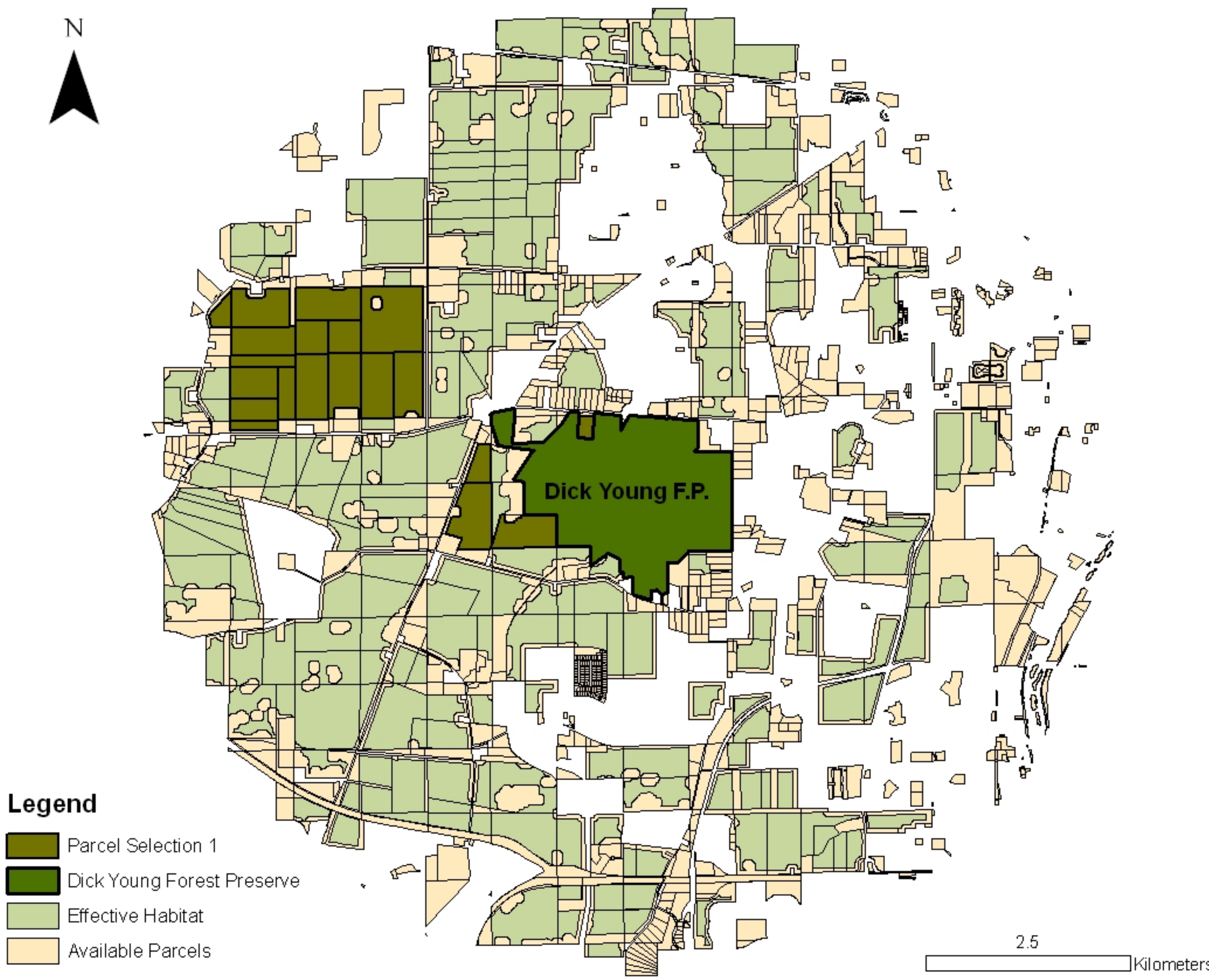
Efficient Site Selections Near the Dick Young Forest Preserve, IL at Different Contiguity Thresholds



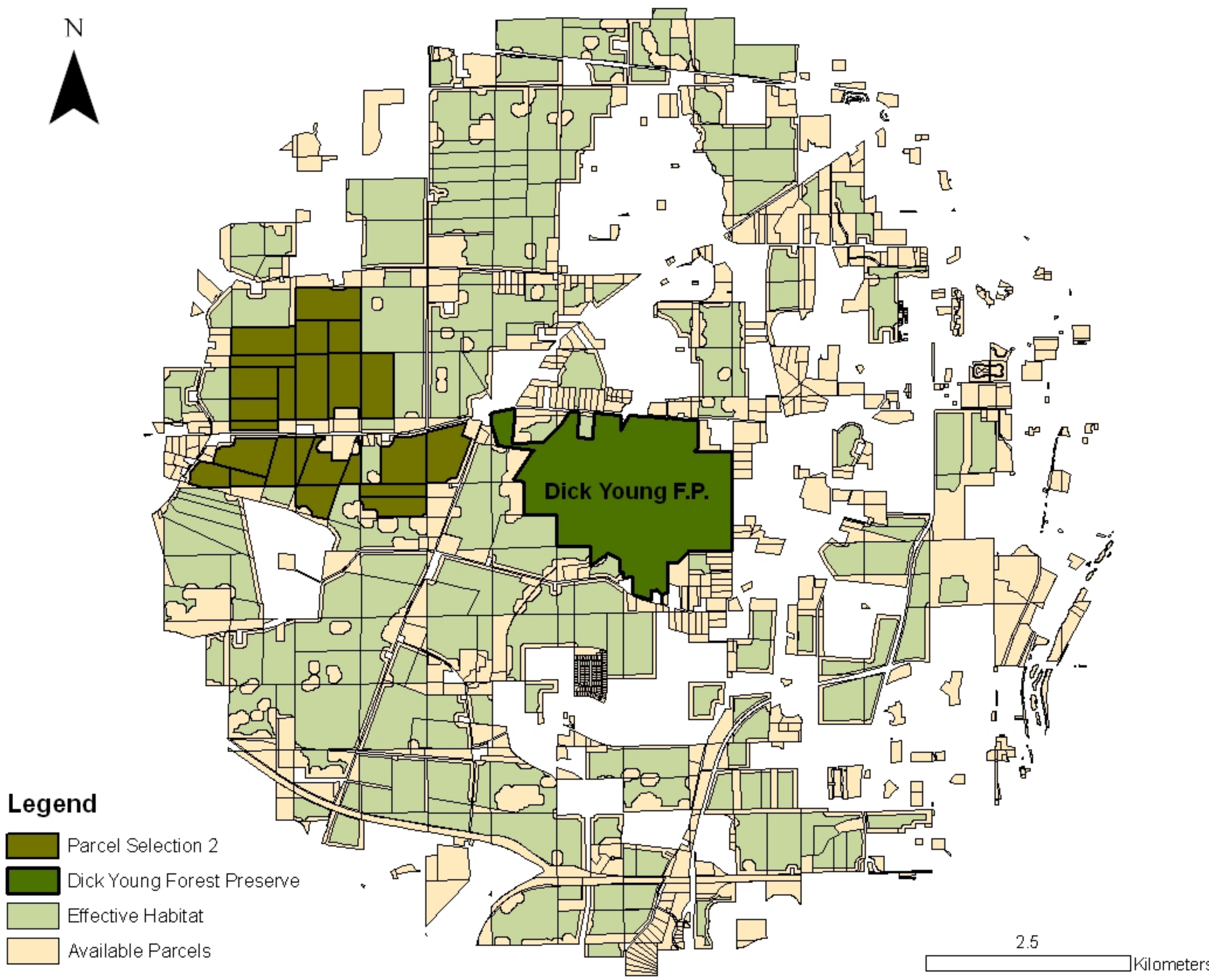


400-450-500ha

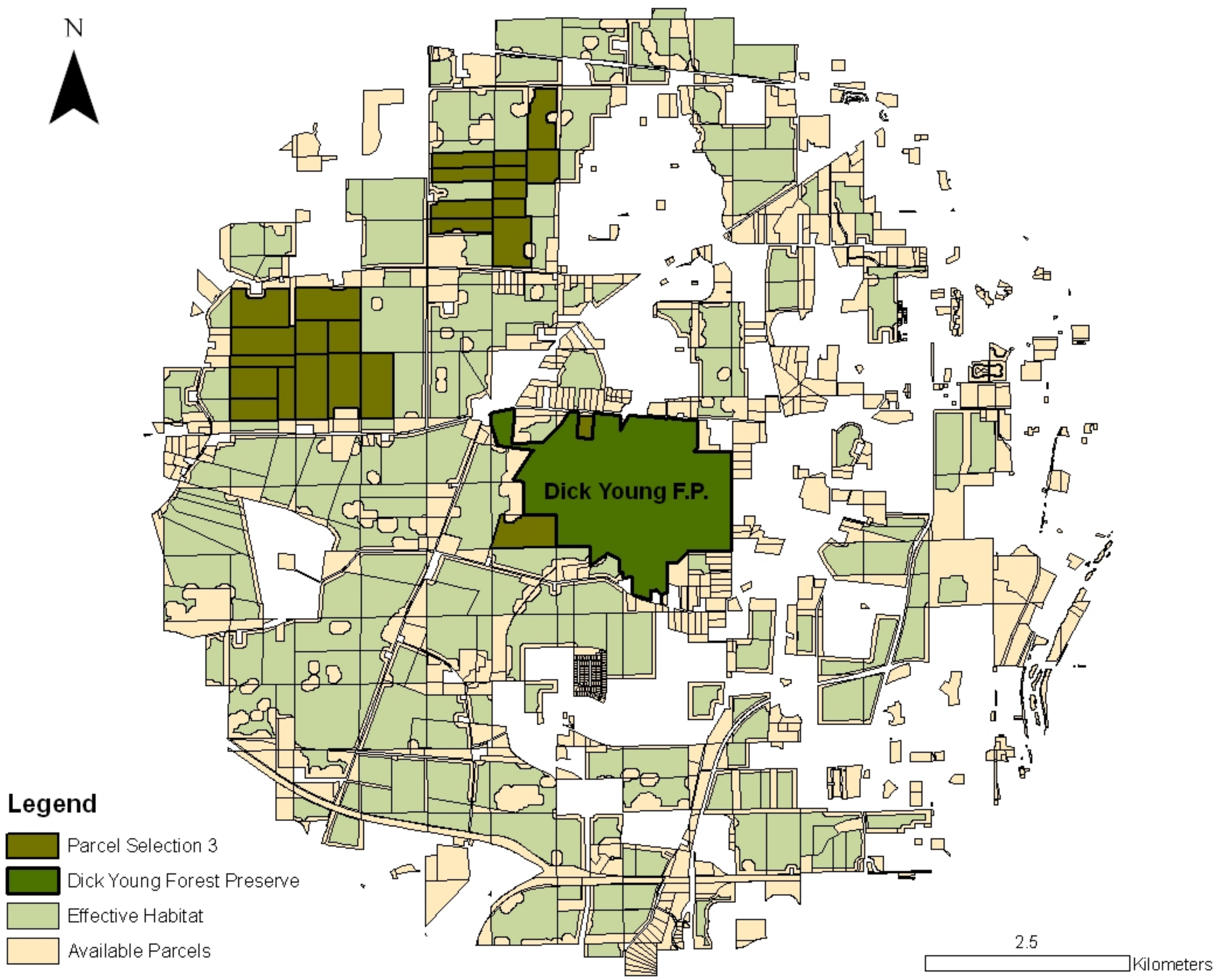
Contiguity Threshold: 250-300-350 ha
Acquisition Cost: $49.91M
Acquisition Area: 460.6 ha



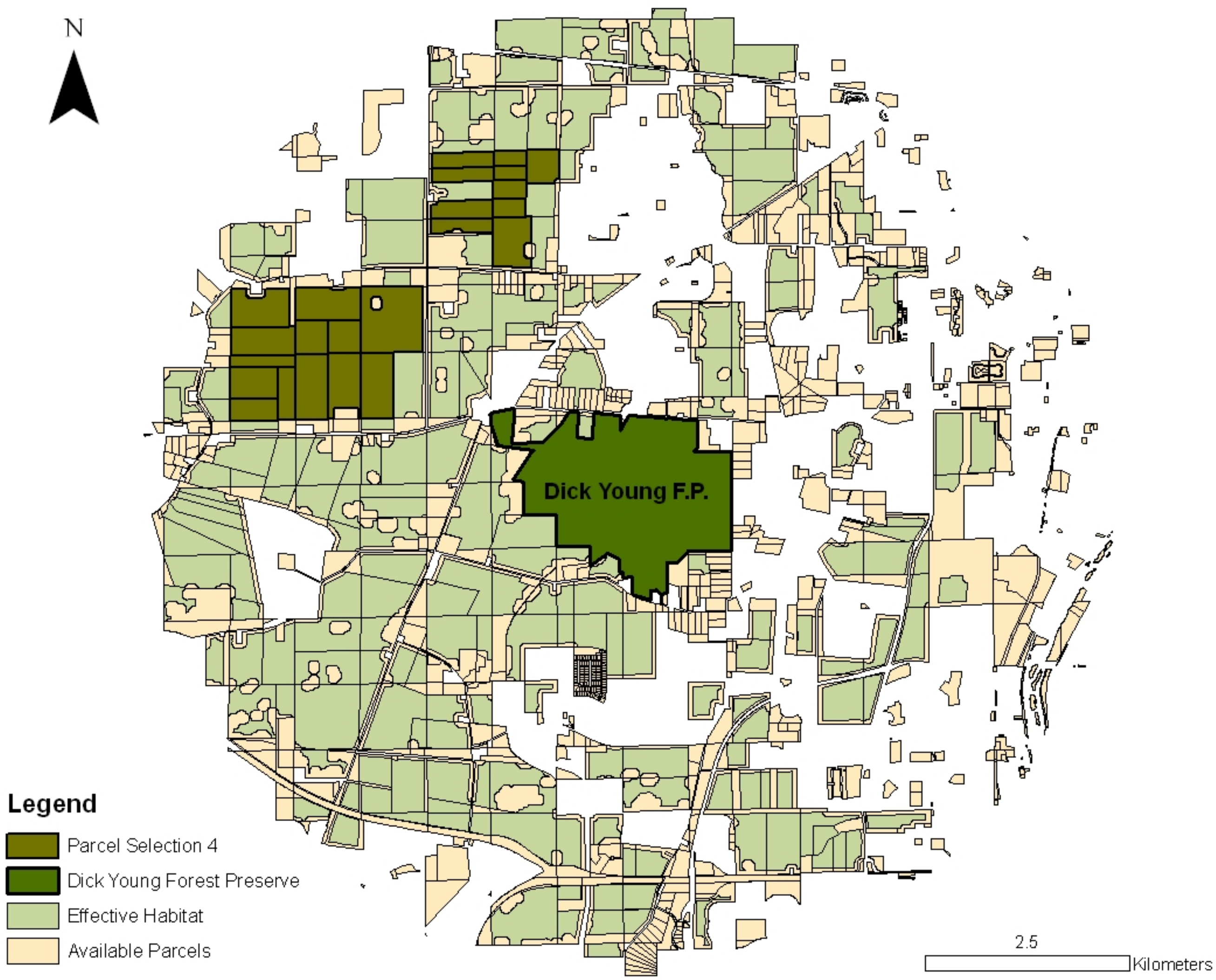
Contiguity Threshold: 200 ha
Acquisition Cost: $50.00M
Acquisition Area: 460.84 ha



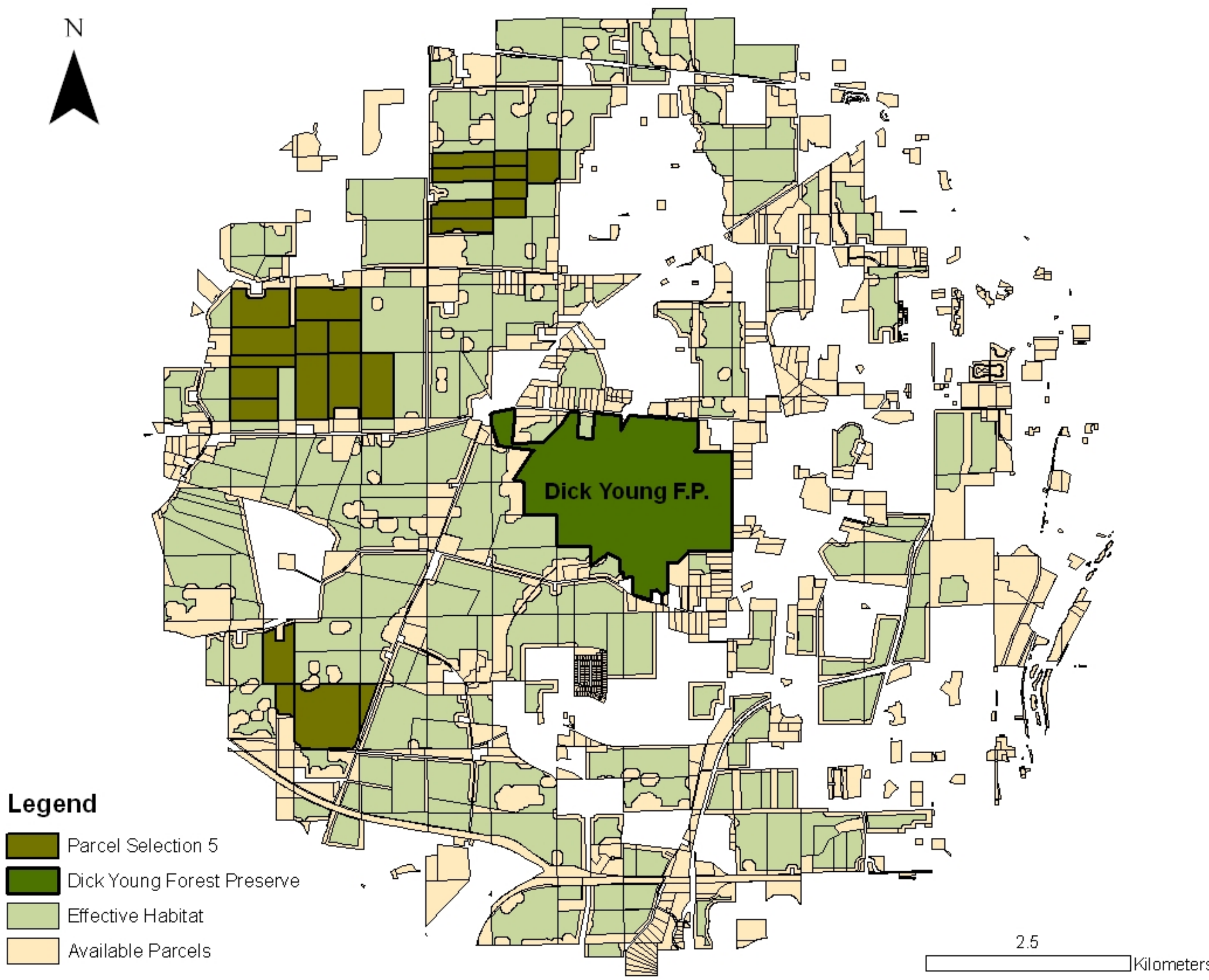
Contiguity Threshold: 150 ha
Acquisition Cost: $49.99M
Acquisition Area: 468.52 ha



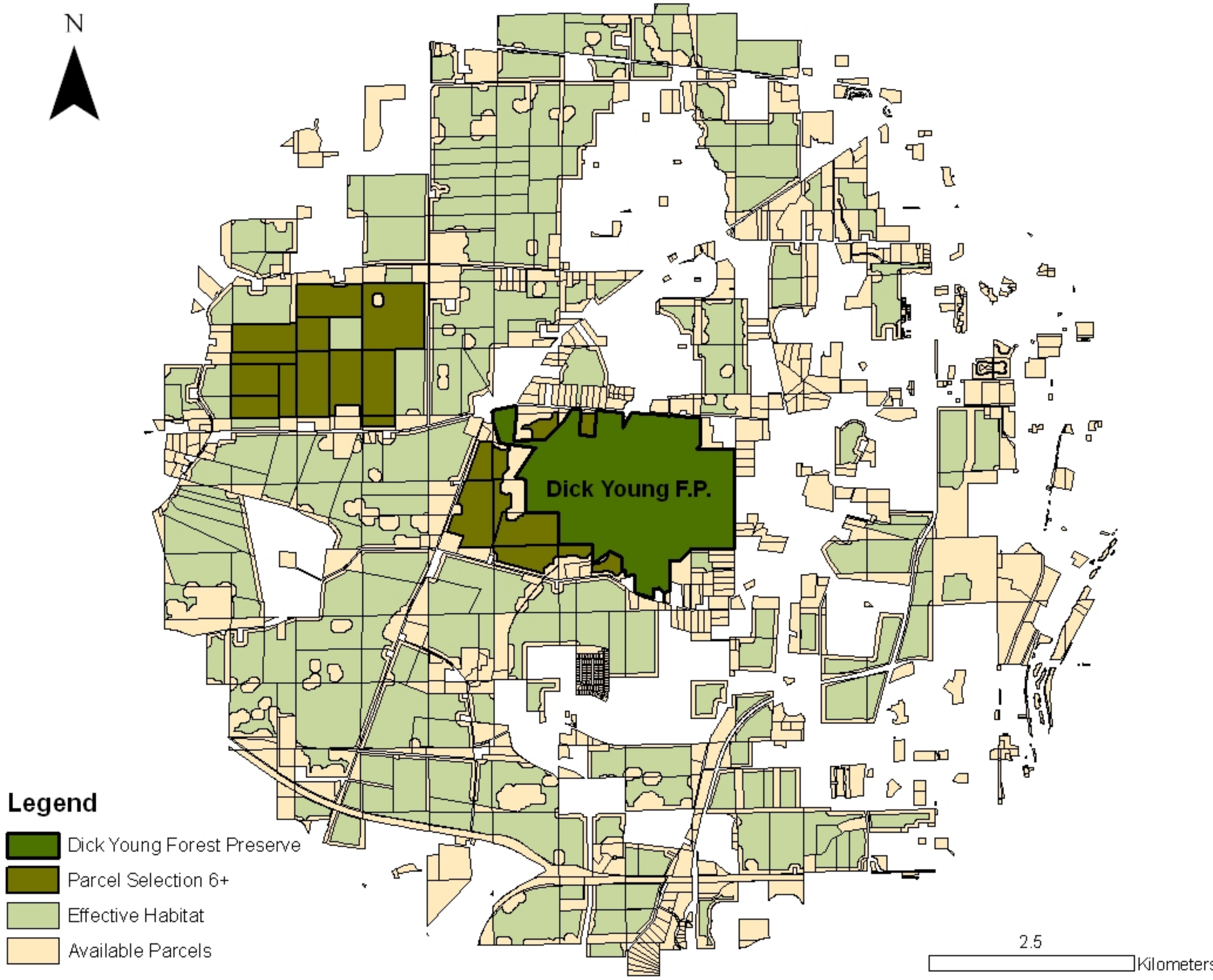
Contiguity Threshold: 120 ha
Acquisition Cost: $49.95M
Acquisition Area: 469.88 ha



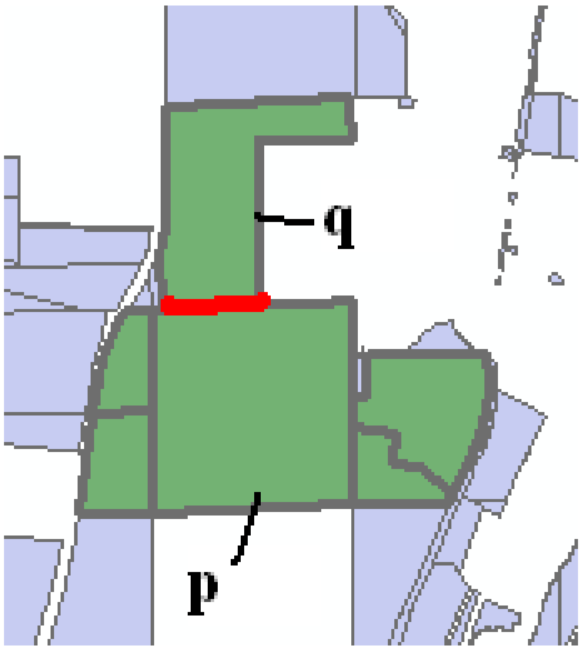
Contiguity Threshold: 100 ha
Acquisition Cost: $49.97M
Acquisition Area: 471.13 ha



Parcel Selection 6+
Contiguity Threshold: N/A
Acquisition Cost: $49.94M
Acquisition Area: 430.6 ha



Shape
– Edge-to-Interior Area Ratio –



Perimeter

$$K_{p+q} = k_p + k_q - 2CB_{pq}$$

$$K_{total} = \sum_{i \in I} k_i x_i - 2 \sum_{pq \in E} \overbrace{x_p x_q}^{\omega_{pq}} CB_{pq}$$

$$x_p + x_q - \omega_{pq} \leq 1$$

$$x_p + x_q - 2\omega_{pq} \geq 0$$

- K_{p+q} = combined perimeter of parcels p and q;
- K_{total} = the total combined perimeter of all protected parcels;
- CB_{pq} = length of common boundary between stands P and Q;
- E = the set of adjacent pairs of parcels
- x_p = 1 if parcel p is to be selected for conservation,
  = 0 otherwise.

- ω_{pq} = 1 if both stand P and Q are part of mature patch.