The Additive Rules and Mutually Exclusive Events

Additive rule of probability-- Given events A and B, the probability of the union of events A and B is the sum of the probability of events A and B minus the probability of the intersection of events A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually exclusive-- Events A and B are mutually exclusive if A and B have no sample points in common; or, if $A \cap B$ is empty. Thus, for mutually exclusive events:

$$P(A \cup B) = P(A) + P(B)$$

Example 1

The outcomes of two variables are {Low, Medium, High} and {On, Off}. An experiment is conducted in which each of the two variables are observed. The probabilities associated with each of the six possible outcome pairs are given as:

Consider the events:

A: {On}
B: {Medium or On}
C: {Off and Low}

	Low	Medium	High
On	.50	.10	.05
Off	.25	.07	.03

Example 1 (continued)

a. Find *P(A)*

b. Find *P(B)*

c. Find *P(C)*

d. Find $P(\overline{A})$

e. Find $P(A \cup B)$

f. Find $P(A \cap B)$

g. Consider each pair of events (A and B, A and C, B and C). List the pairs of events that are mutually exclusive.

Conditional Probability

The probability that event A occurs given that event B occurred is denoted by $P(A|B)$

This is a conditional probability; it is read as "the conditional probability of A given that B has occurred"

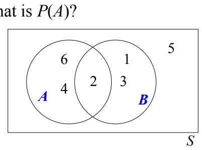
Example

A: {even number on throw of fair die}

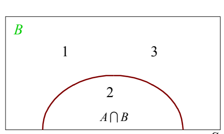
B: {on a particular throw of the die, the result was a number ≤ 3}

Conditional Probability

What is $P(A)$?



What is $P(A|B)$?



Conditional Probability

To find the conditional probability that event A occurs given that event B occurs, divide the probability of that both A and B occur by the probability that B occurs

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Assumption: $P(B) \neq 0$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(2)}{P(1) + P(2) + P(3)}$$

$$= \frac{1/6}{3/6} = \frac{1}{3}$$

Example from Introduction to the Practice of Statistics, 3rd Edition, Moore and McCabe, pp. 350-351

Age and marital status of women (thousands of women)

	Age			
	18 to 24	25 to 64	65+	**Total**
Married	3,046	48,116	7,767	**58,929**
Never Married	9,289	9,252	768	**19,309**
Widowed	19	2,425	8,636	**11,080**
Divorced	260	8,916	1,091	**10,267**
Total	**12,614**	**68,709**	**18,262**	**99,585**

Example from Introduction to the Practice of Statistics, 3rd Edition, Moore and McCabe, pp. 350-351

Choose one women at random ➔ all women have an equal chance of being chosen

What is *P*(married)?

What is *P*(age 18 to 24 and married)?

How about *P*(married | age 18 to 24)?

Example from Introduction to the Practice of Statistics, 3rd Edition, Moore and McCabe, pp. 350-351

There is a relationship among the three probabilities

P(married and age 18 to 24) =

P(age 18 to 24) x *P*(married | age 18 to 24)

Multiplicative Rule and Independent Events

The probability that both of two events A and B happen together can be found by

$$P(A \text{ and } B) = P(A|B)P(B)$$
$$P(A \cap B) = P(B)P(A|B)$$

Derived from the formula for calculating conditional probability

$$\frac{P(A \cap B)}{P(B)} = P(A|B)$$

Multiplicative Rule and Independent Events

Independent events---Events A and B are said to be independent events if the occurrence of B does not alter the probability that A has occurred;

or, events A and B are independent if

$$P(A|B) = P(A)$$

Otherwise, events ***A*** and ***B*** are ***dependent***

Summary-- for independent events, knowing B does not effect the probability of A

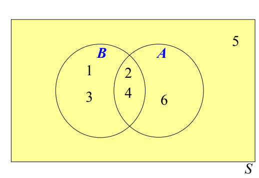
Multiplicative Rule and Independent Events

Example: Experiment consisting of tossing a fair die

Define the events:

A = {observe an even number}

B = {observe a number ≤ 4}



Multiplicative Rule and Independent Events

Calculate:

$$P(A) = P(2) + P(4) + P(6) = \frac{1}{2}$$

Calculate:

$$P(B) = P(1) + P(2) + P(3) + P(4) = \frac{4}{6} = \frac{2}{3}$$

Calculate:

$$P(A \cap B) = P(2) + P(4) = \frac{2}{6} = \frac{1}{3}$$

Multiplicative Rule and Independent Events

Assuming *B* has occurred, calculate

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/3}{2/3} = \frac{1}{2} = P(A)$$

Thus, the probability of observing an even number remains the same, regardless of assuming that event *B* occurs

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/3}{1/2} = \frac{2}{3} = P(B)$$

Example: Three cards are dealt off the top of a well-shuffled deck of playing cards

What is the probability that the first card is a heart?

What is the probability that the second card is a spade?

What is the probability that the first card will be a heart and the second card will be a spade?

What is the probability that the second card will be a spade given that the first card is a heart?

Probability of the Intersection of Two Independent Events

If events *A* and *B* are *independent*, the probability of the intersection of *A* and *B* equals the product of the probabilities of *A* and *B*; or

$$P(A \cap B) = P(A)P(B)$$

The converse is also true---

if $P(A \cap B) = P(A)P(B)$
then events *A* and *B* are independent.