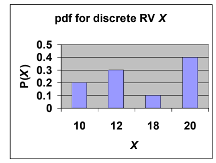
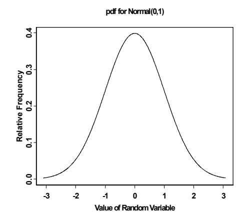
Chapter 6: Normal Probability Distributions

Boxes used to graph probability distributions for discrete RVs are now replaced with smooth curves for continuous distributions

Continuous Probability Distributions





Continuous Random Variables

The curve is a function of x, denoted $f(x)$ and may be called one of several terms: ***probability density function*** (pdf), a ***frequency function***, or a ***probability distribution***

The areas under a pdf correspond to probabilities for X.

$$P(a<X<b)=P(a\le X\le b)$$

$$P(X=a)=0$$

The Uniform Distribution

Used for continuous *RVs* that appear to have equally likely outcomes over their range of possible values.

Suppose $a \le x \le b$ for some continuous *RV*. The height of $f(x)$ is constant in the interval $[a,b]$ and equals $\frac{1}{b-a}$.

The Uniform Distribution

Formally, the pdf is written as:

$$f(x)=\frac{1}{b-a};\quad (a\le x\le b)$$

a and b are parameters of the distribution

$X \sim Uniform(a,b)$ or $X \sim U(a,b)$

The Uniform Distribution

Suppose $c<x<d$ lies within the domain of x;
i.e., $c<x<d$ falls within the larger interval $a\le x\le b$.

The probability that x assumes a value within $c<x<d$ is equal to the area of the rectangle over the interval or $\frac{d-c}{b-a}$

$$P(c<x<d)=\int_c^d f(x)dx$$

$$=\int_c^d \frac{1}{b-a}dx=\frac{d-c}{b-a}$$

The Normal Distribution

A ***normal random variable*** has a bell-shaped probability distribution called the ***normal distribution.***

Important in statistical inference; many processes generate random variables with probability distributions that may be modeled using a normal distribution.

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The Normal Distribution

The normal distribution is perfectly symmetric about its mean, μ.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$-\infty < x < \infty$$

μ and σ are parameters of the distribution

$$X \sim N(\mu,\sigma) \text{ or } X \sim N(\mu,\sigma^2)$$

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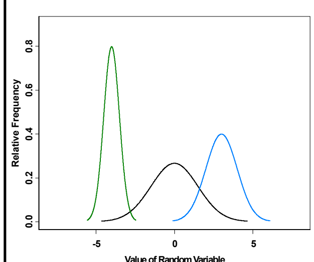
Some Normal Curves



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The Standard Normal Distribution

The ***standard normal distribution*** is a normal distribution with $\mu = 0; \quad \sigma = 1$.

A ***standard normal random variable*** has a standard normal distribution and observations are denoted by *z*.

$$z = \frac{x-\mu}{\sigma}$$

z describes the number of standard deviations between an observed value *x* and the mean.

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The Standard Normal Distribution

If *X* is normally distributed with any mean and standard deviation, then *Z* will always be normally distributed with
$\mu = 0; \quad \sigma = 1$

$P(|Z| > 1.96) = .05$ implies that 95% of the observations of a normally distributed *RV* will lie between ± 1.96 SDs of the mean

50% lies within $\mu \pm .67\sigma$

95% lies within $\mu \pm 1.96\sigma$

99% lies within $\mu \pm 2.58\sigma$

Compare with the Empirical Rule

68.26% lies within $\mu \pm \sigma$

95.46% lies within $\mu \pm 2\sigma$

99.73% lies within $\mu \pm 3\sigma$

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Example

Class lengths are uniformly distributed between 50.0 and 52.0 minutes.

a) Make a sketch of the pdf

b) Randomly select a class length and find $P(X < 51.5$ min)

c) Randomly select a class length and find $P(51.5$ min $<= X <= 51.6$ min)

d) Find μ_X and σ_X

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Example

Assume that adults have IQ scores that are normally distributed with a mean of 100 and a SD of 15. For a randomly selected adult find:

a) The probability that $X < 130$

b) The probability that $90 < X < 110$

c) P_{10} and P_{60}

d) If $P(X > x) = .0643$, find x

e) If $P(X < x) = .4500$, find x

f) If $P(X > x) = .9922$, find x

Example (from McClave and Sincich, 9th Ed., page 237)

A machine used to regulate the amount of dye dispensed for mixing shades of paint can be set so that it discharges an average of μ milliliters (mL) of dye per can of paint. The amount of dye discharged is known to have a normal distribution with a standard deviation of 0.4 mL. If more than 6 mL of dye are discharged when making a certain shade of blue paint, the shade is unacceptable. Determine the setting for μ so that only 1% of the cans of paint will be unacceptable.

Example (from McClave and Sincich, 9th Ed., page 223)

A tool and die machine shop produces extremely high-tolerance spindles. The spindles are 18-inch slender rods used in a variety of military equipment. A piece of equipment used in the manufacture of the spindles malfunctions on occasion and places a single gouge somewhere on the spindle. However, if the spindle can be cut so that it has 14 consecutive inches without a gouge, then the spindle can be salvaged for other purposes.

Assuming that the location of the gouge along the spindle is best described by a uniform distribution, what is the probability that a defective spindle can be salvaged?