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Quantifying Uncertainty: Potential Medical Applications of the Heston Model of Financial Stochastic Volatility

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ABSTRACT

The Heston Model, widely used in financial markets to characterize stochastic volatility, could potentially be useful in accounting for the impact of volatility in the broad field of medicine. This theoretical article highlights the potential uses of the Heston Model to quantify volatility in healthcare, focusing on epidemiology and pharmacology. Conceptually, the ability of the model to quantify unpredictability could provide insight into complex medical processes with variable variability. Rigorous testing would be required to determine the feasibility and validity of applying a financial model to biological processes. Nonetheless, the hypothetical connections between financial market volatility and volatility in medicine merit further exploration. This theoretical article explores a broad overview of possible applications of the Heston Model to the medical field.

Keywords: Stochastic volatility; Heston model; uncertainty; variance; biostatistics.

1. INTRODUCTION

The Heston Model is used to model stochastic volatility in the field of finance. While it has become widely adopted for helping predict price movements in equity options, there have been limited but no rigorous attempts to see if the model could have wider applicability and potentially be useful in the broad field of medicine. This exploratory article looks at potential applications in medicine by looking at disease forecasting in epidemiology and individualized drug dosing in clinical pharmacology. The rationale is that the model's capabilities in quantifying

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unpredictable volatility in financial markets could translate to characterizing volatility in biological contexts.

Advocates for enhanced mathematical applications in medicine have called for a two-way exchange of methodologies between fields, applying quantitative tools like physics and engineering to clinical medicine. As Amig´o and Small detail in their overview of mathematical methods in medicine, ”The ultimate reason for the ubiquity of mathematics in modern science is the necessity of mathematical thinking to understand complex phenomena” [1]. While mathematical finance may seem an unlikely source of healthcare insight, the versatility of techniques like the Heston Model merits open-minded exploration. Of course, rigorous validation is essential before clinical adoption.

2. THE HESTON MODEL

Introduced by Steven Heston in 1993, the Heston Model is a mathematical framework that describes the evolution of volatility in financial markets [2]. Unlike constant volatility models, the Heston Model proposes that volatility follows a stochastic process that fluctuates over time. This widely recognized stochastic volatility model used for pricing European options suggests that volatility undergoes mean reversion, returning to a long-term average. This feature enables the Heston Model to represent the volatility smile, where the implied volatility of options increases as the option is further out of the money [Fig. 1]. This diverges from predictions by the classic Black-Scholes model, which assumes static volatility [3].

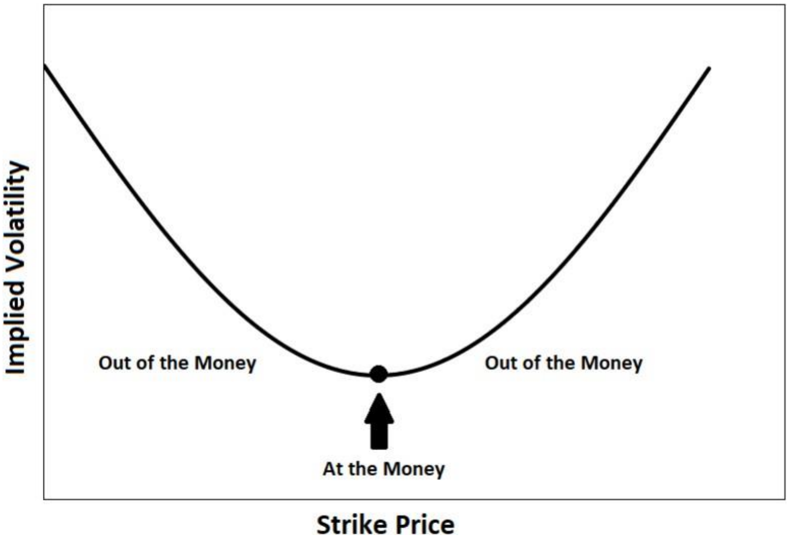

Fig. 1. The volatility smile

The Heston Model is defined by two stochastic differential equations: one for the price of the asset and the other for the variance of the price of the asset. The asset price follows a geometric Brownian motion [4], and the variance follows a mean-reverting square-root process [5]. The model is mathematically expressed as:

$$dS_t = \mu S_t dt + \sqrt{V_t} S_t dW_t^S$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t} dW_t^V$$

Where:

- S_t is the asset price at time t,
- V_t is the variance of the asset price at time t,
- μ is the rate of return of the asset,
- κ is the rate of mean reversion,
- θ is the long-term average variance,
- σ is the volatility of the volatility,
- W_t^S and W_t^v are two Wiener processes with correlation ρ.

The ability of the Heston Model to capture stochastic volatility makes it valuable for traders and risk managers in financial industries, particularly for pricing options on volatile assets like stocks and commodities [6]. The model's parameters, including the mean reversion rate (κ), long-term average variance (θ), and volatility of volatility (σ), are crucial in reflecting financial market dynamics. The correlation parameter (ρ) between the Wiener processes $W^S{}_t$ and $W^v{}_t$ is significant, where a negative ρ suggests that an increase in asset price often coincides with a decrease in volatility, known as the leverage effect [7].

Empirical studies demonstrate that the Heston Model, with its stochastic volatility feature, outperforms the Black-Scholes model, which assumes constant volatility. The model can be adjusted by market data, leading to improved precision in pricing options for different strike prices and expiration periods [8].

Incorporating the Heston Model into trading and risk management strategies can significantly enhance market analysis. For instance, the model's parameters can be estimated from market data, allowing traders to use it for real-time pricing and hedging options [9]. The model is particularly adept at pricing exotic options, such as Asian or barrier options, which are sensitive to the underlying asset's volatility path. Moreover, the Heston Model can be extended to multi-asset options by introducing additional correlated variance processes for each asset.

The calibration of the Heston Model to market data is a non-trivial task that often requires sophisticated numerical techniques, such as the Fourier transform methods or the use of characteristic functions [10]. Once calibrated, the model can be used to generate a volatility surface that is consistent with observed market prices of options, which in turn can be used to price and hedge new option contracts.

Extensions of the Heston Model framework, such as Heston++ [11] and the rough Hawkes Heston [12], also facilitate the incorporation of jumps or spikes in volatility, which can occur due to market events or announcements. These extensions of the model can be particularly useful given that sudden shifts in volatility are common and can have a significant impact on option prices.

In more recent developments, the Heston Model has been combined with machine learning techniques to further enhance its predictive power [13]. Using artificial neural networks to approximate the distribution of the underlying asset, researchers have shown the potential for an even greater accuracy in option pricing.

Applications of the Heston Model may go beyond financial markets, but so far have remained focused on economic forces. For example, in energy economics, it has been utilized to forecast the market volatility of energy generation [14]. This includes adjusting the model to account for seasonal patterns, spikes, or long-term trends in energy generation or consumption. By doing so, the model has shown potential for predicting financial, operational, and strategic risks in various industries (Reichert and Souza, 2022). However, the model's adaptability could allow it to be applied even more widely, outside of finance and economics, to other fields where stochastic volatility is a factor.

The fundamental principle of the Heston Model is straightforward: volatility fluctuates randomly but tends to return to its average level over time. The concept of accounting for random variability (i.e., stochastic volatility) almost certainly is not restricted to financial applications. Here, we look at potential medical applications in epidemiology and pharmacology.

3. STOCHASTIC VOLATILITY IN MEDICINE

3.1 ARCH and GARCH

Stochastic volatility, a concept deeply rooted in financial market analysis, has also been applied in medical research, particularly through the use of ARCH (Autoregressive Conditional Heteroskedasticity) and GARCH (Generalized ARCH) models. Originally developed for economic time series data, these models have been adopted to the unique demands of medical data analysis, offering a sophisticated approach to understanding the erratic nature of biological variables.

The ARCH model, introduced by Robert Engle in 1982 [15], revolutionized how we perceive time-series data by allowing the volatility to change over time based on past shocks or errors. The core premise of this model is that current volatility is a function of the magnitude of previous time periods' errors, making it particularly suitable for datasets where this kind of time-lagged effect is prominent. In medicine, this translates to understanding how past events (like a dose of medication or a stress episode) affect current physiological states, such as blood pressure or heart rate variability.

Building on ARCH, the GARCH model, formulated by Tim Bollerslev in 1986 [16], adds another layer of complexity in that it incorporates not just past errors, as in ARCH, but also past variances. This addition makes GARCH more adaptable to scenarios where the volatility itself has memory. In medical contexts, this potentially provides a more accurate way to model phenomena like the fluctuation of hormone levels or the variability in response to treatment in chronic diseases.

Variability of variability is well known and accounted for in multiple clinical settings. For example, there is variability in the heart rate variability. This is known to fluctuate throughout the day, with the maximum volatility typically occurring at night [17]. On the other hand, the variability of the resting metabolic rate is similar in the morning and at night [18]. This changing variability in heart rates could be more thoroughly understood through stochastic models, offering deeper insights into cardiac health [19].

Other medical areas that have demonstrated stochastic variability include general brain activation, as demonstrated by high-frequency activity [20]. Epilepsy has showed a similar pattern of stochastic volatility [21]. Also, using non-constant variance when modeling population growth has been shown to be helpful in capturing the volatility and heterogeneity in historical population data [22].

3.2 Ornstein-Uhlenbeck

The Ornstein-Uhlenbeck (OU) process is another stochastic model that has been utilized in medicine, in particular neurology [23]. While the OU process and the Heston Model offer frameworks for modeling stochastic processes, they differ significantly in their approach to volatility. The OU process models stochastic volatility with a linear reversion towards the mean, while the Heston Model utilizes a square root reversion. In the OU process, the deviation from the mean decays exponentially over time, which is an attractive feature for modeling biological phenomena where such linear mean-reverting behavior is observed.

The OU process is mathematically characterized by the equation $dX_t = \theta(\mu - X_t)dt + \sigma dW_t$, where θ is the speed of reversion to the mean level μ, σ is the scale of the process, and dW_t is a standard Wiener process.

While the Heston Model's square root reversion is adept at capturing the volatility dynamics observed in financial markets, the OU process's linear reversion is more suited to natural decay patterns. The OU process has been used more frequently in biological contexts because it stabilizes around some equilibrium point, making it suitable for modeling various biological phenomena [24].

However, taking into account a more complex model of volatility, as provided by the Heston Model, can potentially improve the modeling of several dynamic processes. Current models used in healthcare often assume static volatility. By taking into ac- count stochastic volatility, predictions in multiple important healthcare settings could potentially be improved.

3.3 Epidemiological Data Modeling

The incorporation of the Heston Model in epidemiological data modeling represents an innovative and promising strategy, drawing a parallel between the stochastic nature of financial market volatility and the dynamic patterns of disease transmission. Known for its effectiveness in modeling the unpredictable fluctuations of asset prices in finance, the Heston Model provides a potentially valuable framework for comprehending the variable nature of infectious disease spread, which is similarly influenced by a range of factors that can either mitigate or amplify transmission rates. This variability is characterized by its stochastic rather than constant nature. This variability, characterized by its stochastic nature, aligns with the principles of stochastic integration [25], highlighting the fundamental role of stochastic processes in accurately modeling complex systems like disease transmission. This potential for enhanced modeling ac- curacy is further underscored by the methods of quantitative risk management, which provide a comprehensive understanding of managing uncertainties in complex systems [26]. Adapting the Heston Model, or its essential components, for use in epidemiological modeling holds significant potential for enhancing the accuracy of predictive models and the efficacy of public health initiatives. This cross-disciplinary application promises to contribute to more robust epidemiological models, thereby improving preparedness and response to public health challenges.

The Heston Model's ability to incorporate shocks or spikes in volatility aligns well with modeling the effects of public health interventions. Campaigns like vaccination drives or policy changes such as lockdowns can dramatically influence disease transmission rates and variability in a manner akin to market-moving news events in finance. Additionally, the impact of social behaviors on disease transmission is analogous to market sentiment in finance, where changes in behavior, whether spontaneous or policy-induced, significantly affect disease spread. The model could treat these interventions as volatility shocks.

Additionally, the Heston Model can capture dynamic changes in volatility levels, making it suitable for modeling different phases of an epidemic. In the initial stage, there may be high uncertainty and variance in transmission patterns, analogous to an emerging new stock with unpredictable price changes. As immunity builds or mitigation measures roll out, this volatility may stabilize, similar to the maturation of an asset class.

The mean-reverting property also suits infection dynamics that tend to follow a wave-like surge pattern, periodically returning to an endemic level after each outbreak peak. This is analogous to how market volatility spikes around events but ultimately reverts to a mean range in calm periods.

Using the Heston Model to model the changing variability or volatility in disease trans- mission rates can enable public health officials to develop more dynamic strategies. High predicted volatility in spread rates might necessitate more rigorous interventions, while low volatility could indicate a period of stability, allowing for the relaxation of certain measures.

The core of the Heston Model is its mean-reverting stochastic volatility. This contrasts with traditional epidemiological models, which typically employ fixed parameters, lacking the flexibility to adapt to real-time changes in disease dynamics.

3.4 Empirical Study

To demonstrate the applicability of the Heston Model, consider a simulated outbreak scenario. Using historical data on influenza spread, we can parameterize the Heston Model to mimic the observed patterns. The mean-reverting property is particularly insightful here, as it captures the tendency of disease spread to fluctuate around a long-term mean, influenced by factors like seasonal changes, population immunity, and public health interventions.

Utilizing this mean-reverting feature to address variability in volatility of disease spread can potentially offer a more nuanced understanding and management of disease transmission dynamics, potentially leading to more informed and effective public health responses.

While periodicity in epidemiology refers to predictable, regular cycles of disease outbreaks, mean-reversion, as modeled by the Heston Model, offers a more nuanced view. It suggests that while disease spread can fluctuate significantly due to external shocks (e.g., super-spreader events), it tends to return to a baseline level, influenced by long-term factors like herd immunity and seasonal variations. This aspect of the Heston Model could be particularly valuable in predicting the course of diseases that exhibit both sudden outbreaks and periods of relative calm.

3.5 Drug Efficacy and Dosage Optimization

Applying a variability parameter to drug efficacy and dosage optimization accounts for the broad differences among individuals due to genetic, environmental, and lifestyle factors. Similar to the way asset prices vary with market conditions, individual re- sponses to drugs can fluctuate according to numerous factors. The effectiveness of a drug changes over time, influenced not only by the development of resistance or changes in disease pathology but also by alterations in patient behavior and physiology.

Understanding the variance in drug response allows clinicians to optimize dosage sched- ules for individual patients. This optimization involves adjusting the amount of drug administered, the frequency of dosing, or the duration of the dosing interval to maintain efficacy while minimizing side effects and reducing waste. At the population level, accounting for stochastic volatility can inform public health decisions, such as determining appropriate medication stockpiles.

3.6 Empirical Application

Consider a new medication designed to decrease hard cardiac events in high-risk patients. By simulating the varying and unpredictable responses of individual patients, dose titration could potentially be improved. Stochastic volatility metrics could incorporate diverse variables such as the age of the patient, existing health conditions, and genetic predispositions. In addition, it could account for periodic variables such as daily or seasonal cycles as well as more random shocks to the system, such as unexpected stressors (e.g., a death in the family or a turn in world events). In this manner, the Heston Model could potentially enhance patient-specific treatments and optimize healthcare resources.

The potential of the Heston Model in clinical pharmacology lies in its ability to provide a structured approach to understanding and managing the inherent variability in drug response over time and among different individuals, taking into account unexpected, random events that affect their unique physiology. Through improved modeling, there is the potential to reduce costs and side effects while maximizing drug efficacy.

4. DISCUSSION

The Heston Model, a pioneering contribution to stochastic modeling in finance, is distinguished by its incorporation of fluctuating volatility, a defining characteristic of its design. The underlying assumption supporting this model posits that volatility adheres to a mean-reverting process, indicating a propensity to return to a long-term average. This attribute is pivotal to the model's capacity to capture the dynamic essence of volatility, presenting a sophisticated approach compared to models assuming static volatility.

However, the assumption of mean reversion poses certain challenges, especially when utilized beyond the financial domain, such as in health care. In varied applications, the phenomenon being modeled may not conform to the mean-reverting pattern assumed by the model. Consequently, the Heston Model may not accurately reflect the true behavior of the studied variable, potentially resulting in analytical inaccuracies. This is evident in scenarios like epidemiological studies or drug efficacy trials, where the studied variable's variability can exhibit persistent trends or shifts beyond the model's anticipated time frame.

Another aspect to consider is the model's dependence on the theory of mean reversion and specific parameters that define the rate of mean reversion, the long-term average of the variable, and the variance of the variable itself. Accurately estimating these parameters is crucial for the model's effectiveness. However, challenges associated with estimating parameters, especially in the dynamic realm of biological phenomena and the limitations inherent in historical data as a basis for predicting future trends, can make this task challenging. Therefore, while the Heston Model represents a significant advancement in stochastic modeling, its effectiveness is constrained by its dependence on the theory of mean reversion and the challenges associated with estimating parameters. These limitations should be carefully considered when applying the model in diverse fields, particularly in medicine, where atypical patterns of variability are common.

Beyond the potential applications proposed in this article, exploring the adoption of the Heston Model in medicine carries a secondary benefit of spurring interdisciplinary awareness and mathematical literacy. Having healthcare experts grow conversant in sophisticated financial economics tools like stochastic volatility models facilitates better exchange of quantitative methodologies across fields. Just as statistical fragility gauges the reliability of research findings [27], measurements of uncertainty in financial markets could help provide improved interpretations of data variability and stability in biological systems. With the lines blurring between industries in a data-rich world, propagating versatile mathematical frameworks nurtures the communication channels and level field needed for the cross-pollination of ideas. Techniques proven in complex domains like markets can find an innovative second life in medicine. Medical challenges can equally drive the engineering of new solutions. These opportunities rely on a shared vocabulary at the mathematical interface of disciplines.

5. CONCLUSION

The inherent unpredictability and volatility in biological and medical systems at times parallel the stochastic behavior of financial markets that the Heston Model is designed to characterize. Just as this model captures the dynamics of fluctuating asset prices, its techniques for quantifying volatility could shed light on the erratic variations in multiple areas of medicine, including diverse applications such as disease transmission and pharmaceutical dosing. The rigorous validation of the Heston Model in finance lends credibility to its versatile modeling approach being applicable more broadly.

The parallels between financial systems and healthcare provide a reasonable theoretical justification for exploring the utility of the model with empirical research. Just as the model evolved from its options pricing origins to become a widely used financial tool, rigorous validation could transform its medical applications from speculative hypotheses into clinically useful practice. Extending an established, versatile financial model to other domains, such as medicine, represents a worthy interdisciplinary endeavor with promise. Exploration of potential applications requires caution and a rigorous analysis. Nevertheless, the potential benefits are worth the effort.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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