CHAPTER 5: GROWTH AND YIELD

The subject of this class is forest management. Thus, we will not spend a lot of time talking about growth and yield – that is a subject for another class. However, growth and yield models provide some very basic information for forest management decisions. They are the production functions that undergird all timber management, and, to the extent that other values depend on the characteristics of the trees in a forest, other values as well. If the growth and yield projections used in an analyses are unreliable, the results will be similarly unreliable. As the saying goes, garbage in, garbage out.

In forest management the objectives often are achieved by controlling the characteristics of a forest stand or set of forest stands in order to influence the growth and yield of those stands. Thus, Section 1 briefly reviews the key stand characteristics that most affect growth and yield. A growth and yield model should incorporate and predict the relationships between the characteristics of a stand and the growth and yield of the timber in the stand. Sections 2 and 3 discuss some basic properties of growth and yield functions and some terminology related to growth and yield.

1. Stand Characteristics That Affect Growth and Yield.

An obvious stand characteristic affecting growth and yield is the mix of tree species in the stand. The mix of species in a stand not only affects growth and yield, but also the types of products that are likely to be produced and their value. In the case of pure stands of one species, this stand characteristic is easy to account for. However, when stands include a mix of species, it becomes difficult to characterize the species composition of the stand in any simple way. The most common approach is to classify stands into **forest types**, also called **cover types** or **communities**, based on their species composition. However, there is often a great deal of variability in the species composition of stands within a forest type. Individual-tree growth and yield models project the characteristics of a sample of individual trees from a stand. These models account for species composition explicitly by recognizing the species of the trees in the representative tree list. Even so, individual trees are usually classified into species groups for projection purposes, and some information about the species composition of a stand is lost.

Other than the species of the trees in a stand, the stand characteristic that most obviously affects growth and yield is **stand age**. Generally speaking, trees get bigger and taller as they age, and once they reach their peak growth rate – usually while relatively young – their growth tends to slow as they age. The biggest problem with using stand age as a predictor of growth and yield is that it often is not well-defined. It is fairly straightforward to define and measure the age of a tree, but what is a stand's age? If the stand originated as a plantation, then the majority of the trees may be the same age. However, many stands have a mix of trees of different ages. Even in plantations, many of the trees may be volunteers (of natural origin), and the ages of the trees may be quite variable. Obviously, in uneven-aged stands, stand age is virtually meaningless. Thus, stand age may be very difficult to measure. For these reasons, the average stand dbh or stand volume is sometimes used as a proxy for stand age. An important detail to keep in mind when using age in growth and yield models is the difference between the age that is typically measured, which is age at dbh, and the total age of the tree.

Perhaps the second most important stand characteristic determining growth and yield is **site quality**. The measure of site quality that is used almost universally in the U.S. is the **site index**, which is the projected height at an index age (typically 25, 50, or 100 years) of dominant and codominant trees of a given species on the site. Note that site index is only an indirect measure of site quality, which is the potential capability of the site to produce wood fiber and/or specific wood products. It is important to distinguish between site quality and site index, which is a rather imperfect measure of site quality. Furthermore, site index can only be directly observed at the time the dominant and codominant trees in the stand are at the index age. At any other age, the height that the site trees will reach (or did reach) at the index age must be estimated. Because of the inherent inaccuracy of site index as a measure of site quality, it is common to find that the measured site index of an area changes from one time to another. This does not mean that the site quality has changed, only that the measure of site quality has changed. Note also that the site index is species-specific, and the site index of a site for one species may be a poor predictor of the site's site index for other species.

Because of its imperfections, many alternatives to site index have been proposed. These have been based on other site characteristics, such as soil characteristics, topographical factors, such as elevation, slope, aspect, and slope position, and the presence of indicator species. None of these alternatives have proven to be as universally useful as site index, and the measurement of forest site quality remains an inexact science, at best. Ideally site quality would be measured by historical yields, but good records of historical yields are seldom available, and, even if they were, they would be of limited use as it would be difficult to account for the effects of past management on past yields. Site index remains the most widely accepted measure of site quality largely because it works as well or better than most of the alternatives and because it is relatively easy to measure.

The third important stand characteristic affecting forest growth and yield is **density**, or **stocking**. Basal area, number of trees per acre (typically only trees over some size), and volume per acre are the most common measures of density. Growth and yield are obviously related to density; growth will be slow in stands that are either too dense or too sparse. Also, less dense stands will tend to produce larger diameter trees faster than more dense stands, and *value* may be determined as much by tree diameter as by volume. Stocking is a somewhat outdated measure of density based on some pre-defined "normal" density. Stocking is usually expressed as a percent of "normal," where this percent may be anything from zero to 150% – or even more. A stand whose stocking level is substantially greater than 100% would be considered "overstocked," while a stand whose stocking was sufficiently less than 100% would be "understocked." A problem with the concept of stocking is that it assumes that there is one biologically-determined, ideal density for all stands of a given type. However, the ideal density for a stand should also depend on non-biological factors, such as ownership objectives, market factors, etc.

Competition also affects stand development. Competition actually refers to two different factors. The first is competition from undesirable vegetation. For example, hardwoods are considered undesirable in a pine plantation, and the quantity of hardwood present can significantly affect the growth of the planted trees. Especially in the early stages of a pine plantation's development, competition from grasses, shrubs, and hardwoods can seriously reduce the pine's growth. The second factor that competition refers to is competition between individual trees. This is closely related to stand density. However, in this context, competition generally refers to individual trees, while density is a stand-level characteristic. In tree-based growth and yield models, competition refers to the competitive position of a given tree relative to other trees in the stand. Various competition indices have been developed to measure a tree's relative competitiveness in the stand. Two of the simpler measures of a tree's competitiveness are the crown ratio and the ratio of the tree's diameter to the stand quadratic mean diameter. More complex competition indices may require mapping the locations of individual trees in the stand.

Many other factors affect stand development. Stands of trees are complex biological communities. Obviously, the genetic characteristics of the trees in a stand is important. Also, past management practices can affect current stand development. The obvious example of this is stand origin. Planted stands tend to grow faster than natural stands. Another example is fertilization. Fertilization can be thought of as a change in the site quality. However, the effects of fertilization are generally temporary, while the site quality tends to be quite stable.

Forest managers use their understanding of the relationships between these stand characteristics and the stand's growth and development to control the stand in order to achieve whatever end the landowner's objectives dictate. Harvesting is used to control the stand age; thinning is used to control stand density; fertilization to improve site quality; release and prescribed burning to control competition. In order to use these tools effectively, accurate models are needed to predict the specific response that can be expected from different management activities.

2. Basic Stand Growth Terminology

This section discusses a few basic terms related to the growth of a stand of trees that every forester should be familiar with. This section reviews these terms and their definitions. Figure 5.1 shows an example of a typical yield curve, showing the yield of wood for a hypothetical stand, in thousands of cubic feet per acre, for different harvest ages. This yield curve has a few key properties that you should be aware of. The overall shape of the curve is described as **sigmoid**. This means that the growth rate (the slope, or first derivative, of the yield curve) increases initially and quickly reaches a maximum. After this maximum growth rate has been reached, the growth rate begins to decline and continues to decline as the stand ages. The point where growth begins to slow down is called the inflection point of the curve. Before the **inflection point**, the yield curve is convex; after the inflection point, the yield curve is concave.

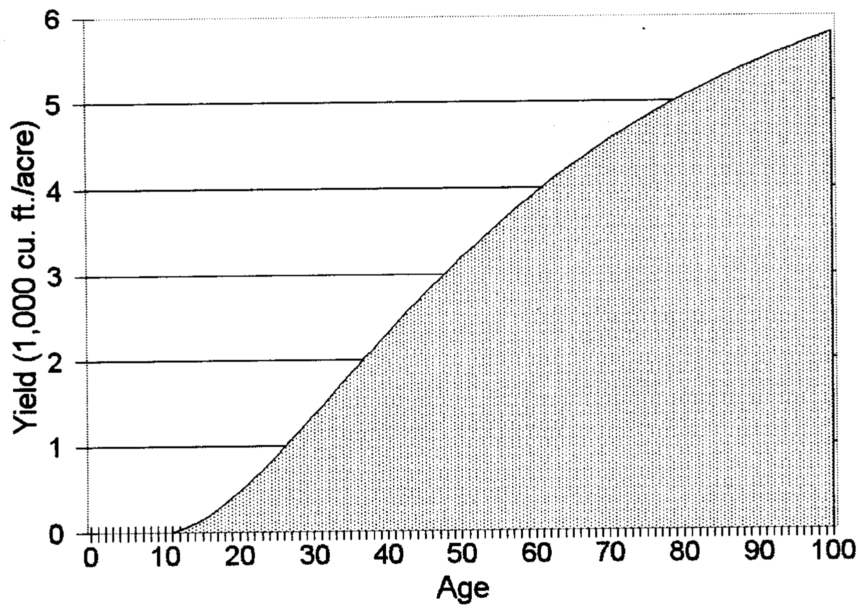

Figure 5.1. Example of a yield curve.

Figure 5.2 shows different measures of growth – the annual increment, the mean annual increment, and the annual compound interest rate of growth – for the example yield curve in Figure 1. Note in Figure 5.2 that the annual increment increases rapidly initially and reaches a maximum when the stand is about 35 years old. After that, the growth rate drops off fairly quickly. The shape of the growth curve differs for different species and forest types. For some species, the growth rate drops off quickly once the maximum has been achieved. For others, a relatively high growth rate is maintained for a longer period and the growth rate drops off more gradually. The common measures of growth and yield defined here include yield, annual increment, periodic annual increment, mean annual increment, and the compound interest rate of growth.

Yield (Y_a) refers to the volume of usable wood fiber per unit area at a given age, *a*.

The **Annual Increment** (ΔY_a) is just the annual growth of the stand per unit of area at a given age. It is the difference between the yield at age *a* and the yield a year earlier, at age *a-1*.

$$\Delta Y_a = Y_a - Y_{a-1}$$

The **Periodic Annual Increment ($PAI_{a1,a2}$)** is the average annual increment per unit area over some period longer than one year. The PAI for ages a_1 to a_2 is:

$$PAI_{a_1,a_2} = \frac{Y_{a_2} - Y_{a_1}}{a_2 - a_1}$$

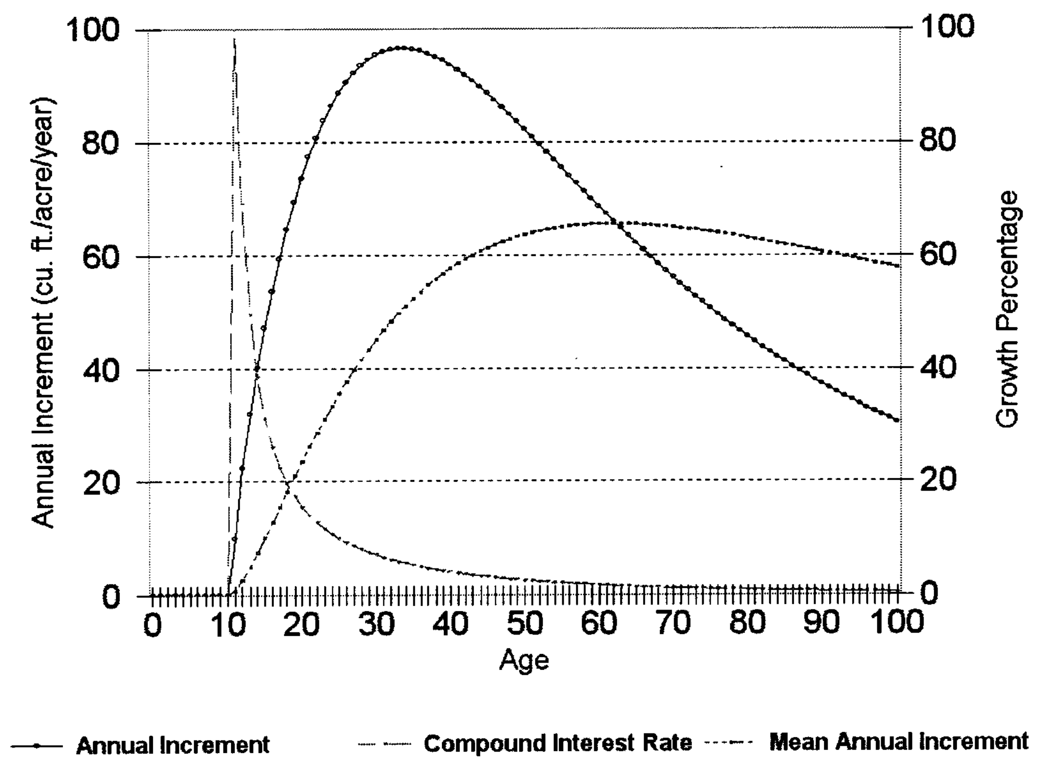

Figure 5.2. Annual increment, mean annual increment, and compound interest rate of growth for the example yield function in Figure 1.

The **Mean Annual Increment (MAI$_a$)** is the average annual growth rate per unit area up to age *a*. It is calculated by dividing the yield at age *a* by the age:

$$MAI_a = \frac{Y_a}{a}$$

The **Compound Interest Rate ($r^y_{a1,a2}$)** of growth for a timber stand gives the average compound volume growth rate between ages a_1 and a_2. The compound interest rate of growth for a timber stand between ages a_1 and a_2 is calculated as follows:

$$r^Y_{a_1,a_2} = \left[\sqrt[a_2-a_1]{\frac{Y_{a_2}}{Y_{a_1}}} \right] - 1$$

Note that the MAI measures the average annual productivity of a stand over the life of the stand. Thus, the age at which the MAI reaches its maximum is the harvest age (rotation) that maximizes the biological volume productivity of the stand over its lifetime. For this reason, the age that maximizes the MAI is sometimes advocated as the optimal biological rotation age. This is true only if the primary objective of the landowner is to maximize the volume production of the forest, with no regard to cost. The age at which the MAI is maximized is important enough to have it's own term: it is called the **culmination of mean annual increment**, or **CMAI**.

The MAI is a special case of the PAI, where $a_1 = 0$. You can estimate the CMAI age from the yield curve in Figure 5.1 by drawing a straight line from the origin of the graph (the point {0, 0}) that is just tangent to the yield curve. The slope of this line is equal to the MAI at the age of CMAI. For the yield curve in Figure 5.1, the CMAI is at age 62, where the MAI reaches 65.6 cu. ft./ac/yr. Note that this is also the age where the annual increment curve crosses the MAI curve. As long as the annual increment is greater than the MAI, the MAI keeps rising. When the annual increment falls below the MAI, the MAI begins to decline. This works just like your grade point average: when your grades for a semester are above your cumulative GPA, then your cumulative GPA rises; when your grades for a semester are below your cumulative GPA, then your cumulative GPA falls.

The compound interest rate of growth measures the average growth rate of a stand as a percentage of the inventory volume of the stand. Timber stands have much in common with many other assets. Like many investments, a timber stand is a valuable asset whose value is growing. One measure of the productivity of other investments is the interest rate at which they grow. The compound interest rate of growth provides a similar measure for a timber stand. If a timber stand is thought of as an asset, then the interest rate of growth of the *value* of the stand, including both the land and the timber and should be at least equal to the alternate rate of return. Otherwise, it would be better to liquidate the stand and put the proceeds into another investment. This is the basic concept on which the optimal economic rotation is based. This will be discussed in the next chapter.

Example: Growth Calculations

Calculate the yield, annual increment, PAI, MAI, and the 5-yr compound interest rate of growth for the following yield equation (cords/acre) for ages 20, 25, 30, 35, and 40.

$$Y_a = \frac{160}{1 + 1999(a-3)^{-2.3}} \quad \textit{for } a>3, \ 0 \ \textit{otherwise.}$$

Answer: Table 5.1 shows the results of these calculations. Sample calculations are provided below.

To calculate the yield at age 35,

$$Y_{35} = \frac{160}{1 + 1999(35-3)^{-2.3}} = 94.664$$

Table 5.1. Alternate measures of growth and yield for the example yield function.

Age	Yield (Cords)	Yield (Age-1)	Annual Increment	P.A.I. (Age-5 to Age)	M.A.I.	Compound Int. Rate
20	40.440	36.373	4.067	---	2.022	---
25	60.745	56.768	3.977	4.061	2.430	0.085
30	79.201	75.733	3.468	3.691	2.640	0.054
35	94.664	91.823	2.840	3.093	2.705	0.036
40	107.077	104.822	2.255	2.483	2.677	0.025

To calculate the yield at age 34,

$$Y_{34} = \frac{160}{1 + 1999(34-3)^{-2.3}} = 91.823$$

To calculate the annual increment between ages 34 and 35,

$$\Delta Y_{35} = Y_{35} - Y_{34} = 94.664 - 91.823 = 2.840$$

To calculate the PAI between ages 30 and 35,

$$PAI_{30,35} = \frac{Y_{35} - Y_{30}}{35 - 30} = \frac{94.664 - 79.201}{5} = 3.093$$

To calculate the MAI at age 35,

$$MAI_{35} = Y_{35} / 35 = 94.664 / 35 = 2.705$$

To calculate the annual compound interest rate of growth between ages 30 and 35,

$$r^{Y}_{35,30} = \left[\sqrt[[35-30]]{\frac{Y_{35}}{Y_{30}}}\right] - 1 = \left[\sqrt[5]{\frac{94.664}{79.201}}\right] - 1 = 0.036 = 3.6\%$$

3. Study Questions

1. What are the key stand characteristics that affect forest growth and yield?

2. What is site quality? What is a site index? How is site index related to site quality?

3. If the measured site index is different at two points in time, which site index is "correct?"

4 What are some problems with the concept of forest stocking?

5 Why is there not single, biologically-determined level of stocking that is appropriate for a given type of forest stand?

6 What is the difference between stand density and competition?

7 What is the difference between growth and yield?

8 What is the age that maximizes the MAI called? Why is this age often considered the optimal biological rotation for a stand?

9 Why is it that the MAI is maximized at exactly the point where the annual increment curve falls below the MAI curve?

10. Explain why the annual compound rate of growth should ideally be based on the increase in the value of a stand and not on the increase in volume.