Chapter 4: Probability

Forms the foundation for the inferential methods we will learn

Rare Event Rule for Inferential Statistics

Give a particular assumption, if the probability of a particular observed event is extremely rare, we conclude that the assumption is probably not correct.

Ex: Consider tossing a fair coin.

What assumption are we making?

Rare Event Rule

Ex: Consider tossing a fair coin.

For an individual toss of the coin we are assuming that probability of heads = probability of tails = 0.5

If after 100 tosses we observe:

TTTTHHTTHTHTTHHTHHTTHTTTTTTTHH
THTHHHHTTTTHHTTTHHTHHHTTTHHT
THTTHTTTHHHHHTHHHTHTTHHHHHHT
HHHTTTHHHHTTHTT

What might we conclude?

Rare Event Rule

Ex: Consider tossing a fair coin.

For an individual toss of the coin we are assuming that probability of heads = probability of tails = 0.5

If after 100 tosses of a different coin we observe:

THHTTTTHHTTTTTTTTTTTTTTHTTHTH
TTTTTHHHTTTTTHTTTTTTHTTTTTTTT
TTTTTTTTHTTTTTHTTTTHTTTTHTTTT
THTTTTHTTTHTT

What might we conclude?

Basic Definitions

Toss a coin three times and record the sequences of heads (H) and tails (T)

HHH HHT HTH THH
HTT THT TTH TTT

Event: a collection of outcomes—all outcomes resulting in two heads and one tail

Simple event or ***sample point*** – one possible outcome that cannot be broken down into simpler components

S = ***sample space***; The set of all possible outcomes from a procedure — all simple events

Notation

- P denotes probability
- Capital letters such as A, B, C, … denote specific events
- $P(A)$ denotes the probability of event A occurring

Rule 1

The Relative Frequency Interpretation of Probability

Define the **probability** of a specific outcome as ***the proportion of times it would occur over the long run*** (**relative frequency** of that particular outcome)

Applies to situations that you can imagine repeating many times

Rule 1 Example

$$P(A) = \frac{\text{number of times } A \text{ occurred}}{\text{number of times the trial was repeated}}$$

Define A to be the event "2 tails in a toss of three coins"

We toss the three coins 25 times and observe 7 sets that have 2 tails

$$P(A) = ?$$

$$P(A) = \frac{7}{25} = 0.28 \text{ or } 28\%$$

Rule 2

Classical Approach to Probability

- Assume that a procedure has n different simple events (n possible outcomes)
- Each of the n outcomes are ***equally likely***
- If event A can occur in s of the n ways, then

$$P(A) = \frac{\text{number of ways } A \text{ can occur}}{\text{number of different simple events}} = \frac{s}{n}$$

Rule 2 Example

Consider the tossing three coins experiment

Define A to be the event "2 tails in a toss of three coins"

How many ways can A occur?

HTT THT TTH

How many different simple events are there?

HHH HHT HTH THH HTT THT TTH TTT

$$P(A) = ?$$

$$P(A) = \frac{s}{n} = \frac{3}{8} = 0.375$$

Law of Large Numbers

As a procedure is repeated many, many times, the relative frequency probability (from Rule 1) of an event tends to approach the actual probability (Rule 2 for equally likely outcomes)

Assigning Probabilities

- A value between 0 and 1 written either as a fraction or as a decimal fraction.
- For the complete set of distinct possible outcomes of a random circumstance, the **total of the assigned probabilities must equal 1.**

Rule 3

- **The Personal Probability Interpretation**
- **Personal probability** of an event--the degree to which a given individual believes the event will happen--***sometimes termed subjective probability***
- **Restrictions** on personal probabilities:
  - Must fall between 0 and 1 (or between 0 and 100%)
  - Must be **coherent**

Why Study Probability?

- Want to be able to make inferences about a population from a sample or samples
- Probability will allow inferences with a measure of reliability (or uncertainty) for the inferences
- Initially, we will assume that the population is known and will calculate the probability of observing various samples from the population; ***i.e., use the population to infer the probable nature of the sample***