

Estimating and Plotting Logarithmic Error Bars

Eric M. Stuve

Department of Chemical Engineering
University of Washington

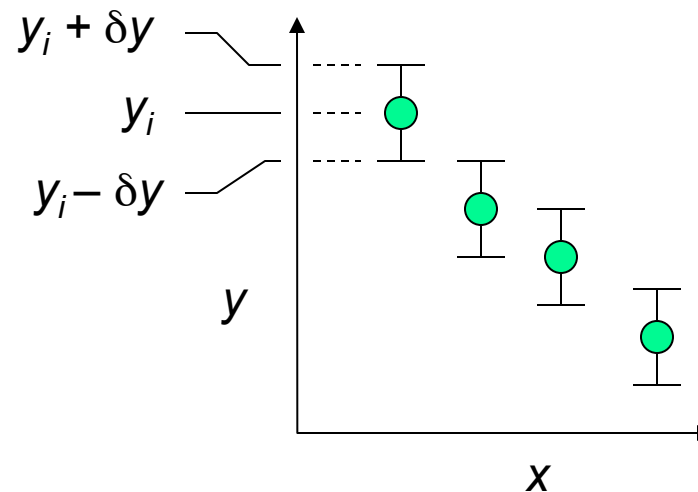
Box 351750, Seattle, WA 98195-1750, USA
stuve@uw.edu

http://faculty.washington.edu/stuve/uwess/log_error.pdf

©2004–12

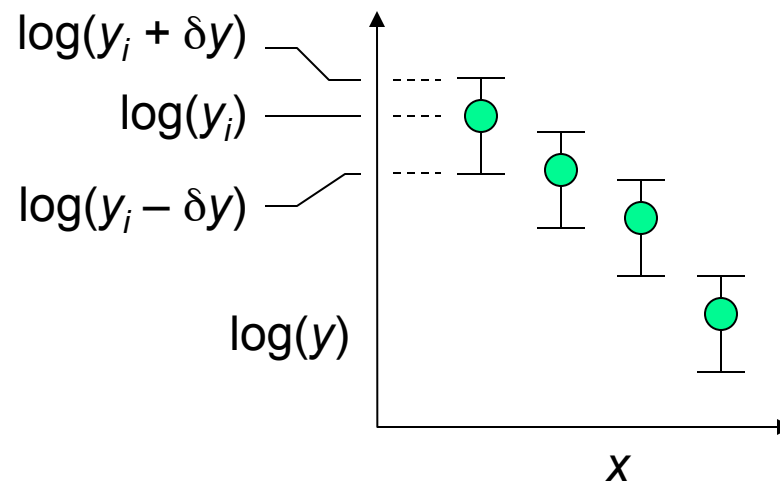
Absolute Error Bars

- Suppose that one has a sufficient number of measurements to make an estimate of a measured quantity y and report its absolute error, $\pm\delta y$.
- The absolute error $\pm\delta y$ is represented on a Cartesian plot by extending lines of the appropriate size above and below the point y .



Absolute Error Bars on a log Plot

- If plotted on a logarithmic plot, however, absolute error bars that are symmetric on a y vs. x plot become asymmetric; the lower portion is longer than the upper portion.



- This gives a misleading view of measurement precision, especially when measured quantities vary by several orders of magnitude.

Error in Logarithmic Quantities

- To represent error bars correctly on a log plot, one must recognize that the quantity being plotted, which we call z , is different than the measured quantity y .

$$z = \log(y)$$

- The error δz is

$$\delta z = \delta[\log(y)]$$

log *Error is Relative Error*

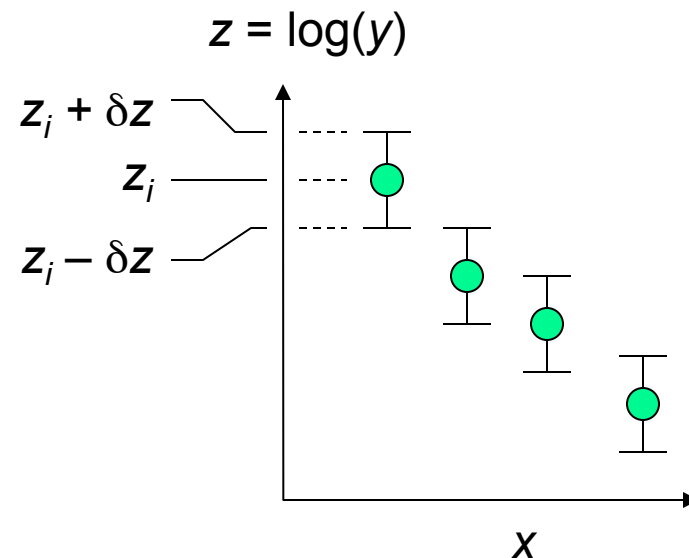
- On the assumption of small errors, a differential analysis can be used

$$\delta z \approx dz = d[\log(y)] = \frac{1}{2.303} \frac{dy}{y} \approx 0.434 \frac{\delta y}{y}$$

- The error δz is thus given by the *relative error* in y

$$\delta z \approx 0.434 \frac{\delta y}{y}$$

- The error bars now display correctly on a logarithmic plot.



Example: log Error Bars

- Plot the following data with error bars on a *log-log* plot

x	y	δy
0.03	0.011	0.003
0.1	0.042	0.006
0.2	0.093	0.018
0.5	0.21	0.02
1	0.28	0.05
2	0.53	0.12
5	0.77	0.12
20	1.88	0.3
50	3.56	0.4
100	8.10	1.58

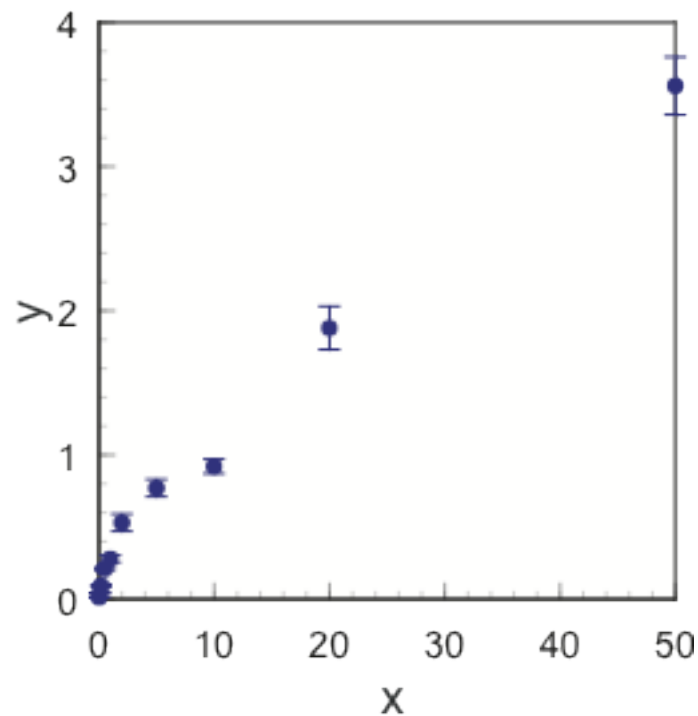
Example: log Error Bars

- First we calculate the quantities we need to plot:

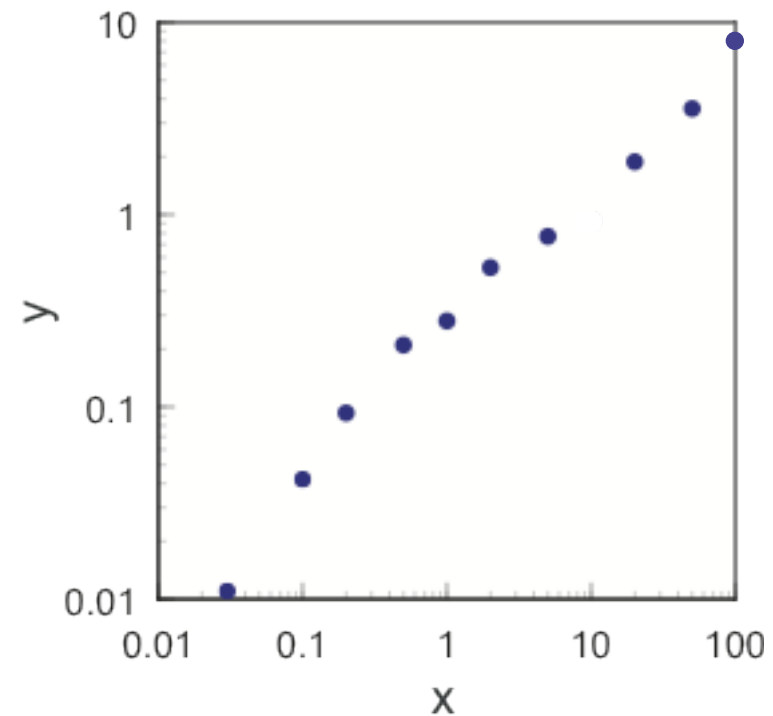
x	y	δy	$\log(x)$	$\log(y)$	$\log(y - \delta y)$	$\log(y + \delta y)$	$\delta y/y$	$0.434 \delta y/y$
0.03	0.011	0.003	-1.523	-1.959	-2.097	-1.854	0.273	0.118
0.1	0.042	0.006	-1.000	-1.377	-1.444	-1.319	0.143	0.062
0.2	0.093	0.018	-0.699	-1.032	-1.125	-0.955	0.194	0.084
0.5	0.21	0.02	-0.301	-0.678	-0.721	-0.638	0.095	0.041
1	0.28	0.05	0.000	-0.553	-0.638	-0.481	0.179	0.078
2	0.53	0.12	0.301	-0.276	-0.387	-0.187	0.226	0.098
5	0.77	0.12	0.699	-0.114	-0.187	-0.051	0.156	0.068
20	1.88	0.30	1.300	0.274	0.199	0.338	0.160	0.069
50	3.56	0.40	1.699	0.551	0.500	0.598	0.112	0.049
100	8.10	1.58	2.000	0.908	0.814	0.986	0.195	0.085

Example (cont.)

- Compare the Cartesian (left) and *log-log* (right) plots.
- The *log-log* plot displays the data better.
- Many data points are lost in the lower left corner of the Cartesian plot



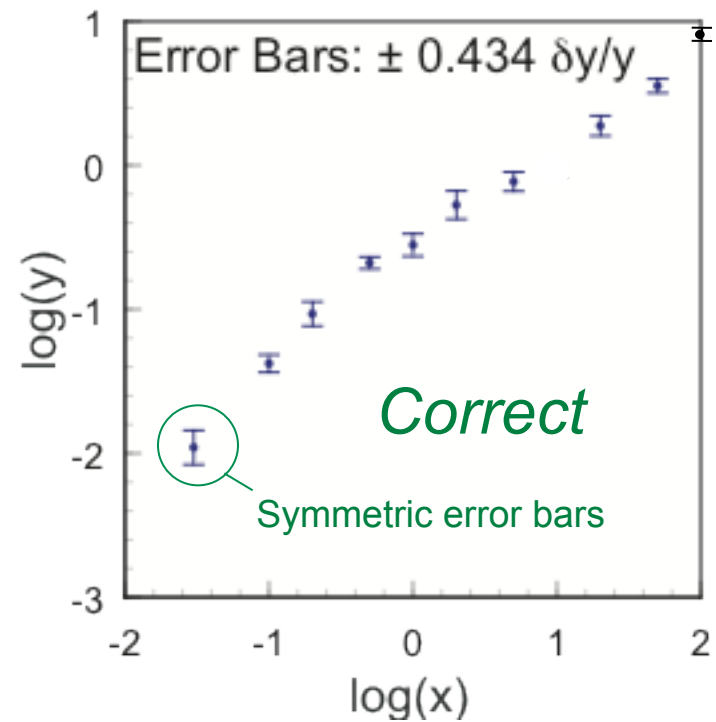
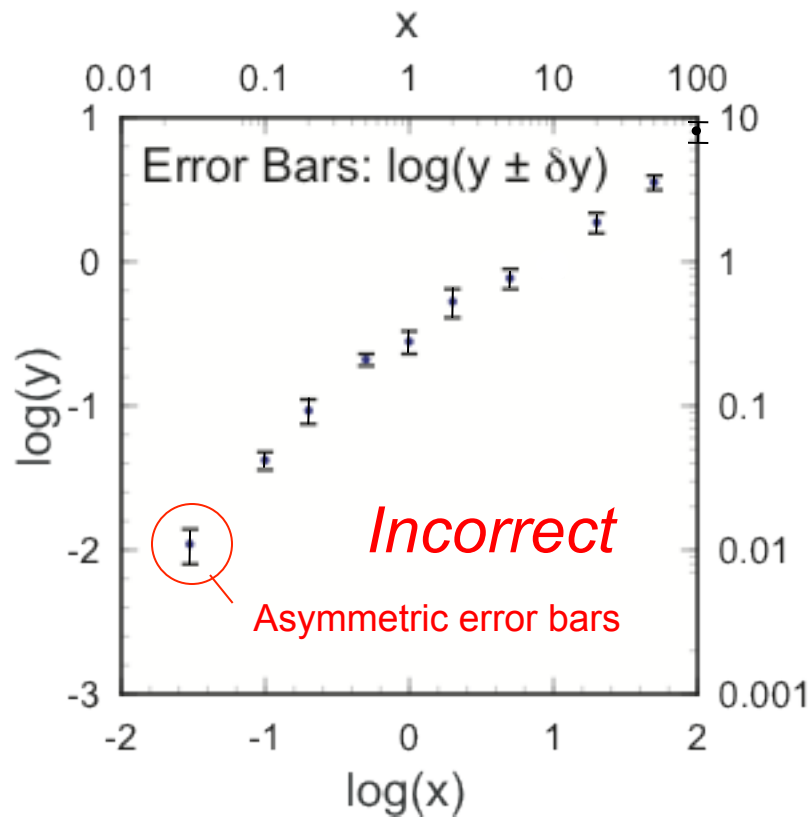
Cartesian plot



log-log plot

Example (cont.)

- Plot on left shows absolute error bars
- Plot on right shows relative error bars



Comments on Example

- The column $\delta y/y$ is the *relative error*. It varies from 10–27% in this example. The relative error is used for the error bars on a logarithmic plot.
- The asymmetric error bars on the Cartesian plot are best seen for the points with large errors, like the first point.
- The logarithmic error bars are plotted on the *log(y) scale*. That means on the scale that reads $-3, -2, -1, 0, 1$; *not* on the scale that reads $0.001, 0.01, 0.1, 1, 10$.

Comments (cont.)

- The data in the example represent measurements of the amount (y) of methanol electrooxidation as a function of time (x) taken from:

S. Sriramulu, T. D. Jarvi and E. M. Stuve, "Reaction mechanism and dynamics of methanol electrooxidation on platinum(111)," *Journal of Electroanalytical Chemistry*, Vol. 467, pp. 132–142 (1999).

- It was necessary to show that a straight line cannot be drawn through all of the points, which required correctly drawn error bars.
- The points can only be fit by a curved line, which meant that the reaction mechanism was more complex than thought: there were four rate determining steps instead of just one.

Reference

- The method for calculating *log* error bars can be derived from discussions of measurement error as appear in texts on analytical chemistry.
- For a specific reference on this material, see:

D. C. Baird, Experimentation: An Introduction to Measurement Theory and Experiment Design (3rd Ed.), Benjamin Cummings (1994). ISBN 978-0133032987