Chiral perturbation theory with physical-mass ensembles

Steve Sharpe
University of Washington

ChPT for LQCD: Does it have a future?

Steve Sharpe
University of Washington

LQCD for ChPT?

Steve Sharpe
University of Washington

Outline

- Brief history of ChPT for LQCD
- Will ChPT continue to be useful for LQCD?
- LQCD for ChPT

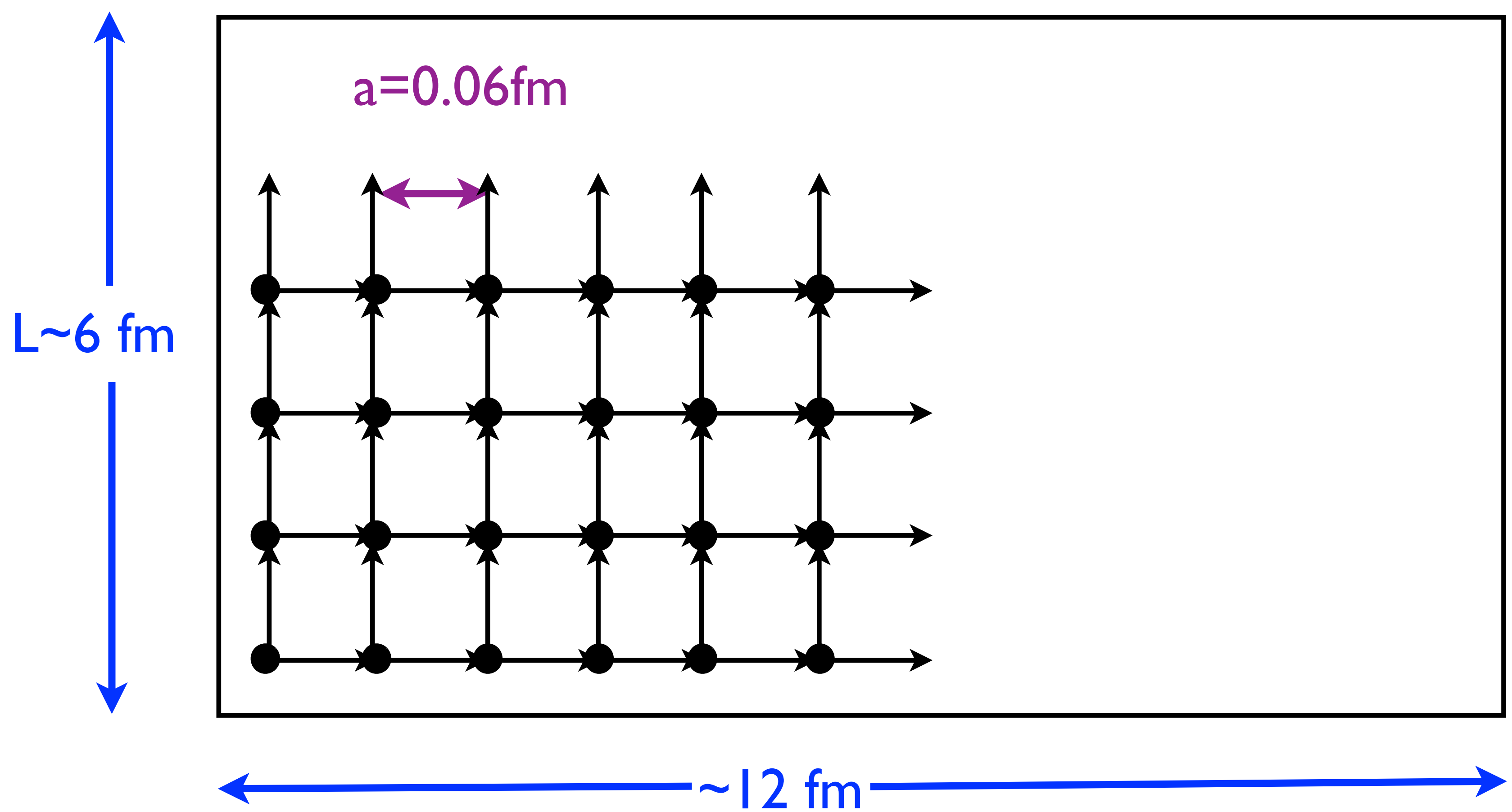
LQCD calculations need help

- Cannot simulate directly with physical theory
  - But can adjust knobs to approach the desired theory

m_u, m_d

a

L



LQCD calculations need help

- Cannot simulate directly with physical theory
  - But can adjust knobs to approach the desired theory

m_u, m_d

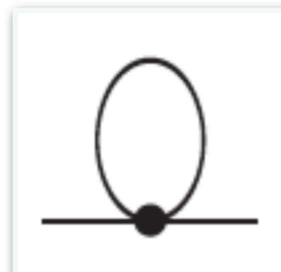
a

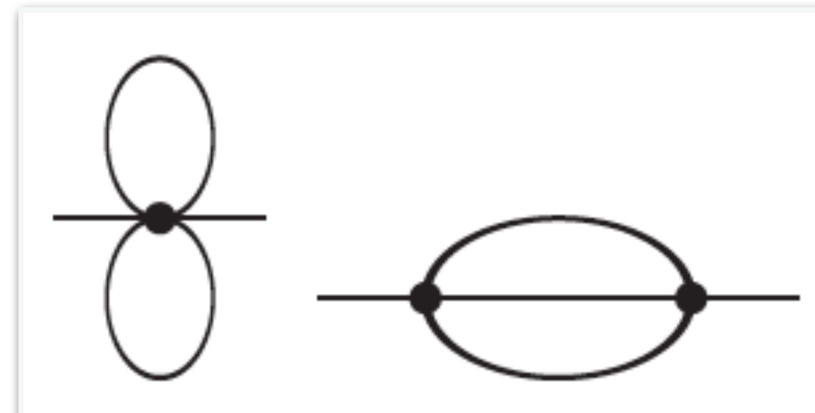
L

- Need ChPT to determine how to extrapolate
- ChPT systematically incorporates long-distance physics
  - PGBs dominate, and loops lead to non-analytic dependence on m_q and to leading dependence on L [$\exp(-M_\pi L)$]
  - Discretization errors break continuum symmetries, distort the vacuum, and alter the PGB spectrum (and thus impact long-distance physics)

A simple example: m_π vs m_q

- Continuum SU(2) ChPT at NNLO for $m_u=m_d=m_q$





$$\frac{M_\pi^2}{m_q} = 2B\left[1 + x\ln(M/\Lambda_3) + \frac{17}{2}x^2\ln^2(M/\Lambda_M) + x^2 k_M + \mathcal{O}(x^3)\right]$$

$$x = \frac{2Bm_q}{4\pi F^2} \qquad M = 2Bm_q \qquad \bar{\ell}_3 = \ln(\Lambda_3^2/M^2)$$

[Colangelo et al., 2001]

- Coefficients of logs are known, while analytic terms involve (*a priori* unknown) LECs

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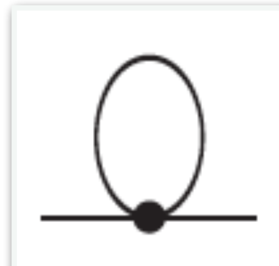

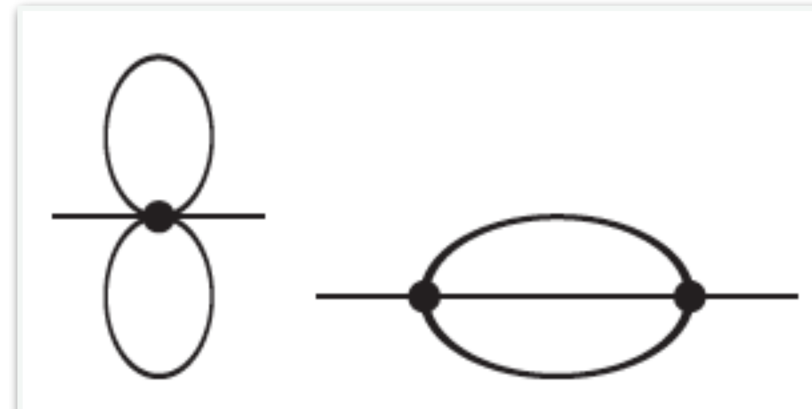

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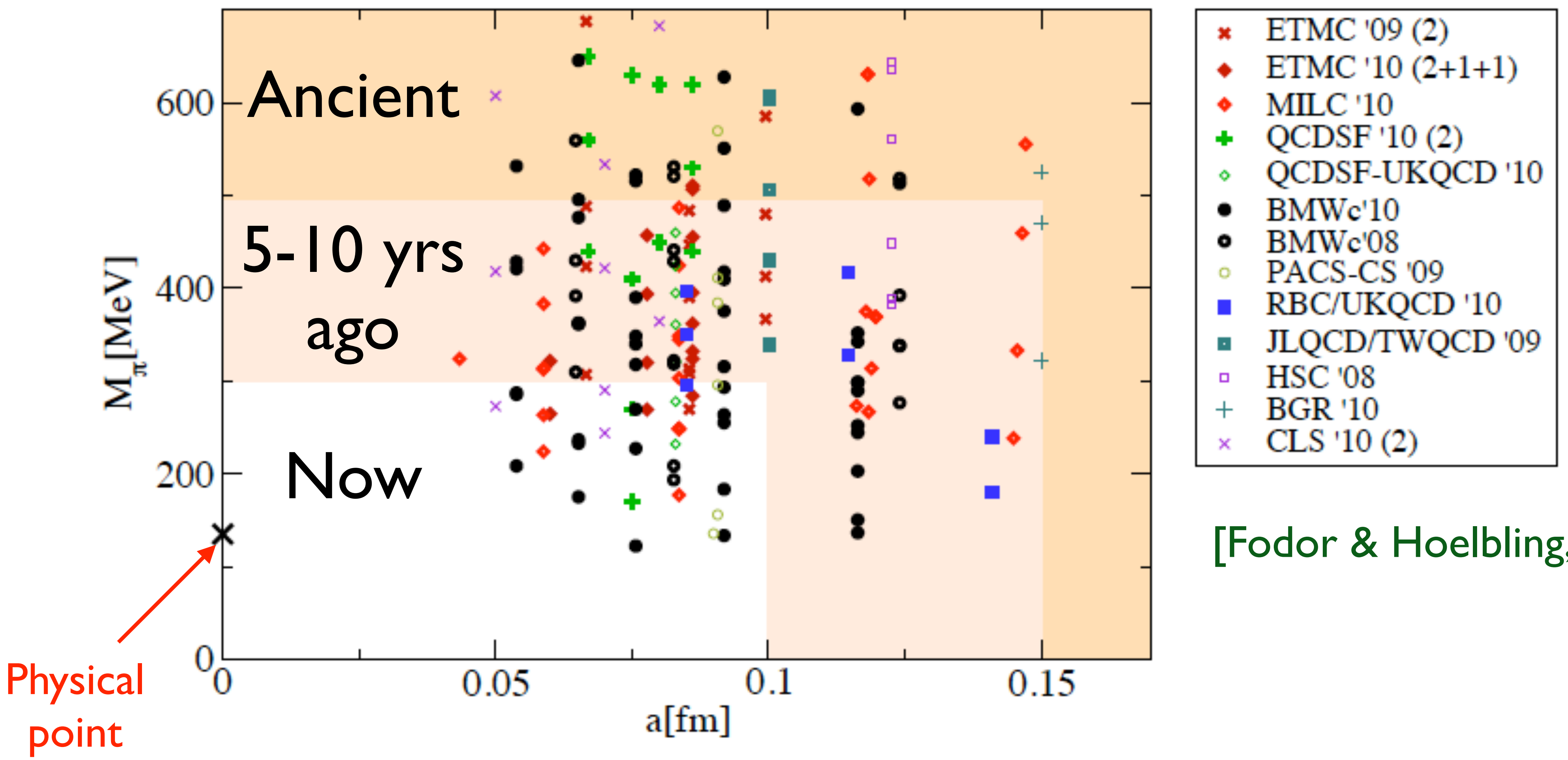
- Replacing loop integrals with finite-volume sums gives leading L dependence
- Including flavor/taste breaking in loops gives non-analytic dependence on a

LQCD calculations need help

- Historically needed to extrapolate in $m_u = m_d = m_q$



[Fodor & Hoelbling, RMP12]

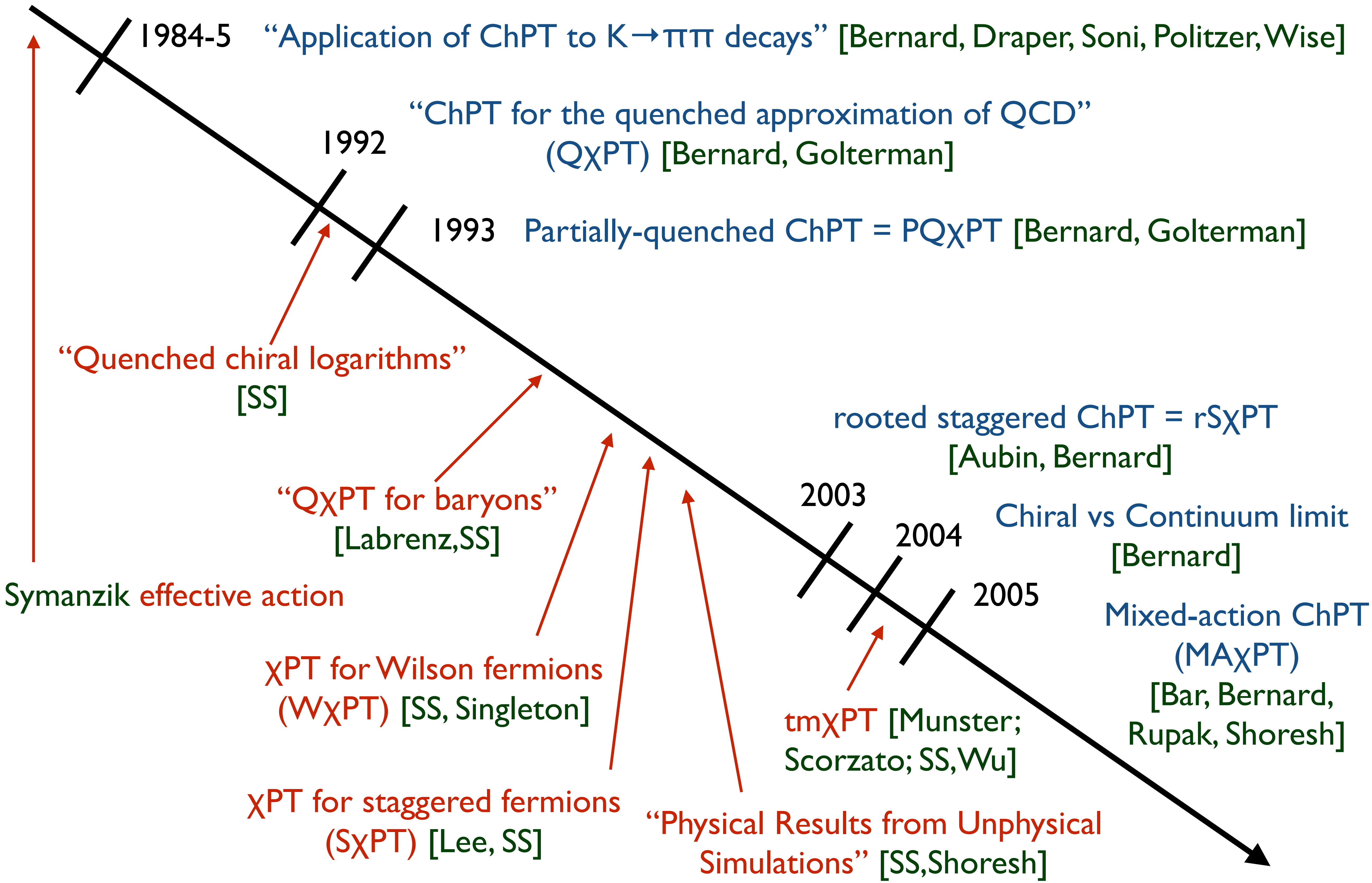
Use of partial quenching

- Valence & sea masses can be tuned independently
  - Cheaper to lower valence masses; improves chiral extrapolation

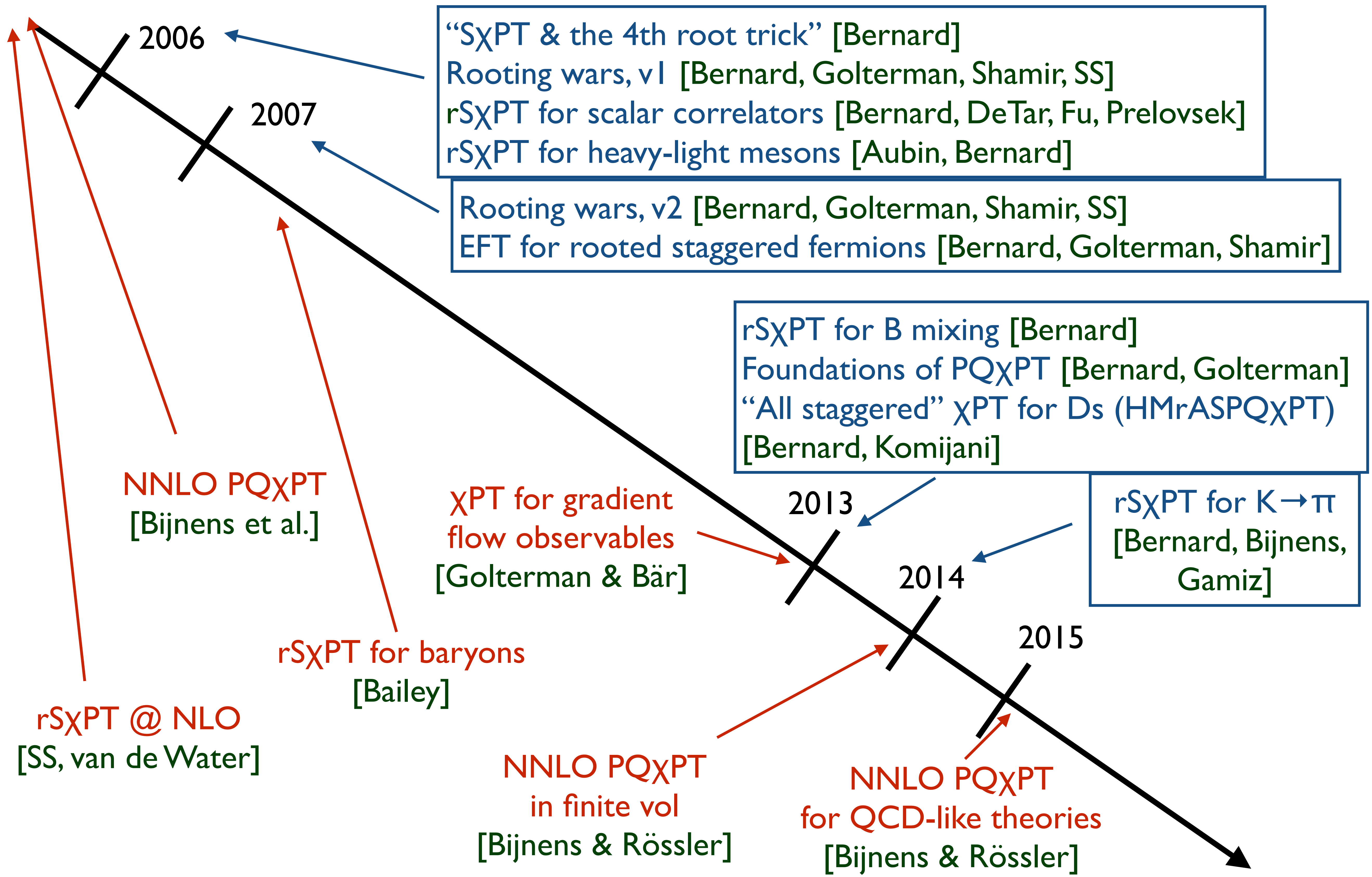
$m_{q,sea}$ $m_{u,val}$ $m_{d,val}$ a L

- Need PQChPT to determine how to extrapolate
  - Introduces few additional LECs (so PQing can be powerful)
- ChPT can also account for other approximations
  - Rooting (staggered fermions), mixed actions, twisted BC, Wilson-flow
  - Wilson, twisted-mass, staggered discretization effects

(Partial) timeline of ChPT for LQCD



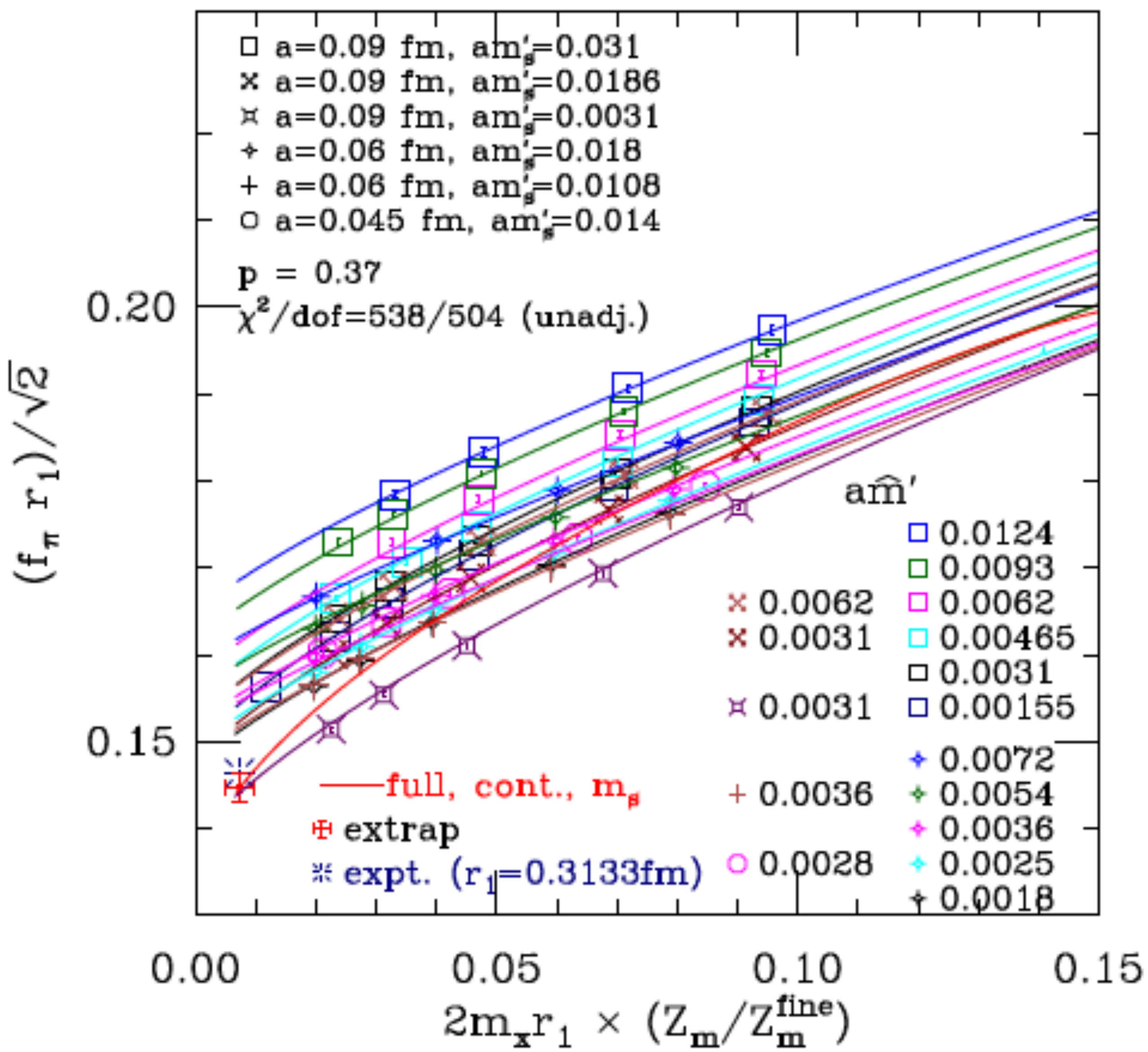
(Partial) timeline of ChPT for LQCD



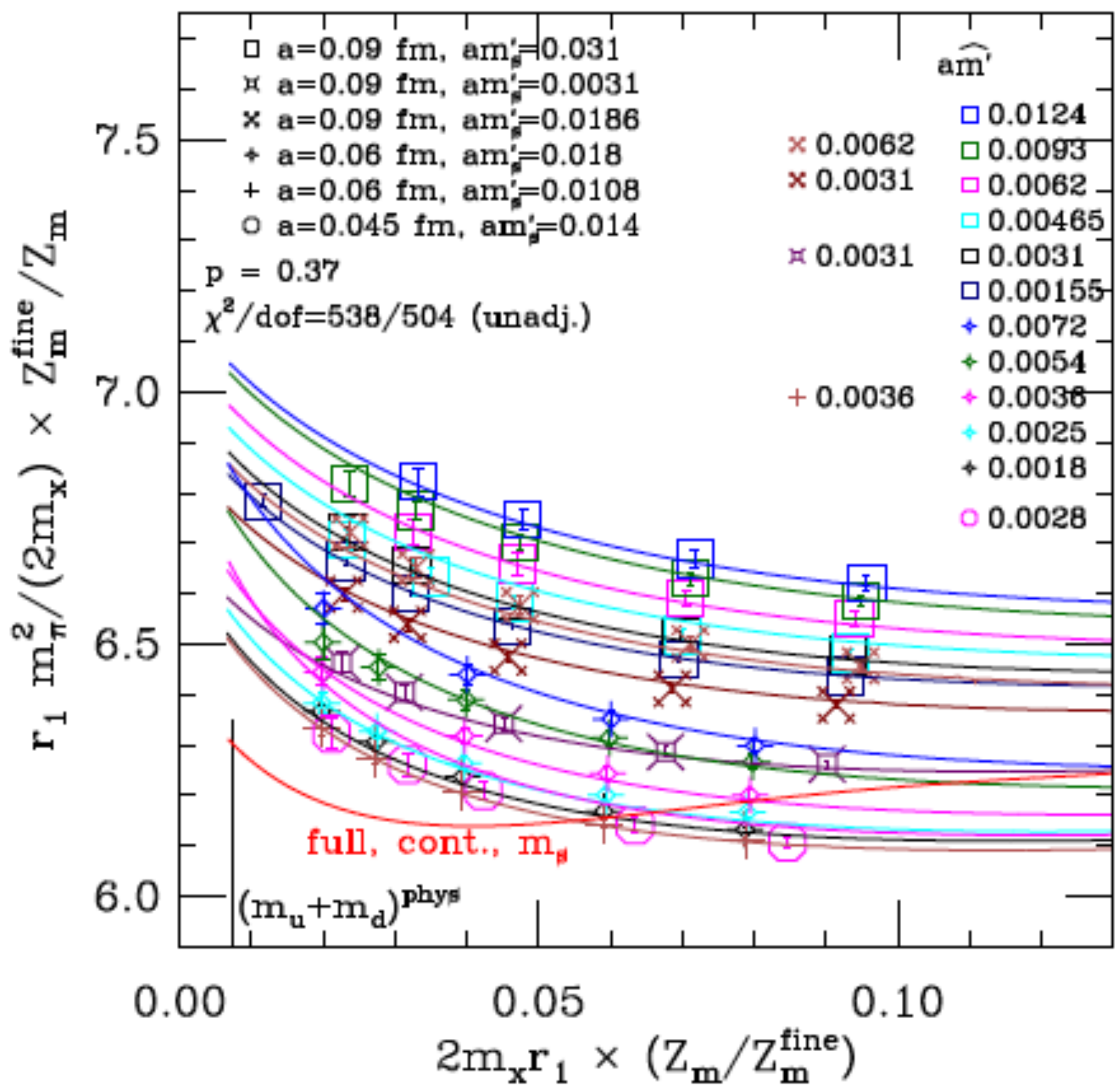
Success of r(ooted)S(taggered)PQχPT

[Bazavov et al., 1012.0868] HISQ fermions

f_π vs m_q



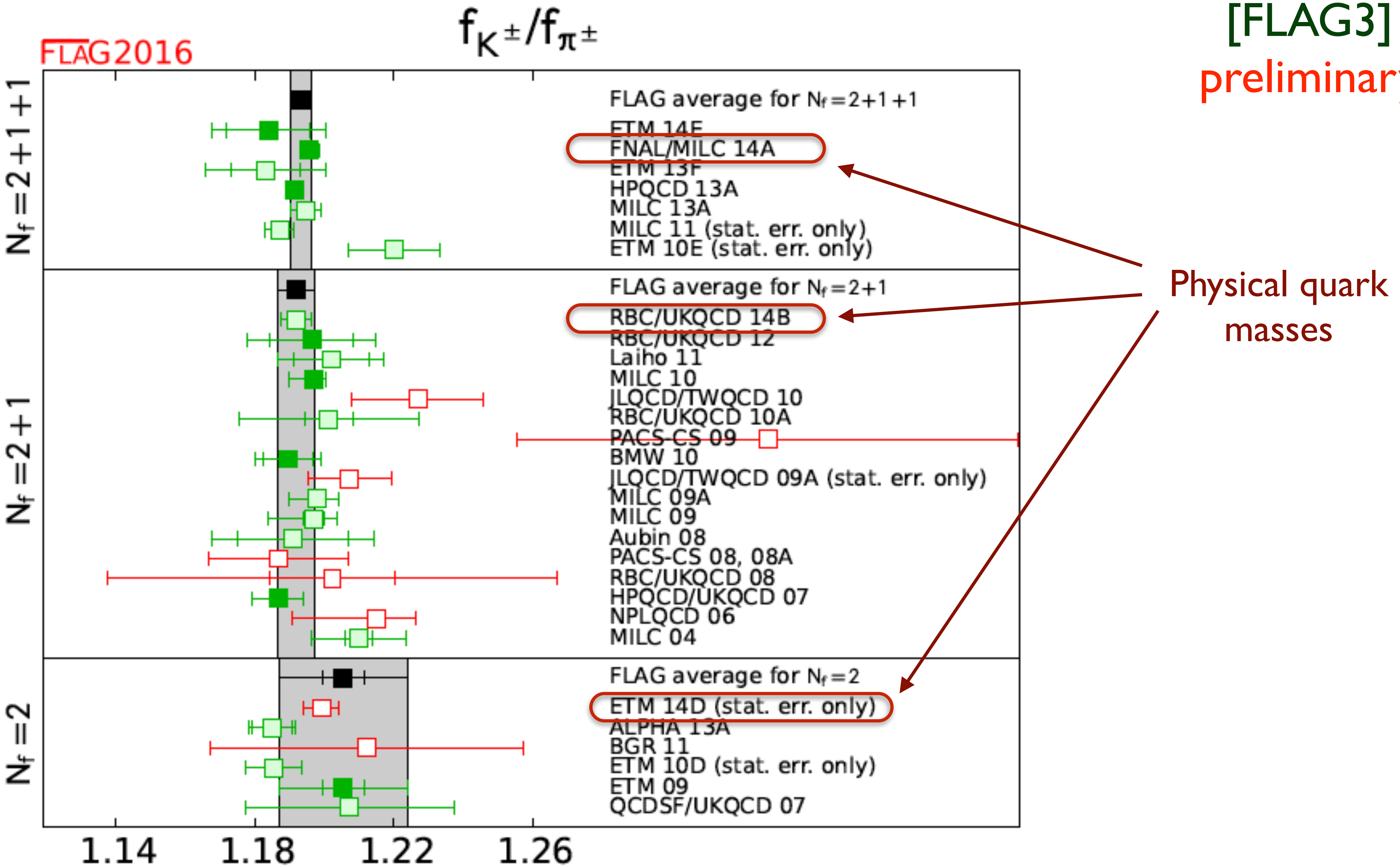
M_π^2/m_q vs m_q



Uses SU(3) rSPQChPT

Summary of present status

[FLAG3]
preliminary

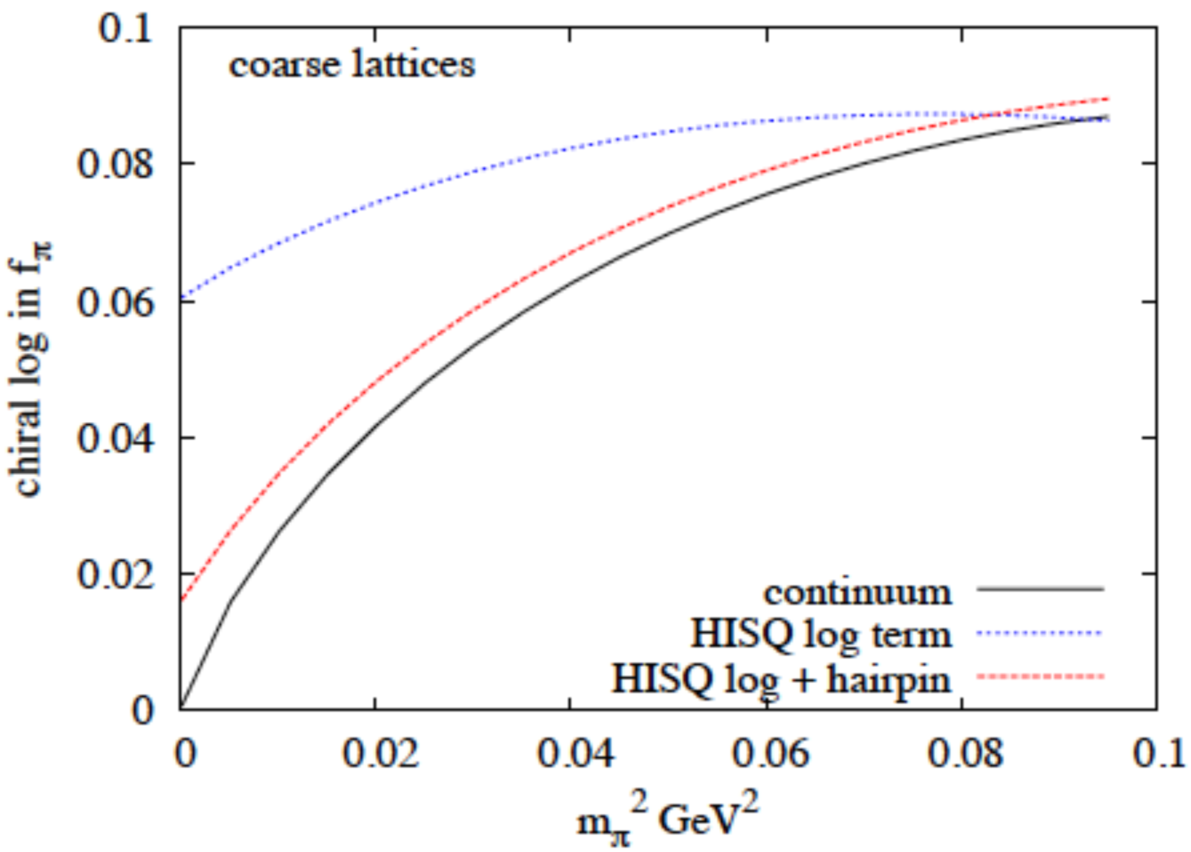


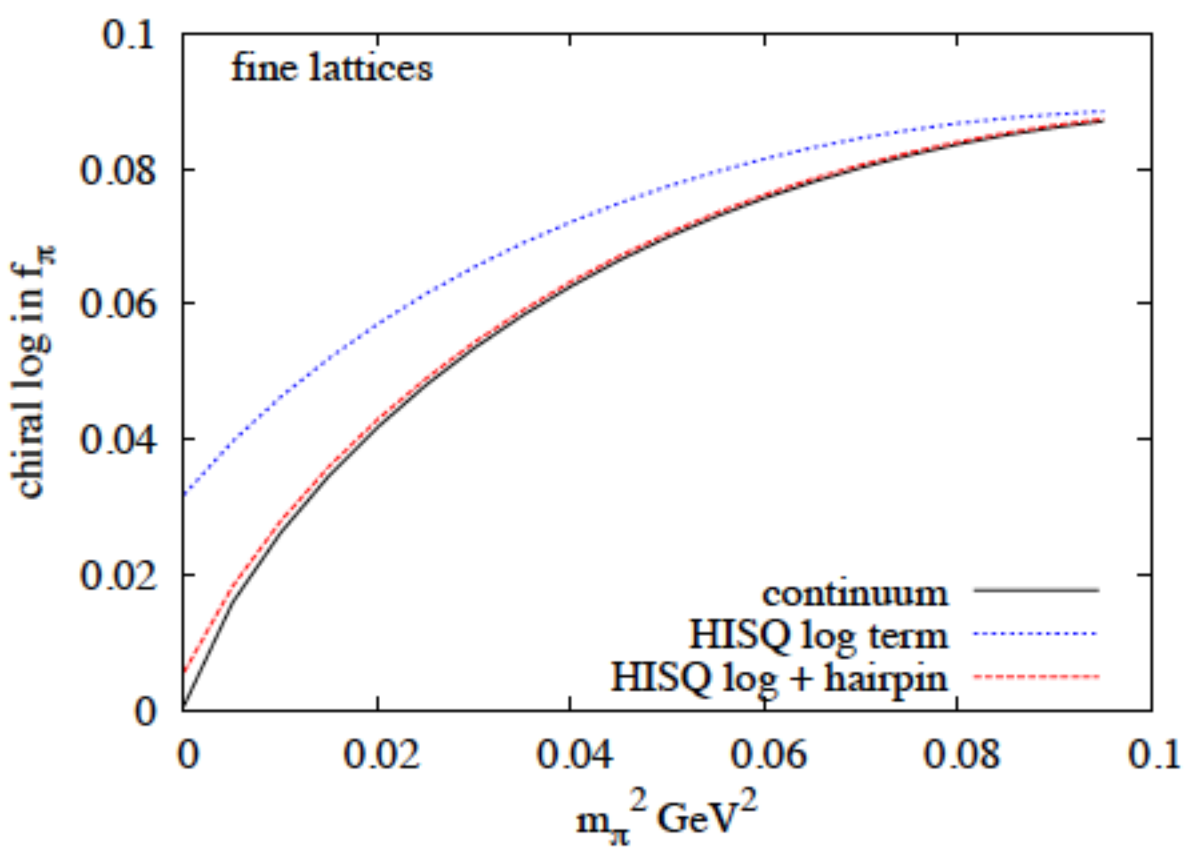
Almost all results rely on ChPT

Summary of history

- ChPT has played a crucial role in extrapolations
  - Particularly SU(2) ChPT: expansion in $(m_\pi/4\pi f_\pi)^2$
  - Convergence of SU(3) ChPT fails close to physical m_s
  - Including discretization errors particularly important for staggered fermions*
- Consistency with chiral logs gave confidence in LQCD
- Hopes of simplifying calculation of $K\to\pi\pi$ weak decay amplitudes did not pan out
  - ChPT relates to simpler $K\to\pi$ and $K\to 0$ amplitudes [Bernard et al. 1984, Laiho & Soni 2002/2005]
  - SU(3) ChPT simply not accurate enough, even at NNLO

Efficacy of HISQ fermions





[HPQCD 1510.07446]

- $a^2 \ln(a)$ terms from SChPT cancel to good numerical accuracy for HISQ fermions!
    - Continuum ChPT works almost as well for f_π, f_K, m_π, f_D and $B \to \pi$
    - Normal logs and logs from hairpin vertices cancel

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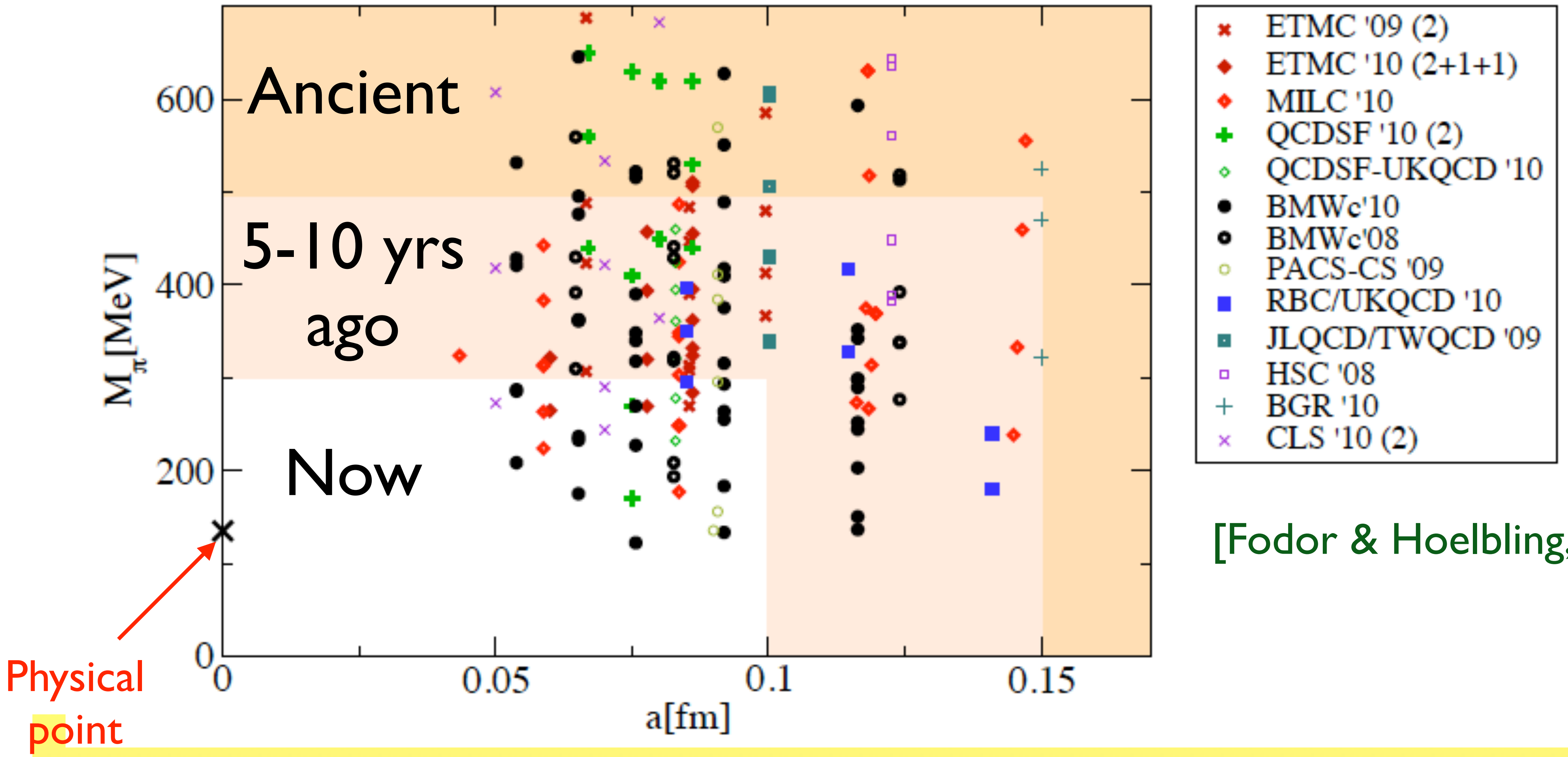
Era of physical quark masses



[Fodor & Hoelbling, RMP12]

- No longer need to extrapolate in quark masses
- Combined with use of improved actions, simple analytic expansions in a^2 sufficient

Era of physical quark masses

m_u, m_d

a

ChPT not needed

L

ChPT still useful

Is this the situation?

Era of physical quark masses

m_u, m_d

a

ChPT not needed

L

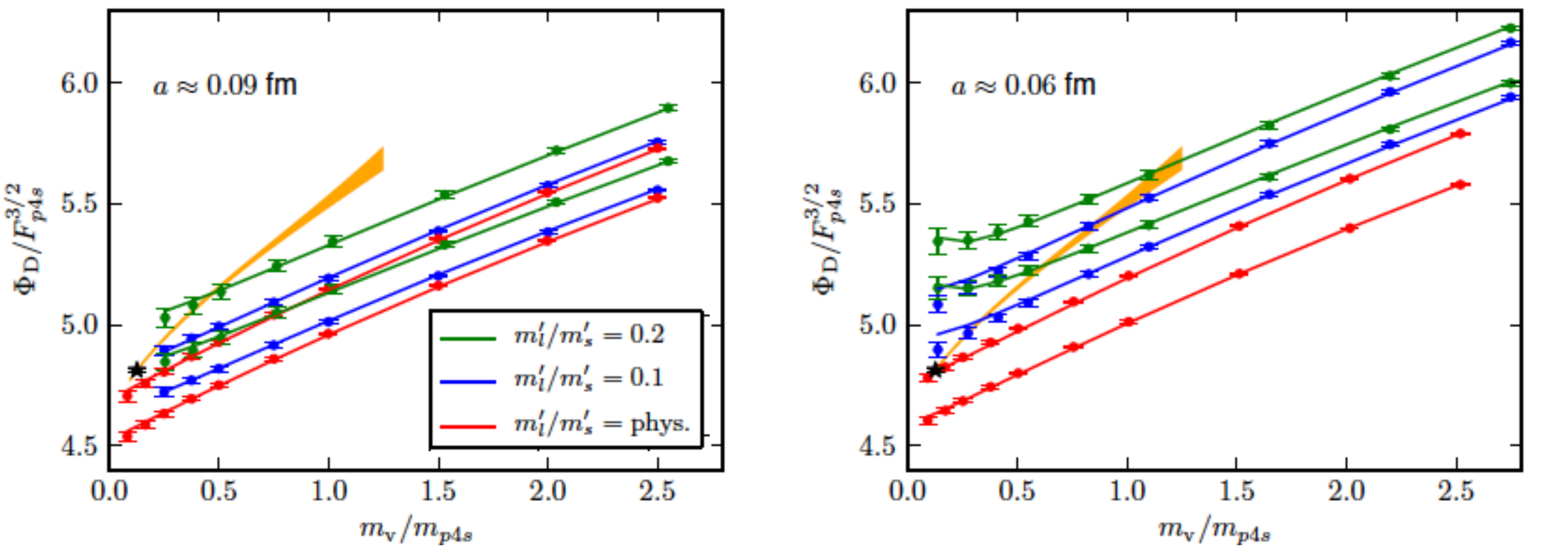
ChPT still useful

- We are headed in this direction, but not there yet
  - Many calculations not yet done at physical masses (e.g. baryon properties)
  - Errors at physical masses are larger, so combining with higher masses improves errors
  - Need to interpolate to physical quark masses

Combining physical & heavier m_q

$f_D\sqrt{M_D}$ vs m_q (HISQ fermions)

[Bazavov et al., 1407.3772]



- Use <u>either</u> physical mass ensembles only <u>or</u> full PQ analysis (using HMrASPQχPT !)
    - Latter has smaller statistical and continuum extrapolation errors

Other ongoing uses of ChPT for LQCD

- Extrapolating results for nuclei (pionfull EFT)
- Providing expressions for small volume (ε & δ) regimes & for simulations at fixed topological charge
  - Alternative methods for obtaining LECs
- Determining possible unphysical phases
  - So as to know how to avoid them (for Wilson-like & staggered fermions)
- Estimating systematic errors
  - FV effects in hadronic vac. pol. for g_μ-2 [Aubin et al. 2015]
- Providing checks of LQCD results & methods
  - ππ phase shifts at threshold, low-energy theorems for proton decay amp, …

Phase structure when $m_u \neq m_d$

[Horkel & SS, 1409.2548, 1505.02218, 1507.03653]

- Present frontier: simulations including isospin breaking
  - Aim for physical values: m_u~2.4 MeV, m_d~5.0 MeV and α_{EM}=1/137

m_u m_d a L α_{EM} (θ_{QCD})

- Discretization effects more important as m decreases
  - m_u becomes comparable to $a^2\Lambda^3 \approx$ 3MeV ($1/a \approx 3$GeV, $\Lambda \approx 0.3$GeV)
  - Particularly relevant for Wilson-like fermions where unphysical phases exist

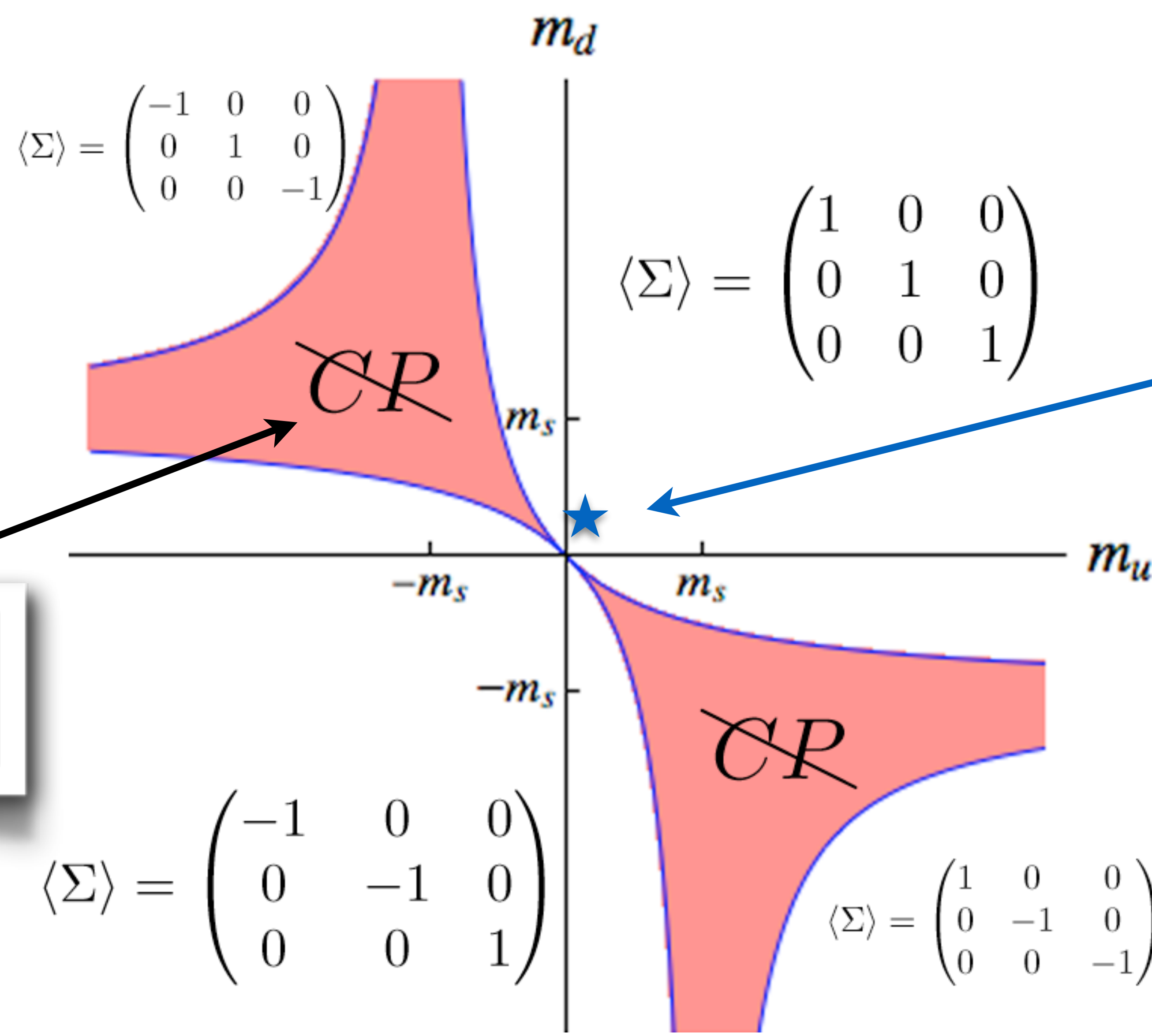
Unphysical phase also in continuum

CP-violating phase [Dashen, 1971]

[Creutz, 2004]

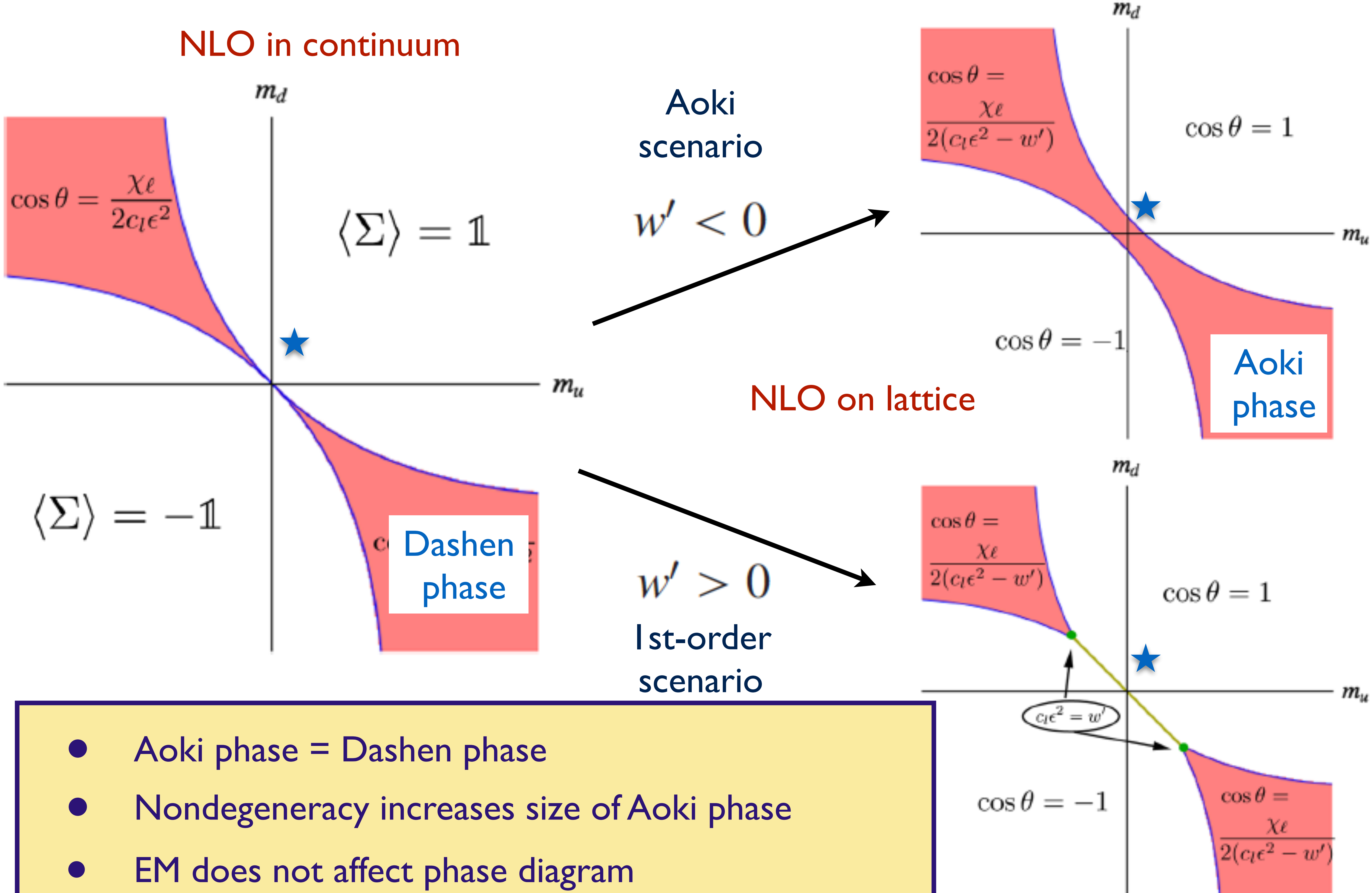
Prediction from leading-order SU(3) ChPT

$$\langle\Sigma\rangle = \begin{pmatrix} \exp i\phi & 0 & 0 \\ 0 & \exp i\psi & 0 \\ 0 & 0 & \exp -i(\phi+\psi) \end{pmatrix}$$



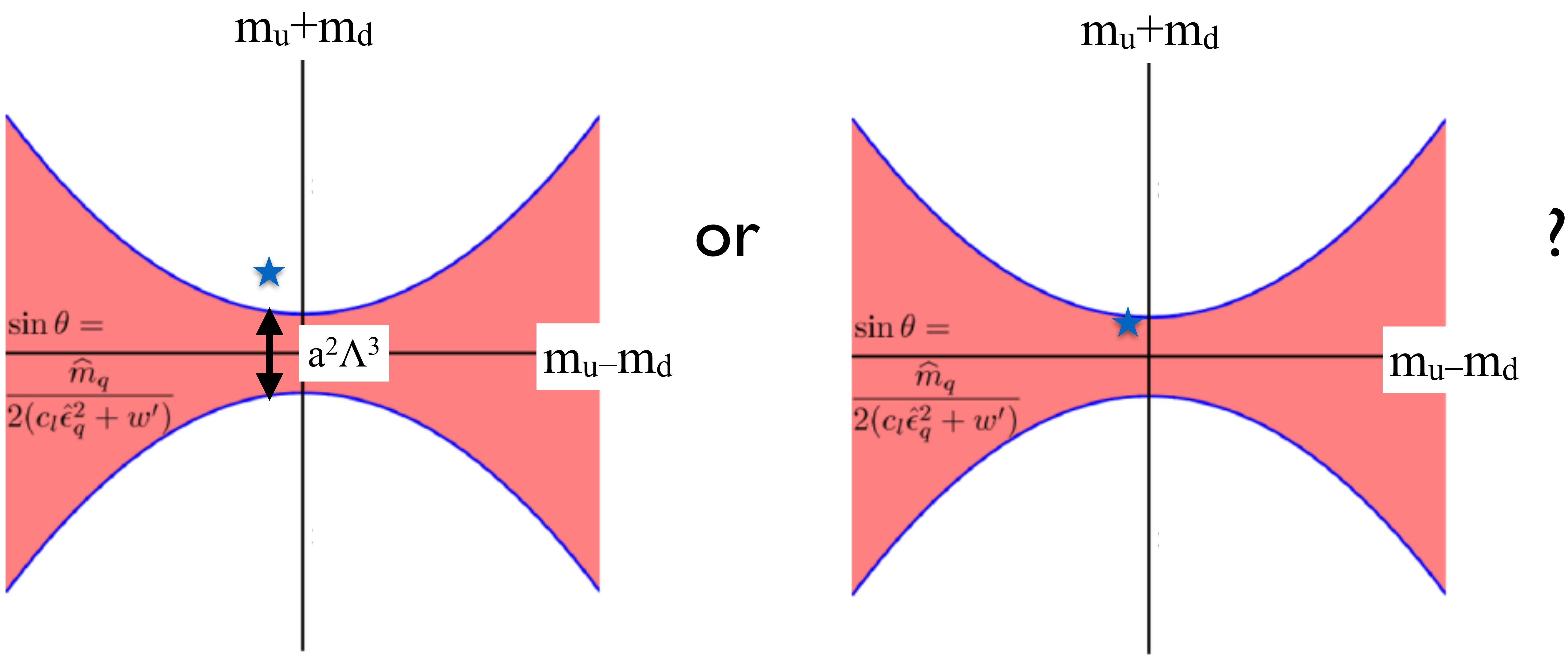
physical point

WχPT: SU(2) with $m_u \neq m_d$ & $\alpha_{EM} \neq 0$



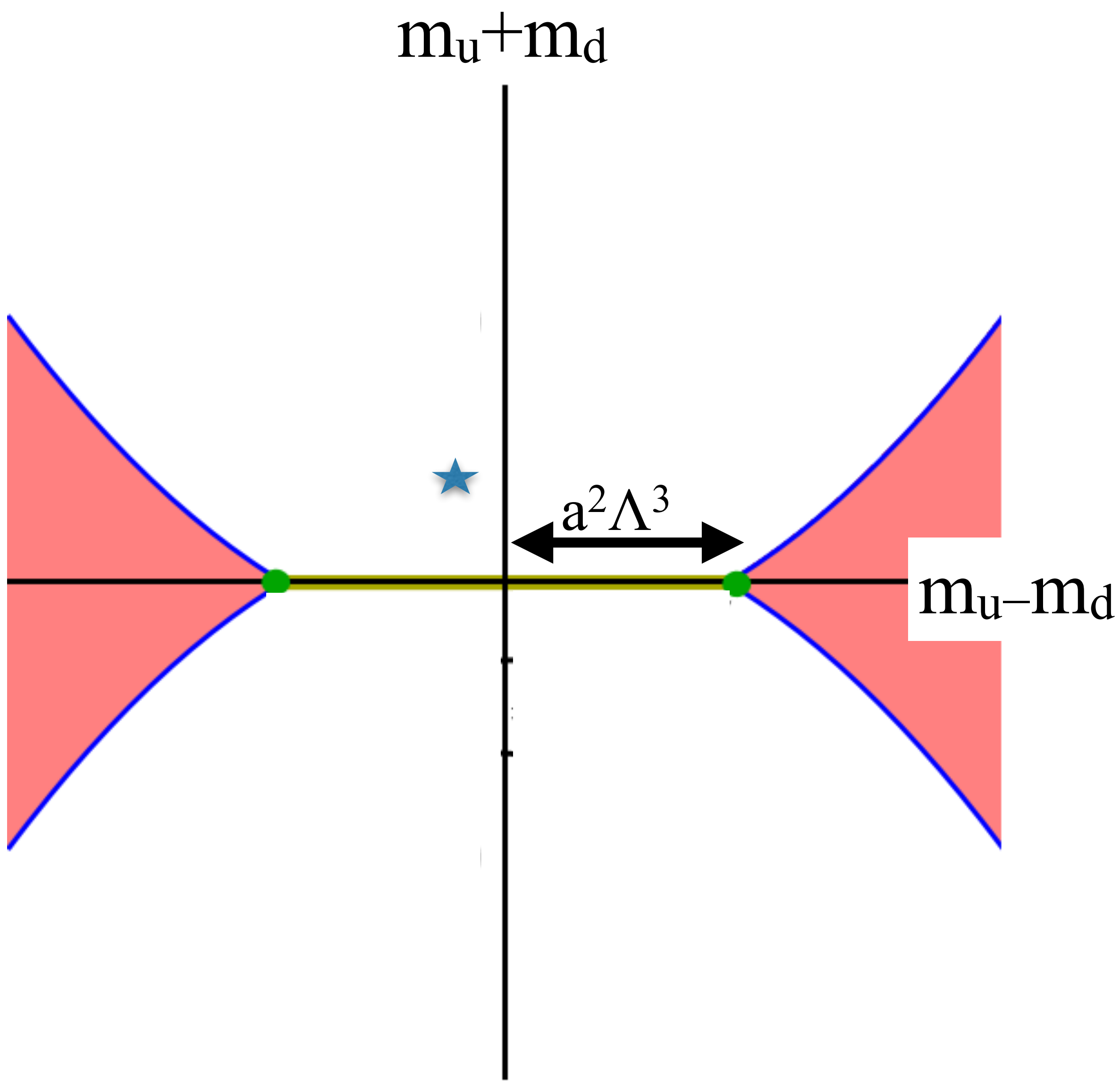
- Aoki phase = Dashen phase
- Nondegeneracy increases size of Aoki phase
- EM does not affect phase diagram

Issue for simulations

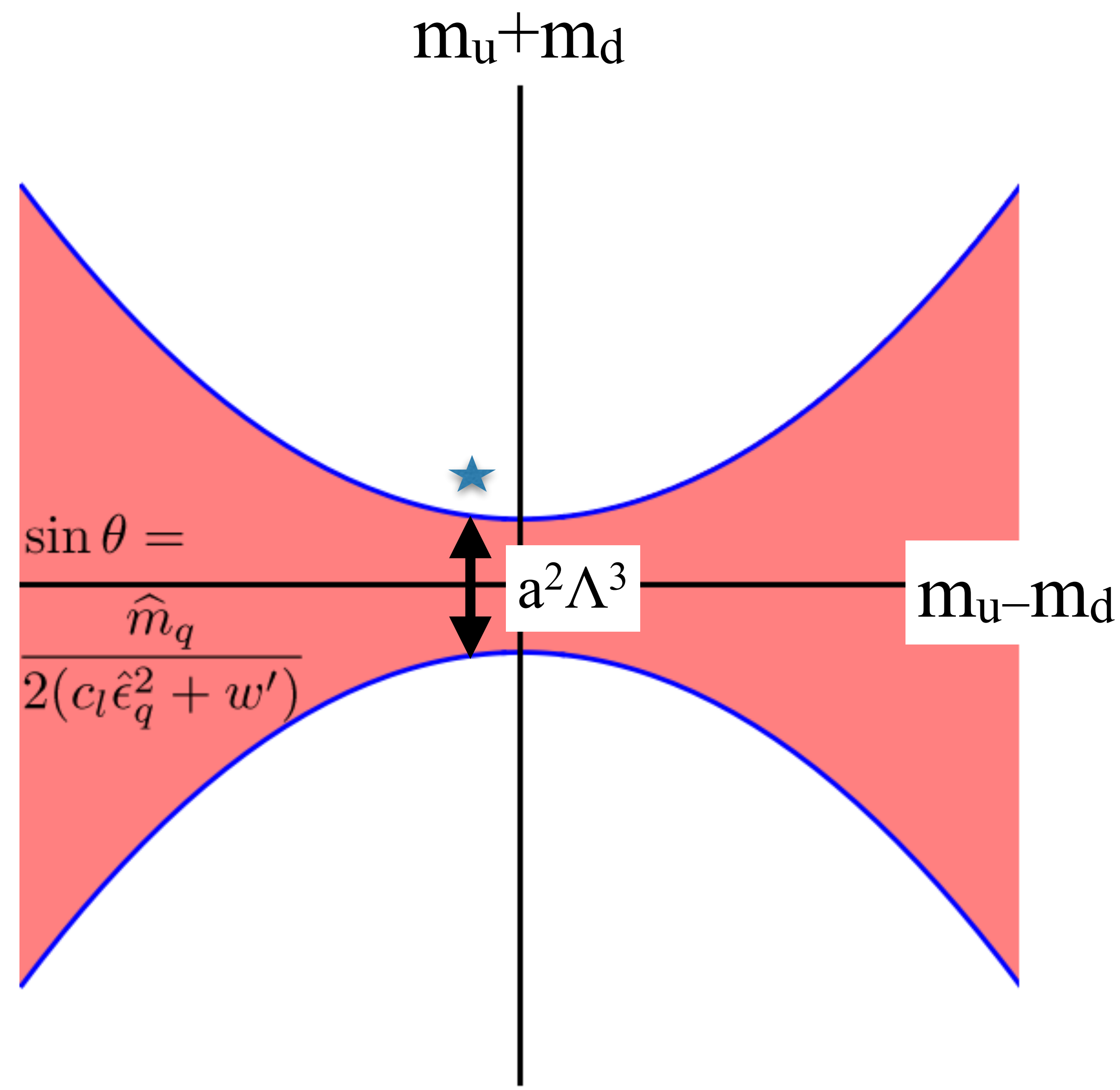


Infact, simulations appear to be outside unphysical phase

tmχPT at max. twist: $m_u \neq m_d$ & $\alpha_{EM} \neq 0$



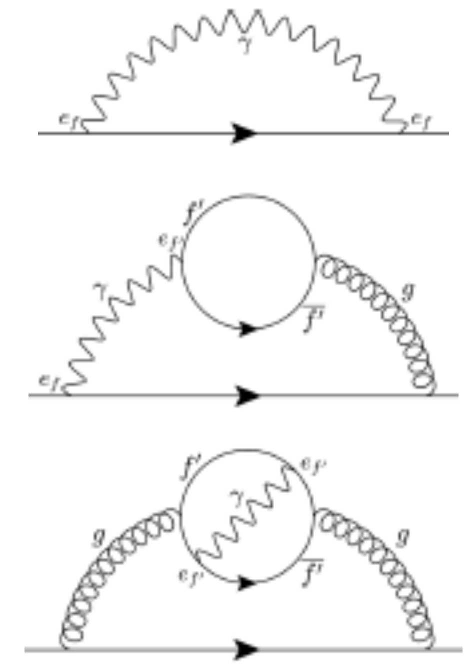
Aoki Scenario (**w'<0**)



First-Order Scenario (**w'>0**)

- Roles of two scenarios interchanged
- Again, simulations appear to lie outside unphysical phase

Tuning to max twist with $\alpha_{EM} \neq 0$



- Up & down critical masses differ by $O(\alpha_{EM}/a)$
- "$m_{PCAC}=0$" method of tuning fails
- RM123 collab. use PQ variant of $m_{PCAC}=0$
- Untuned theory has $\theta_{QCD} \neq 0$
- To study tuning, need PQtmχPT for $m_u \neq m_d$ & $\theta_{QCD} \neq 0$!
  - We find that PQ $m_{PCAC}=0$ method fails (only tune one linear combination)
  - We propose an alternative method (for the distant future when such simulations are possible!)
  - RM123 avoid our criticism since they use expand perturbatively about the isospin-symmetric theory and use the electroquenched approximation

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How can LQCD help (continuum) ChPT?

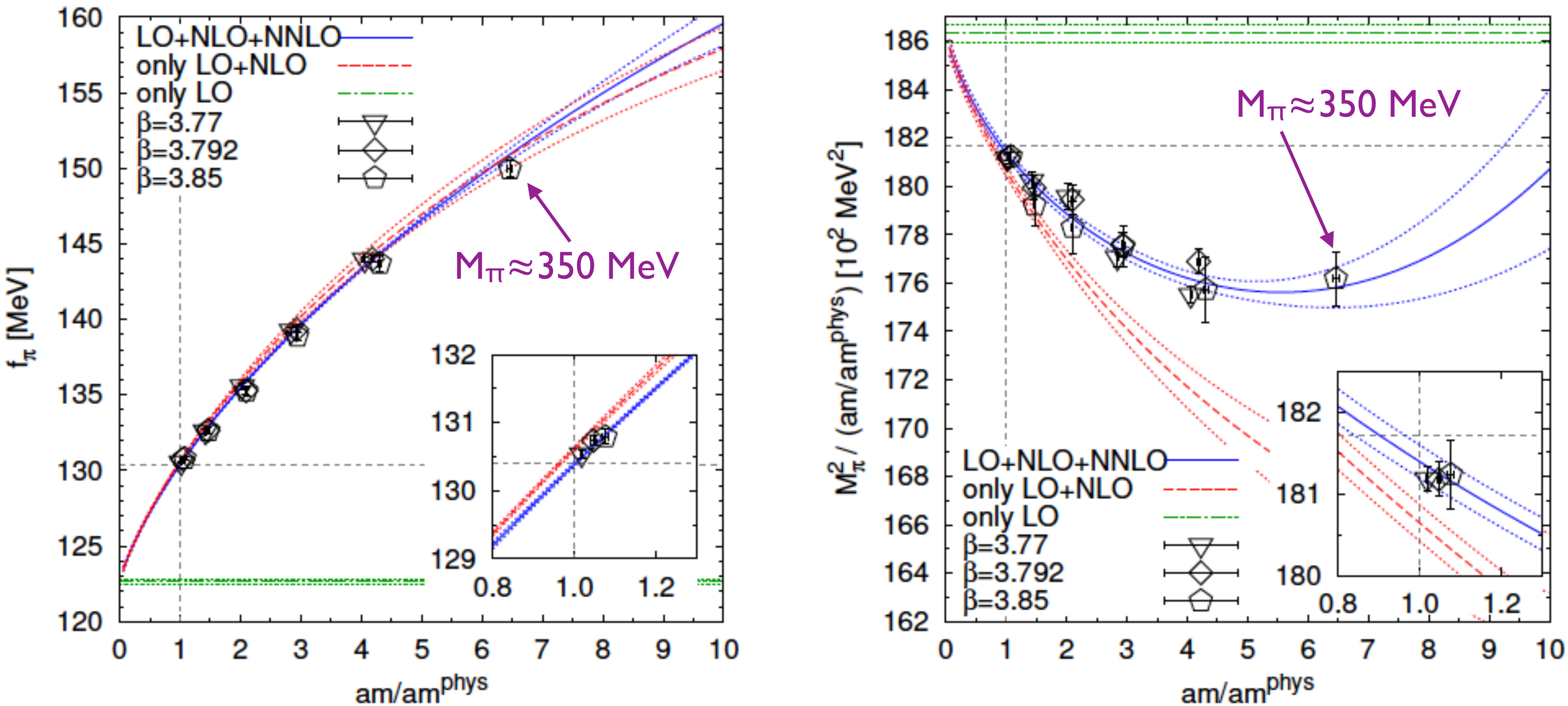
- Providing LECs
  - Both for SU(2) ChPT (with present simulations) and for SU(3) ChPT (with dedicated simulations having $m_s < m_s^{phys}$)
  - Particularly needed for those describing quark mass dependence
- Studies of convergence (since can turn dials)
- Checking continuum approximation methods
  - e.g. for $\pi\pi$ phase shifts, nucleon σ-term, eventually for $\eta \to \pi\pi\pi$
- What else?

Studying convergence

- Careful studies with staggered & Wilson fermions
  [BMWc 1205.0788, 1310.3626, Dürr 1412.6434, Bernard 1510.02180]

Studying convergence

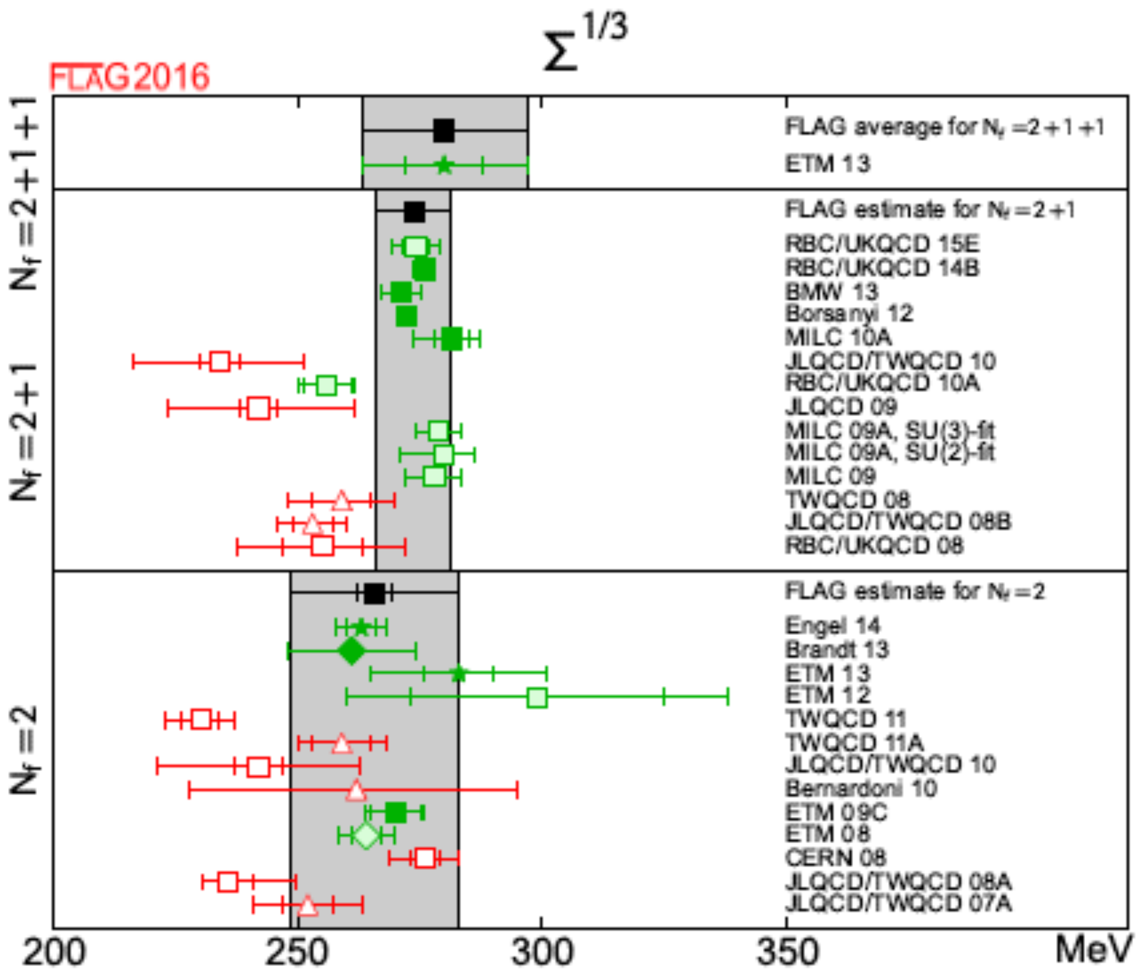
staggered quarks [BMWc 1205.0788]



- SU(2) χPT converges for $M_\pi \lesssim 350$ MeV
- Chiral logs strongly favored over polynomial fits
- If $M_{\pi,min} > M_{\pi,phys}$, NLO χPT fits can work but mislead

Providing LECs

[FLAG3] Preliminary

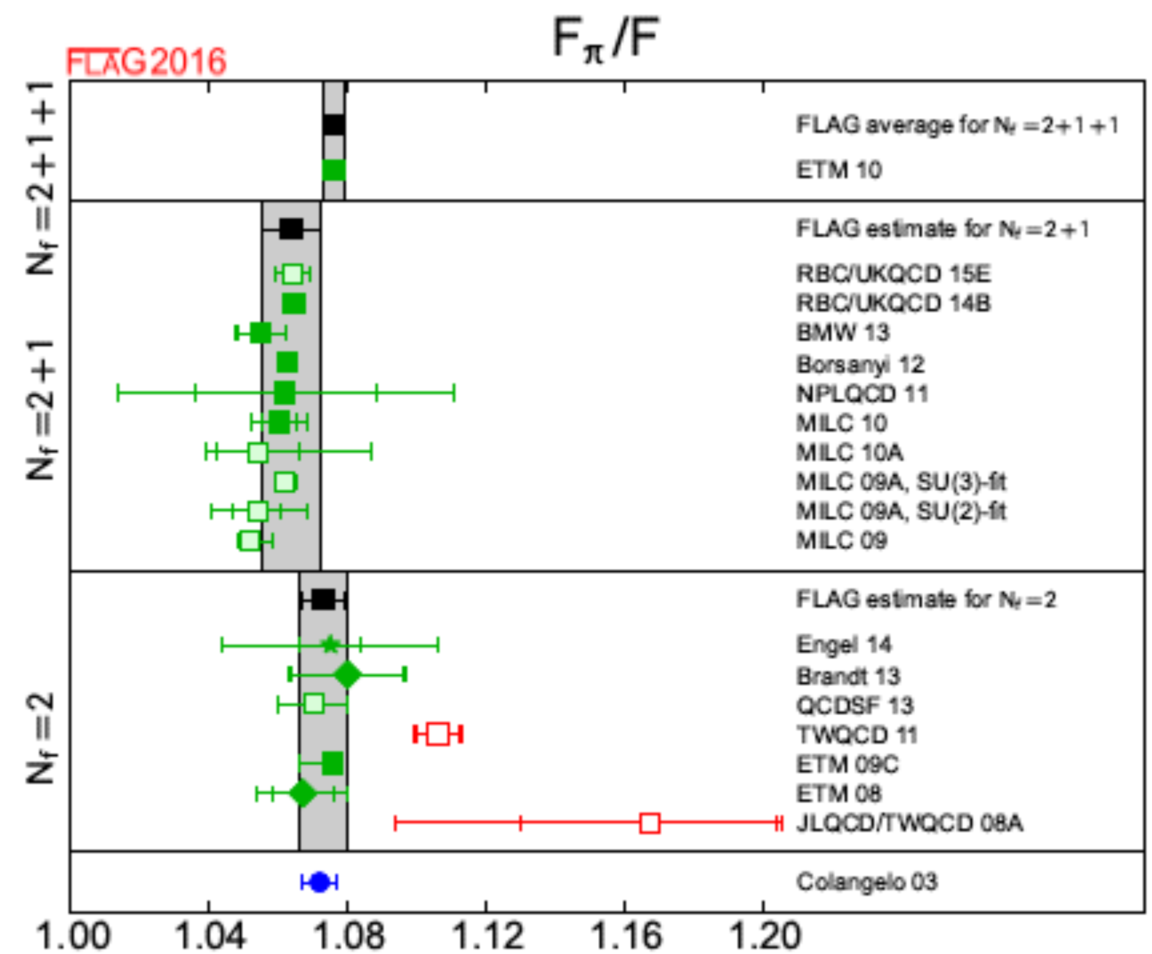


FLAG3 estimate

$$N_f = 2+1: \qquad \Sigma^{1/3} = 274(8)\,\text{MeV}$$

FLAG3 estimate

$$N_f = 2+1: \qquad \frac{F_\pi}{F} = 1.0637(87)$$



Providing LECs

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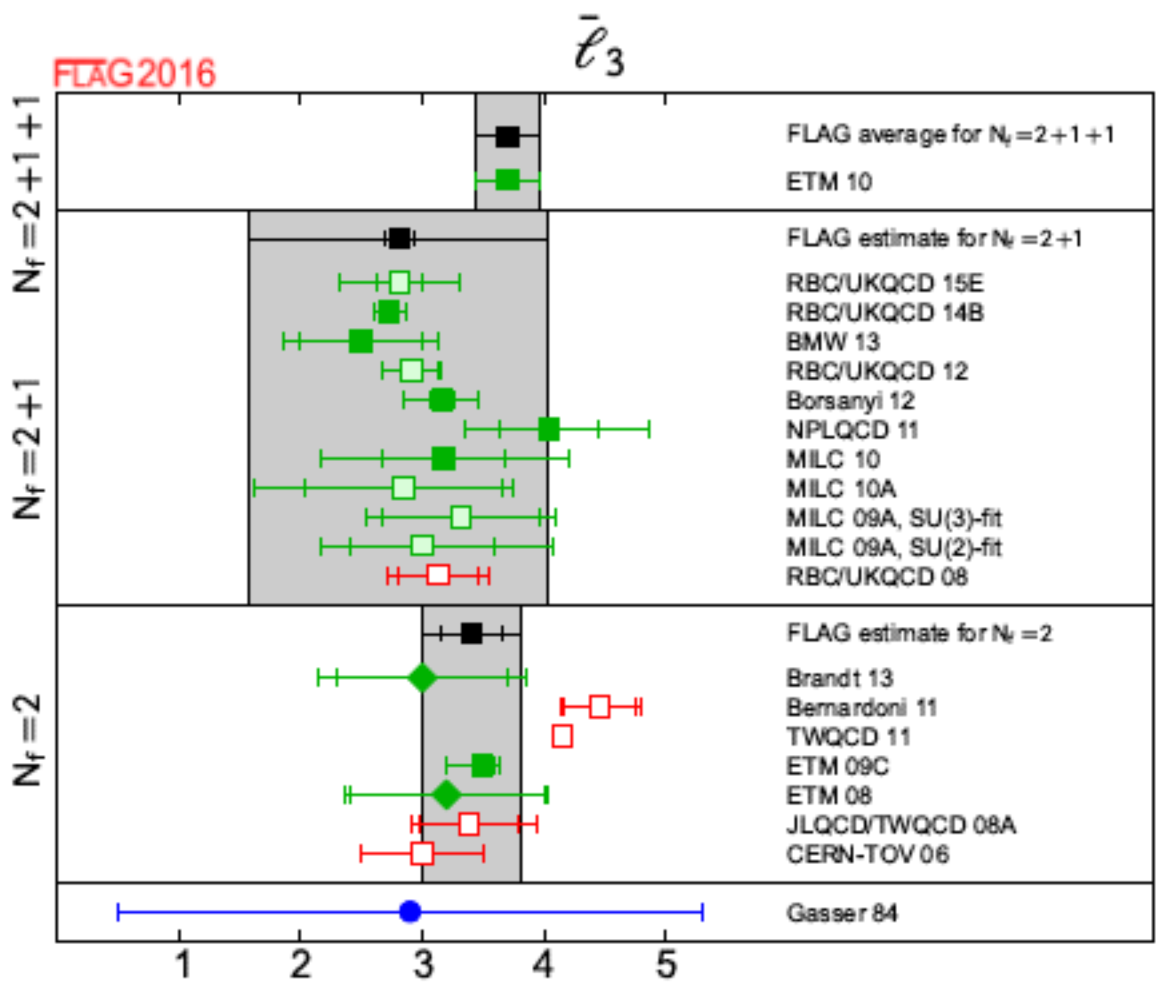
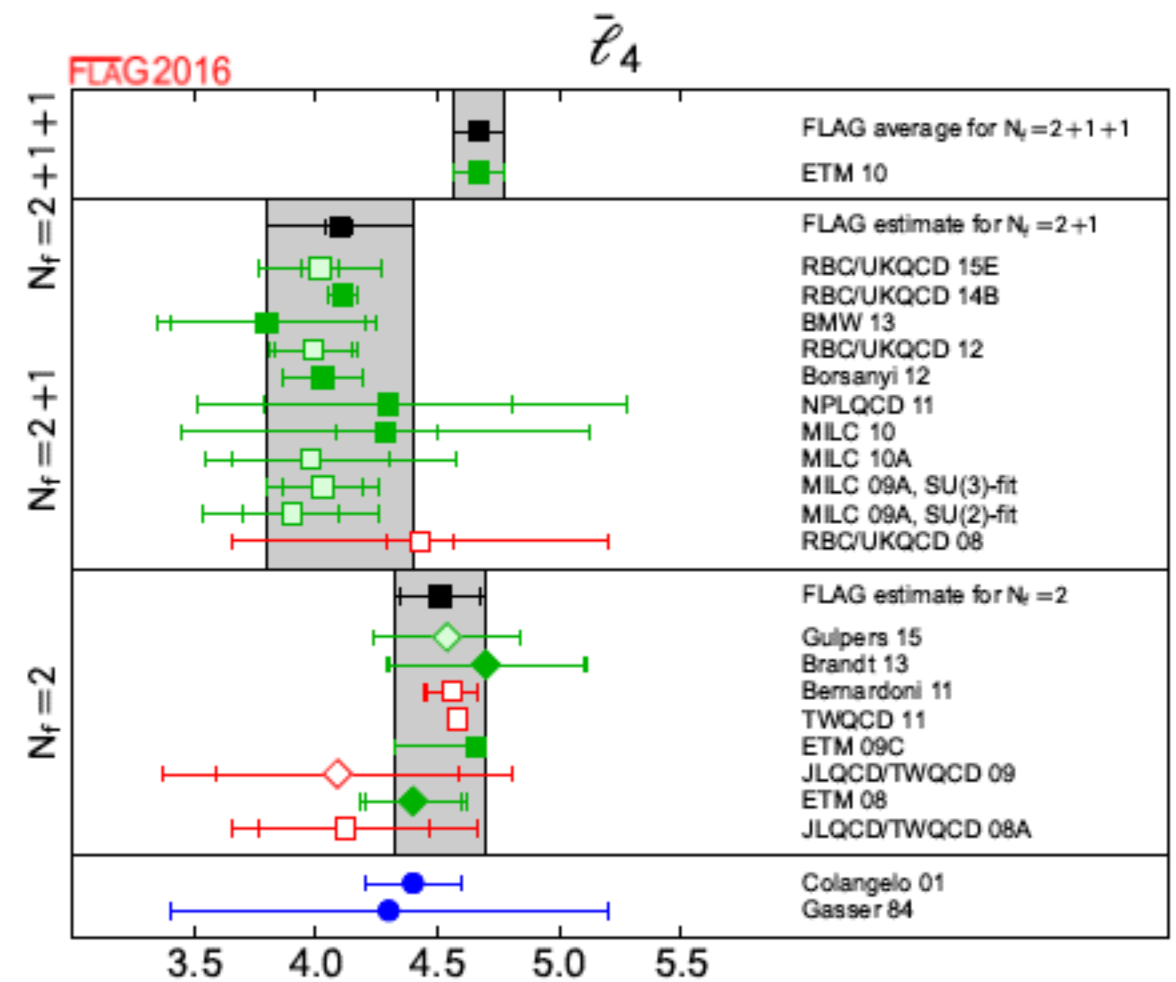


FLAG3 estimate

$$N_f = 2+1: \qquad \bar{\ell}_3 = 2.81(1.23)$$

FLAG3 estimate

$$N_f = 2+1: \qquad \bar{\ell}_4 = 4.10(30)$$



Checking continuum (or maybe checking lattice?)

[Leutwyler, 1510.07511]

- $\pi\pi$ scattering amplitudes
  - ChPT + general properties of amplitudes + dispersion relations give precise description up to ~1GeV
  - E.g., at $s=M_K^2$, $\delta_0-\delta_2 = 47.7(1.5)^\circ$ [Colangelo et al, 2001]
  - Lattice result, $35.4(5.8)^0$ [RBC/UKQCD 1505.07863], differs by ~2σ

- Nucleon sigma term
  - Expt+ChPT+disp. rels. give: $\sigma_N = \frac{\hat{m}}{2M_N}\langle N(p)|\bar{u}u+\bar{d}d|N(p)\rangle = 59.1(3.5)\,\mathrm{MeV}$
  - Lattice result [BMWc 1510.08013] differs by ~4σ: $\sigma_N = 38(3)(3)\,\mathrm{MeV}$

Thank you!

Questions?