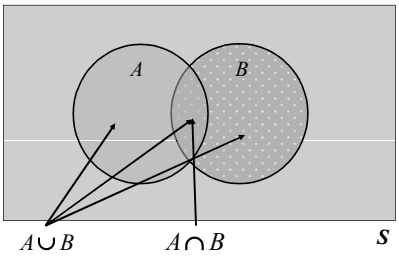
Unions and Intersections

Compound events---defined as a composition of two or more other events

They can be formed in two ways:

- ***Union***---the union of two events A and B, denoted as $A \cup B$, is the event that occurs if either A ***or*** B or both occur on a single performance of an experiment
- ***Intersection***---the intersection of two events A and B, denoted as $A \cap B$, is the event that occurs if both A ***and*** B occur on a single performance of the experiment

Unions and Intersections



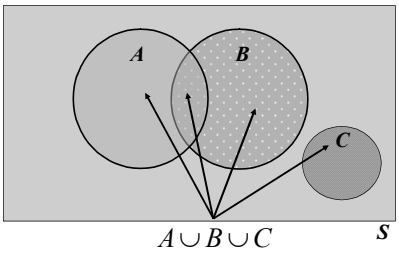
Formal Addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Roll two standard dice and record the sum of up faces
Define ***C*****: {A 5 appears on *at least* one of the dice**
What is *P*(*C*)? ***P*(*C*) = 11/36**

	1	2	3	4	5	6
1	1,1 (2)	1,2 (3)	1,3 (4)	1,4 (5)	1,5 (6)	1,6 (7)
2	3	4	5	6	7	8
3	4	5	**6**	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

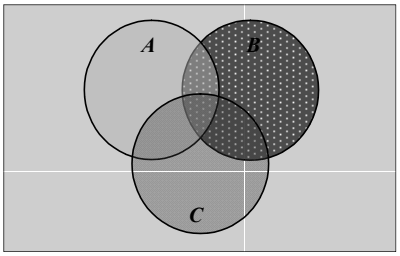
Unions and Intersections



What is $A \cap C$?

$A \cap C$ is empty ➔ P(A and C) = 0

Unions and Intersections



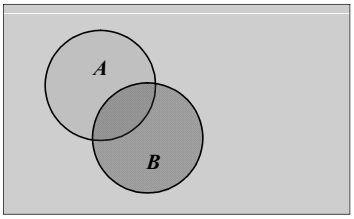
What is $A \cup B \cup C$?

$$P(A \cup B \cup C) = P(A) + P(B) + (C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

Disjoint and Mutually Exclusive Events

Events *A* and *B* are **disjoint** or **mutually exclusive** if they cannot occur at the same time.

Two or more events are mutually exclusive if no two of them have outcomes in common.



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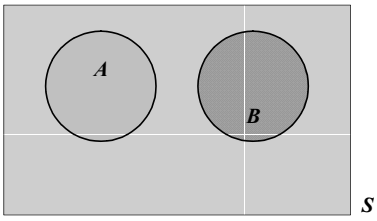

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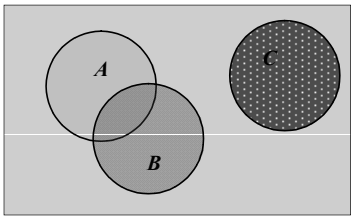

Three non-mutually exclusive events

Unions and Intersections

Applet with a Venn diagram containing three events. You can play around with the 3 events—intersecting or not

http://stat-www.berkeley.edu/~stark/Java/Html/Venn3.htm

Lets you visualize various situations and displays probabilities for the situations. Also includes some conditional probabilities (definition next time).

Complementary Events

Complement---the complement of event A, denoted as $\bar{A}$ or A^c, is the event that A ***does not occur*** ; the event consisting of all sample points **not in *A***

Rules: $P(A)+P(\bar{A})=1$

$P(\bar{A})=1-P(A)$

$P(A)=1-P(\bar{A})$

May be useful for events with large numbers of possible outcomes

Complementary Events

Example

Toss a coin ten times and record the up face after each toss. What is the probability of event

***A*: {Observe *at least* one head}?**

The list of possible sample points is long:
HHHHHHHHHH, HHHHHHHHHT,
HHHHHHHHTH, HHHHHHHTHH, etc.

How many possible outcomes are there?

$2^{10} = 1{,}024$

Complementary Events

Example

Consider the complement of A,

$\bar{A}$: {No heads are observed in 10 tosses} =

{TTTTTTTTTT}

What is P($\bar{A}$)?

P($\bar{A}$) = 1/1,024

Now, $P(A)$ = 1 – P($\bar{A}$) = 1 – 1/1,024

= 1,023/1,024 = 0.999

Example

A study of binge alcohol drinking by college students was published by the Amer. Journal of Public Health in July '95. Suppose an experiment consists of randomly selecting one of the undergraduate students who participated in the study.

Consider the following events:

A: {The student is a binge drinker}
B: {The student is a male}
C: {The student lives in a coed dorm}

Describe each of the following events in terms of unions, intersections and complements

$\left(A \cup B,\ A \cap B,\ \overline{A},\ \text{etc.}\right)$

a. The student is male and a binge drinker -- $A \cap B$

b. The student is not a binge drinker -- $\overline{A}$

c. The student is male or lives in a coed dorm -- $B \cup C$

d. The student is female and not a binge drinker -- $\overline{A} \cap \overline{B}$

***A*: {The student is a binge drinker}**
***B*: {The student is a male}**
***C*: {The student lives in a coed dorm}**