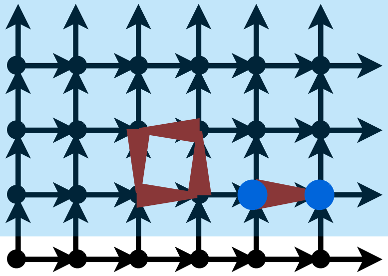
Lattice QCD: successes, challenges & future outlook

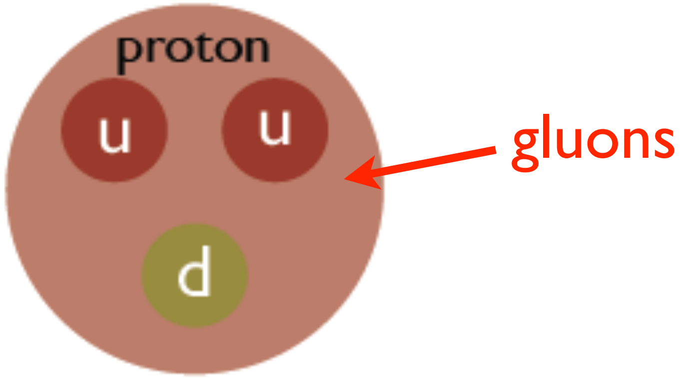
Stephen R. Sharpe
University of Washington

U. Adelaide colloquium, Feb. 15, 2019



Overview

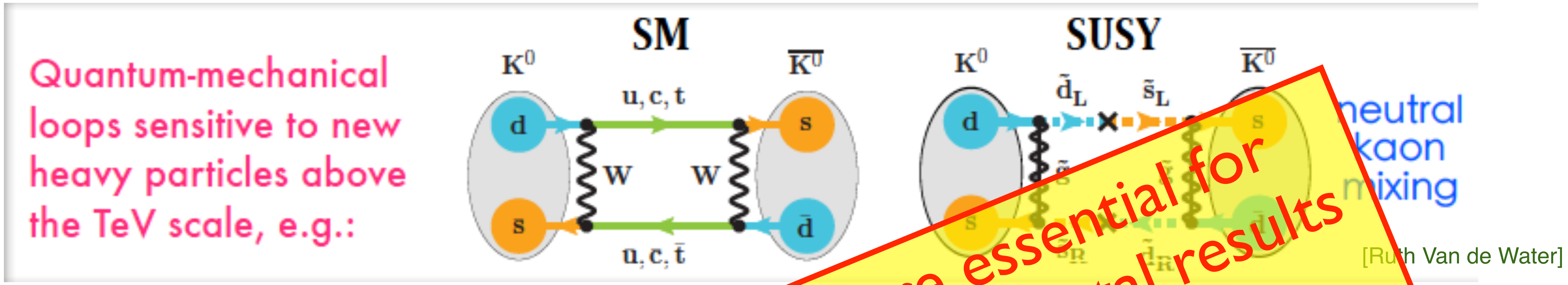
- Quantum ChromoDynamics is a peculiar theory
  - Quarks are absolutely confined
  - Quark properties are obscured



- QCD is a "background" in search for new physics
  - Must understand "old physics" of the standard model (SM) to find "new physics"
  - True both at "energy frontier" (LHC) and at "intensity frontier" (rare decays)
- QCD is strongly coupled, non-perturbative
- Lattice QCD is now a mature method that allows us to make precise predictions of properties of QCD

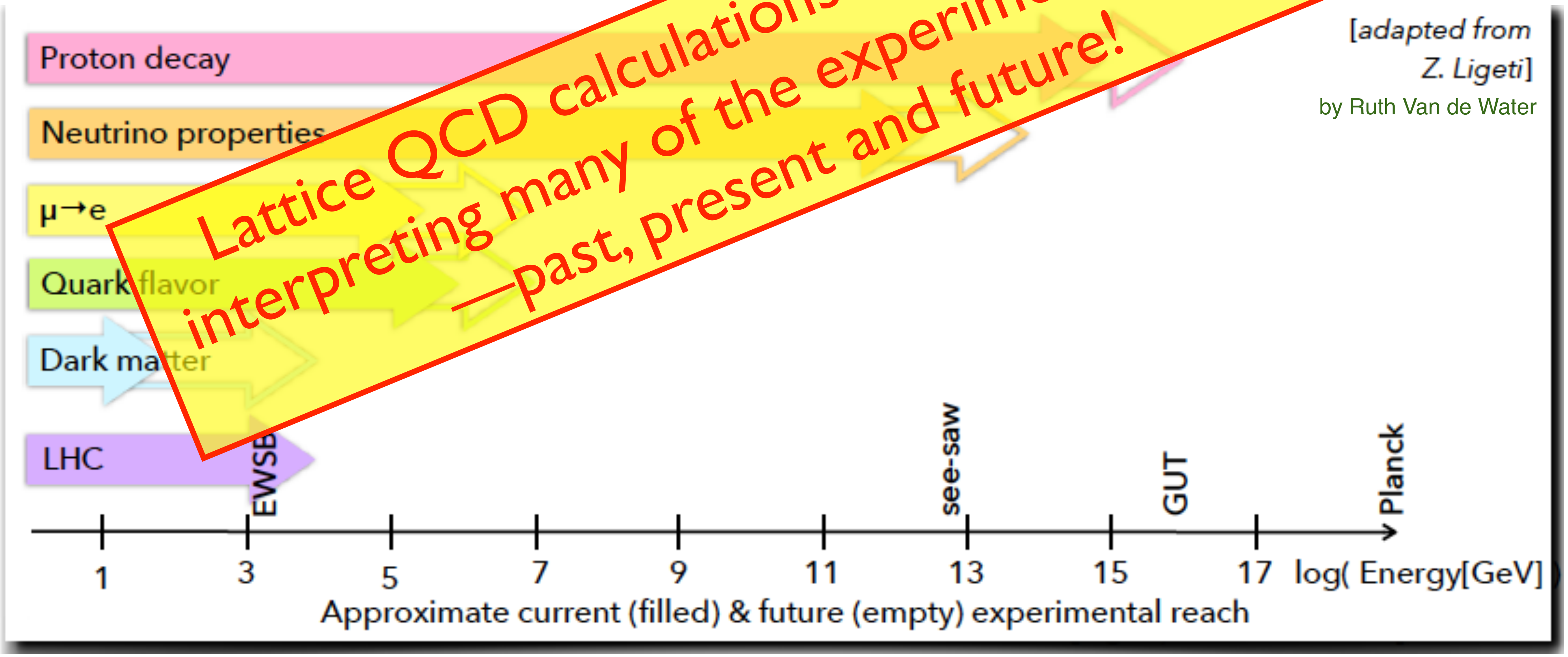
Overview

- Intensity frontier probes extremely high scales



Quantum-mechanical loops sensitive to new heavy particles above the TeV scale, e.g.:

[Ruth Van de Water]



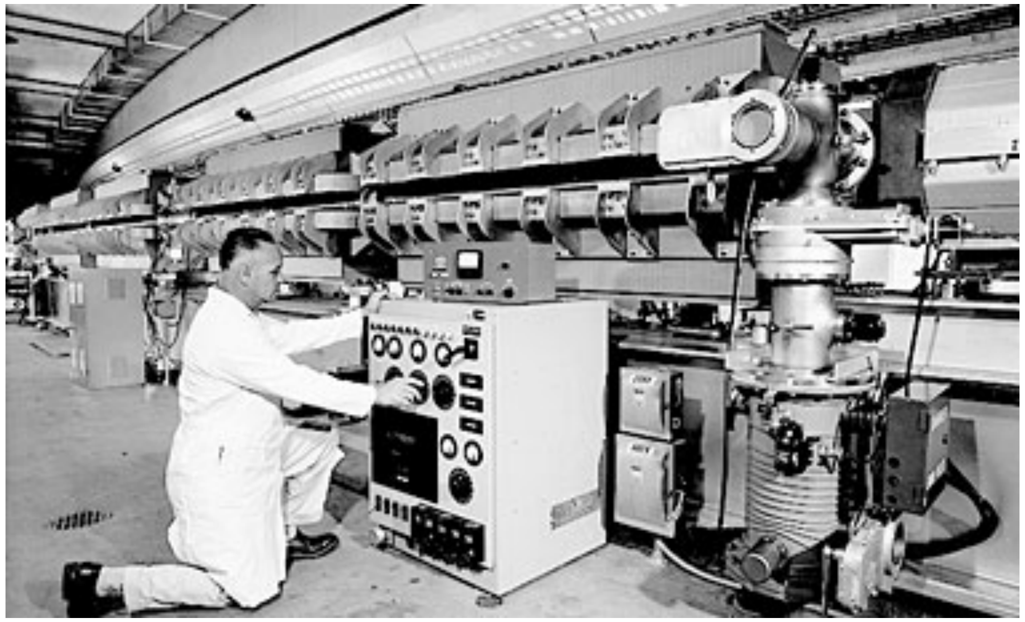
Approximate current (filled) & future (empty) experimental reach

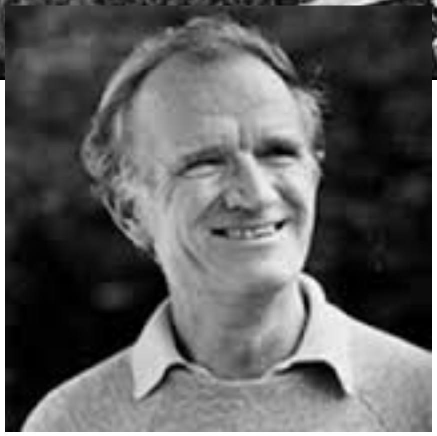
[*adapted from* *Z. Ligeti*]

by Ruth Van de Water

Lattice QCD calculations are essential for interpreting many of the experimental results —past, present and future!

Overview





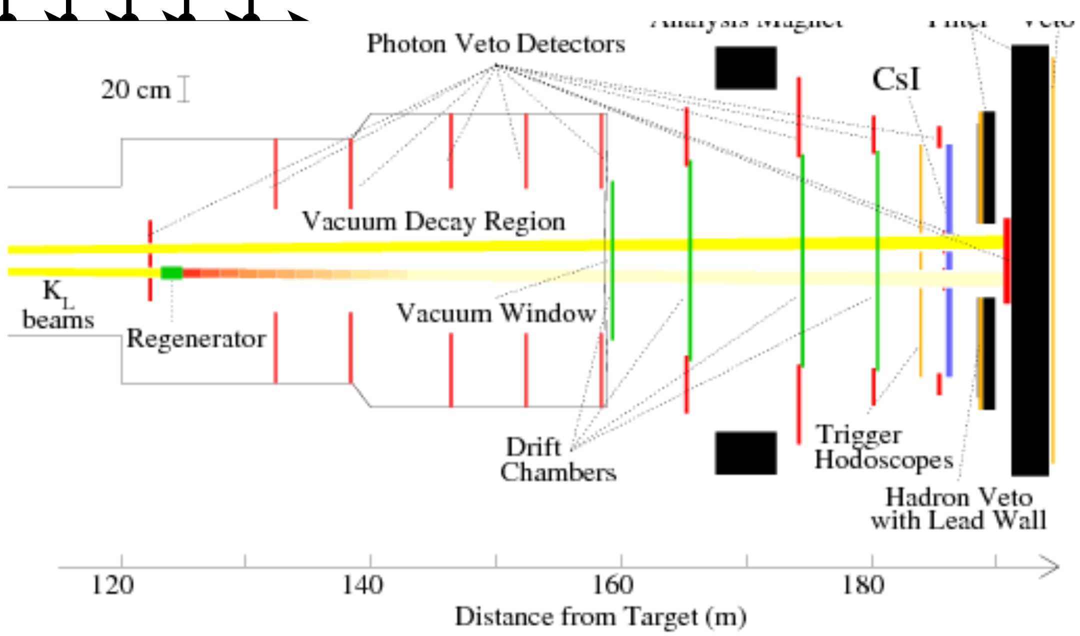
www.bnl.gov/about/history/accelerators.php

AGS @ BNL (E=33 GeV/c^2 protons)

- In 1961, Fitch and others measured the K_L-K_S mass difference: $\Delta M \approx 3.5\ 10^{-12}$ GeV

We still do not know whether this result is consistent with the SM !

Overview



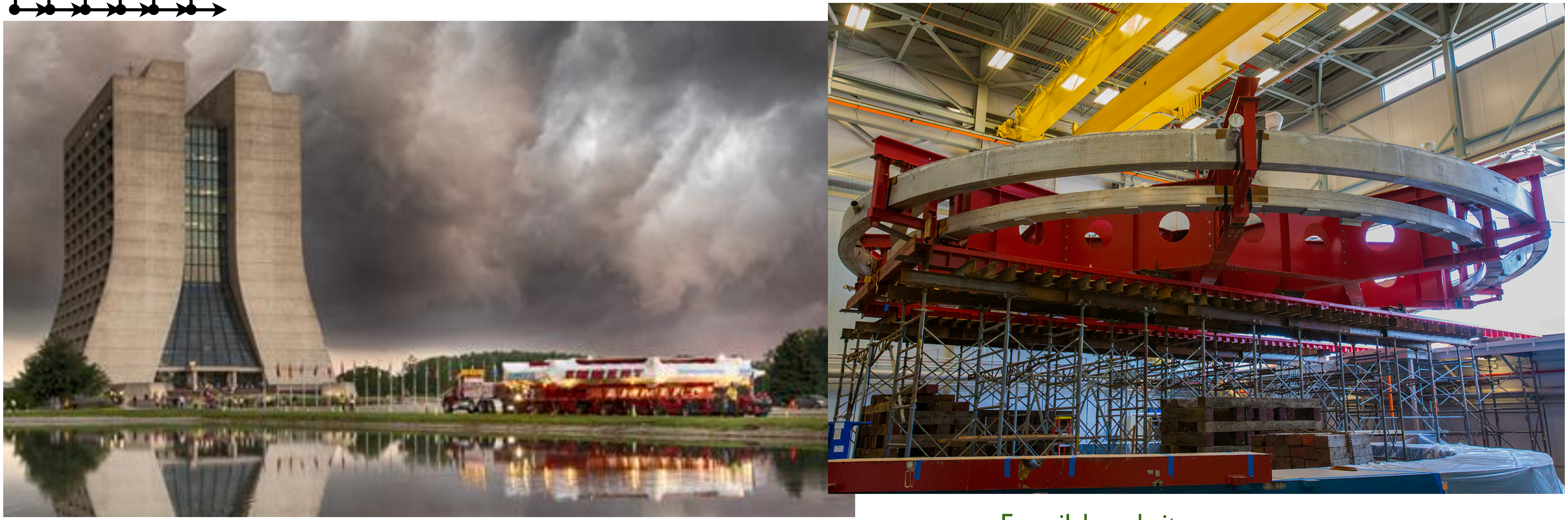
KTeV @ Fermilab: arXiv:0805.0031

NA48 @ CERN: Cern website

- In 1999, KTeV & NA48 measured CP violation in K→ππ decays [$\varepsilon'/\varepsilon=1.63(26)\ 10^{-3}$]

We still do not know whether this result is consistent with the SM !

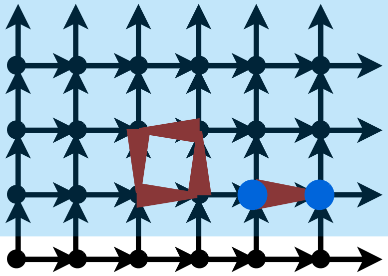
Overview



Fermilab website

- Fermilab's muon g-2 experiment will have first results soon
  - Will reduce error by a factor of ~5

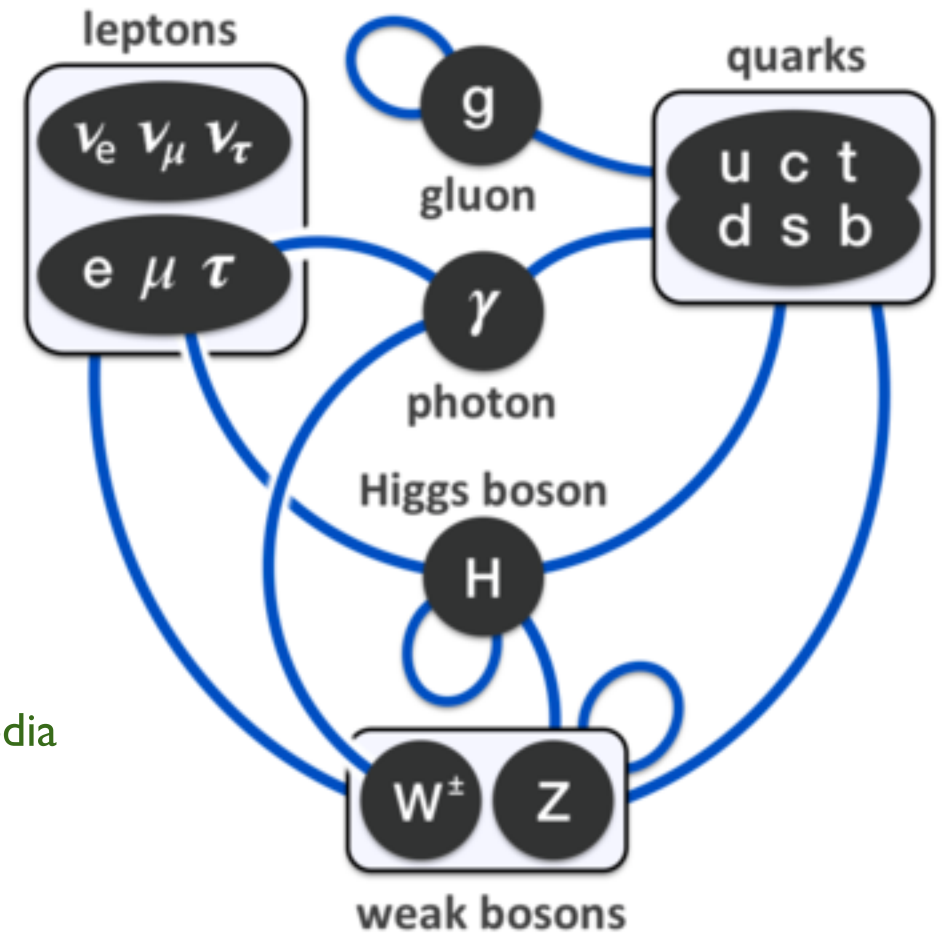
We want to know whether the result obtained is consistent with the SM !



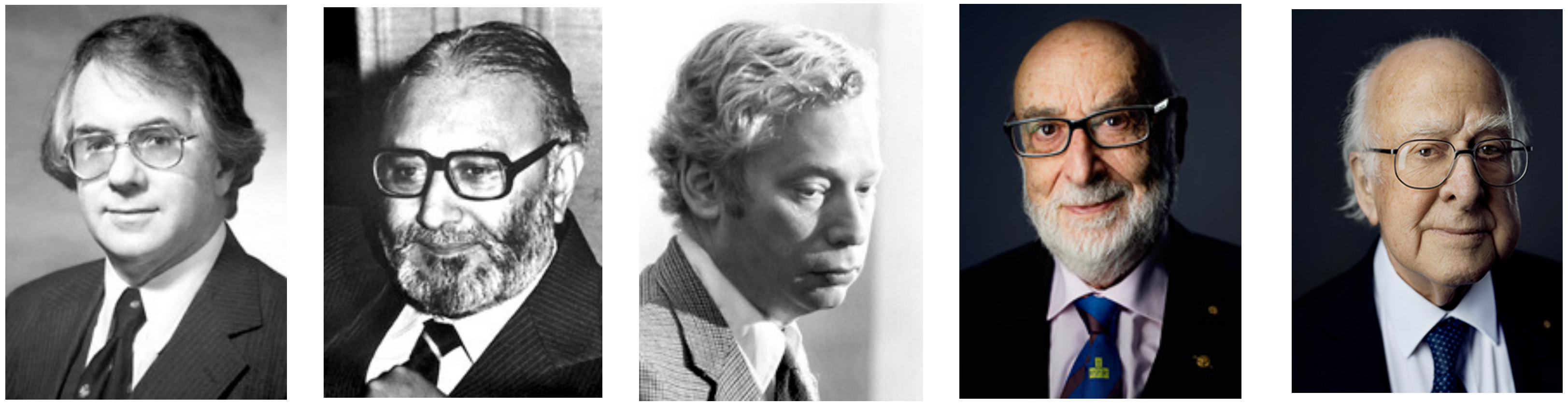
Outline

- Standard model & searching for physics beyond
- QCD & Lattice QCD (LQCD)
- High precision lattice QCD
- Constraining the Standard Model with LQCD
- Extending the LQCD frontier
- Future outlook

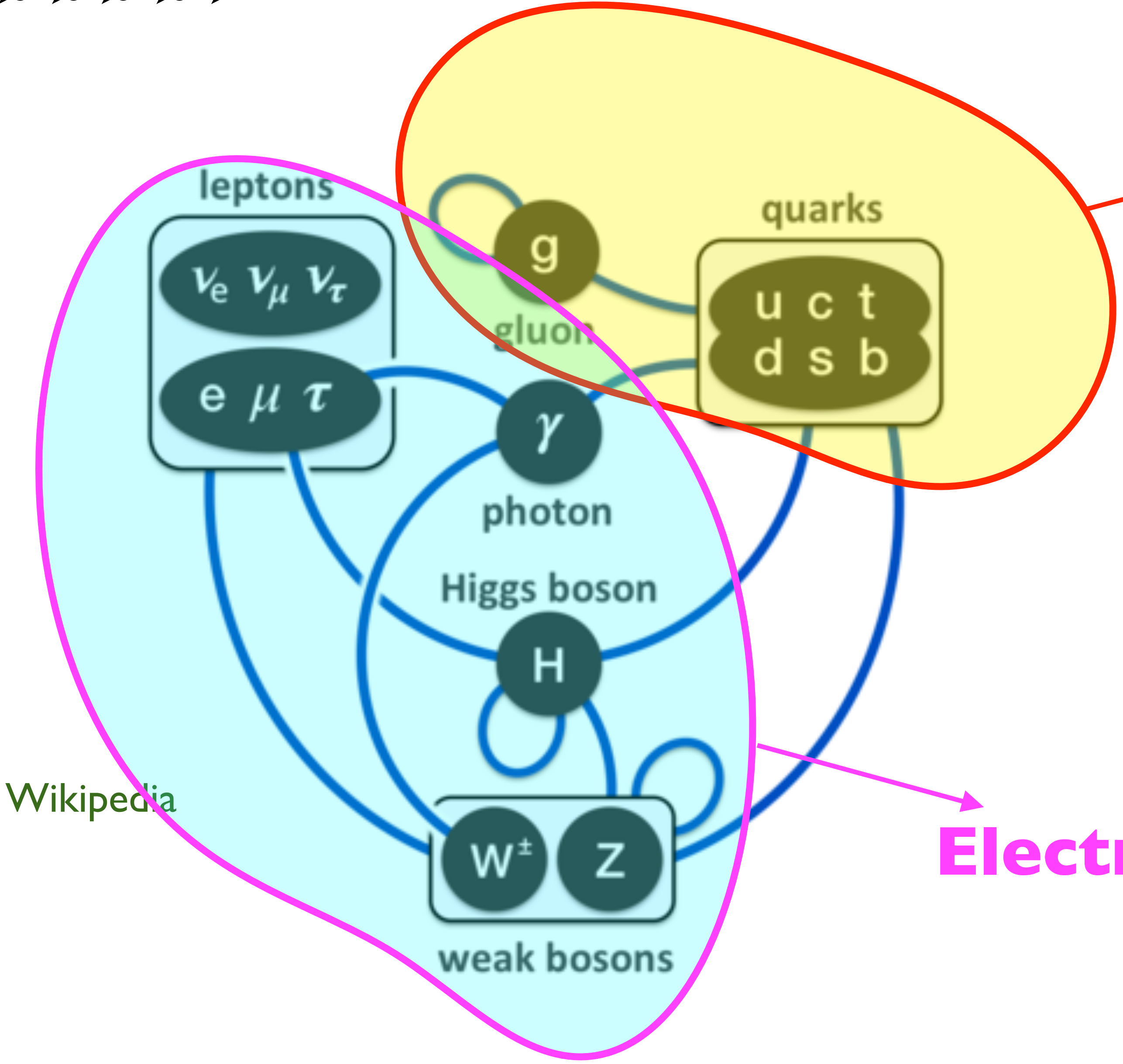
Standard Model (SM)



Wikipedia

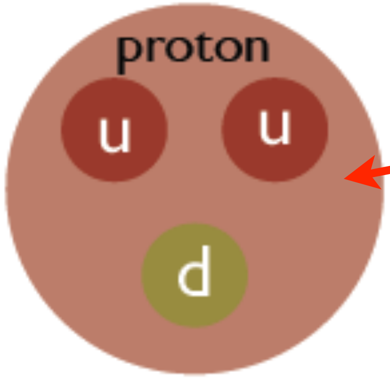


Standard Model (SM)



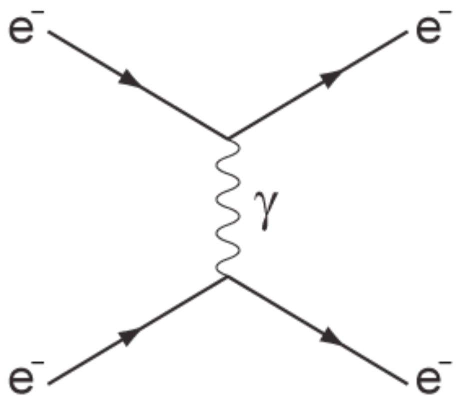
Wikipedia

QCD is non-perturbative



gluons &
sea quarks:
“brown muck”

Electroweak sector is perturbative



SM is extremely successful

- EM sector tested to extraordinary precision

$$4\pi/e^2 = \alpha^{-1} = 137.035\,999\,084(55)$$ Electron g-2, 2008

$$4\pi/e^2 = \alpha^{-1} = 137.035\,998\,78(91)$$ Rb, 2006

- Weak sector tested to few parts in 1000

This is possible because couplings are weak enough to use perturbation theory

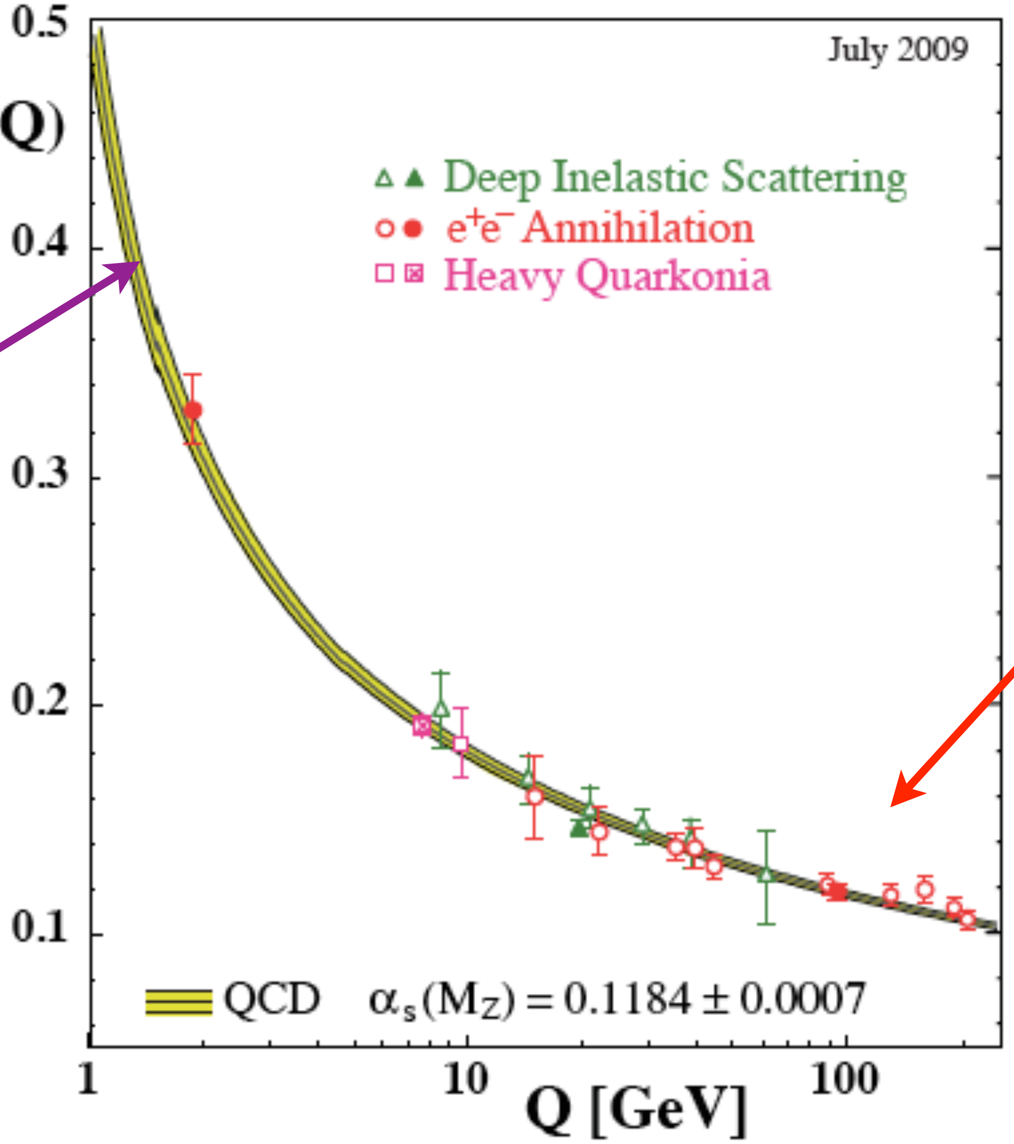
$$g_e - 2 = \frac{\alpha}{\pi} + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + C_5\left(\frac{\alpha}{\pi}\right)^5 + \dots$$

QCD is more challenging

$$\frac{g^2}{4\pi} = \alpha_s(Q)$$

Need non-perturbative method here

PDG, 2010

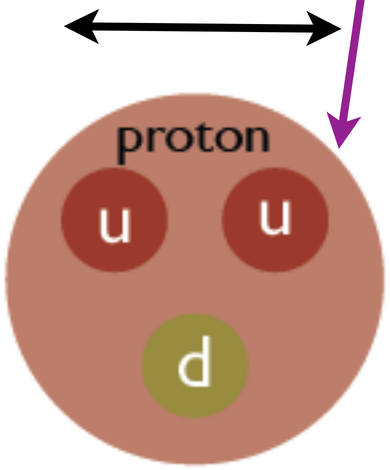


a=0.2 fm

a=0.002 fm

Asymptotic freedom: can use perturbation theory at high energies

$1fm=10^{-15}m$



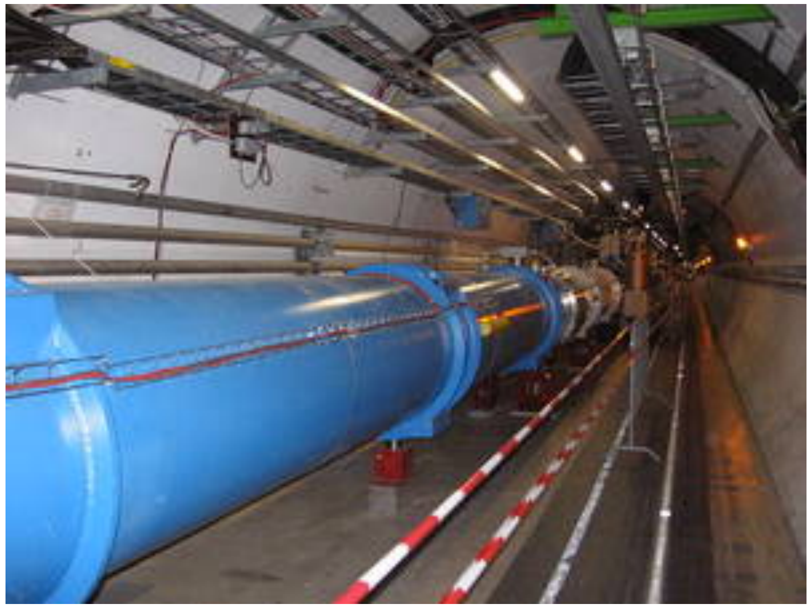
Shortcomings of the SM

NASA

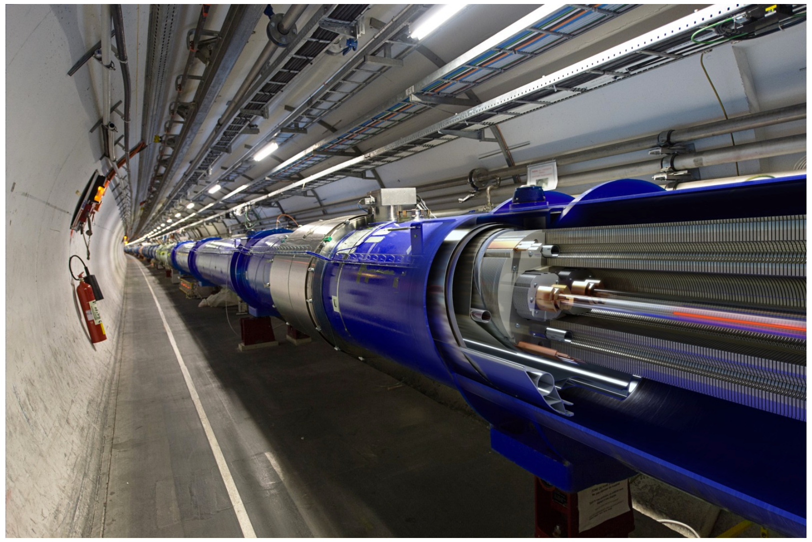
- No dark matter or dark energy
- Predicts insufficient baryogenesis
- Why 3 generations? Why the observed pattern of quark & lepton masses and weak couplings?
- Weak scale relative to Planck scale
- …

Searching for new physics

- At the highest energies—LHC

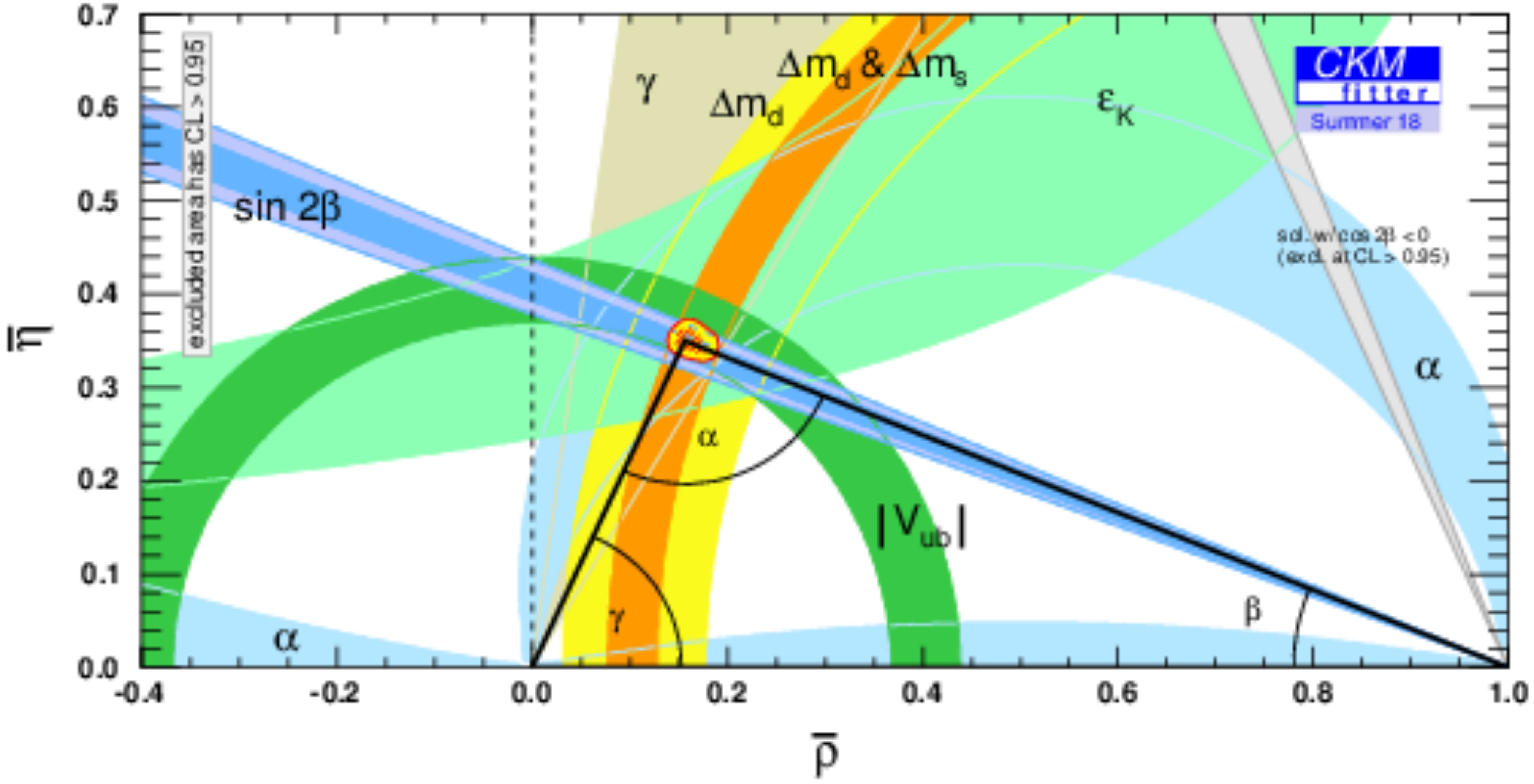


coffeeshopphysics.com



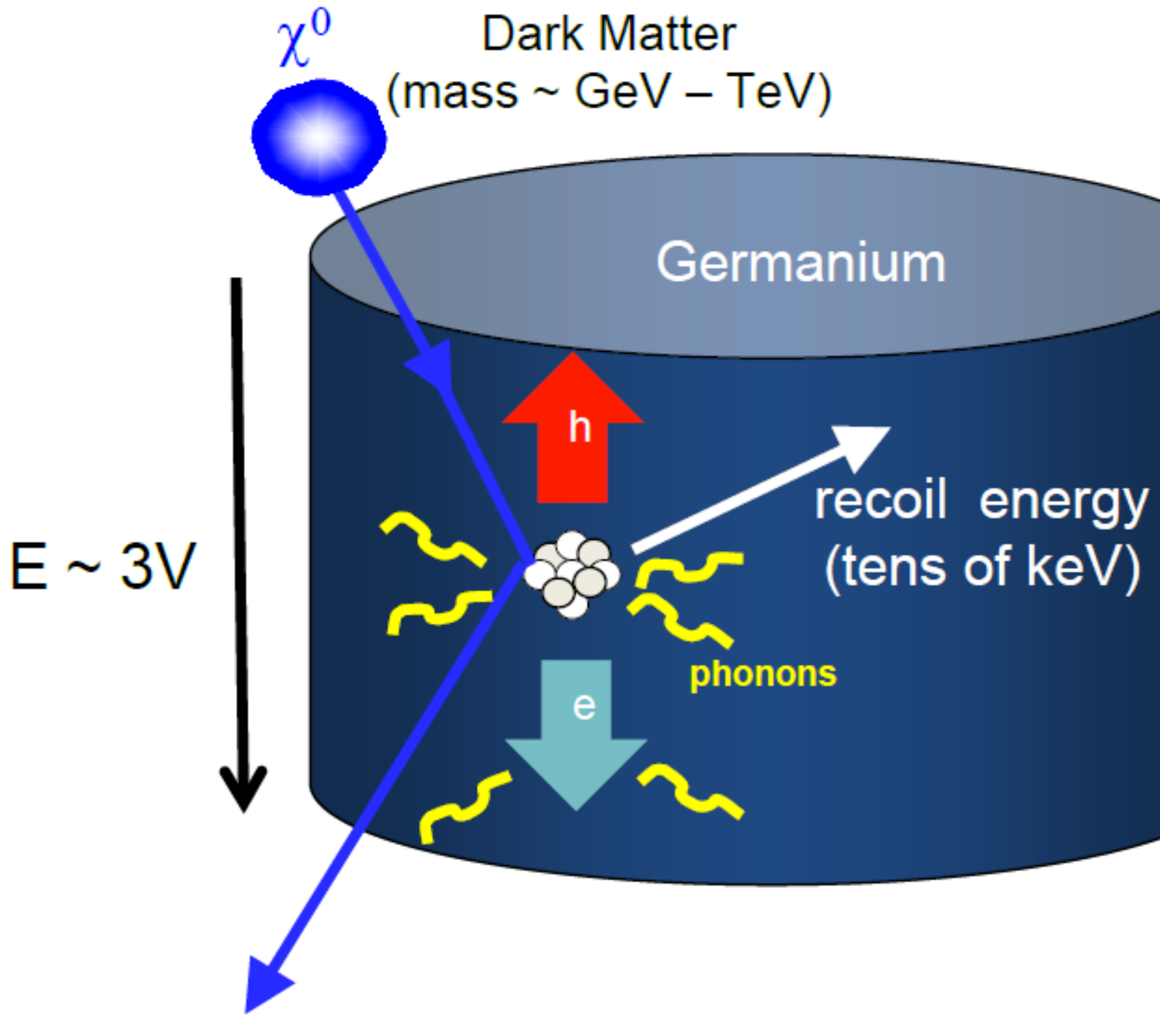
Searching for new physics

- In rare decays or precision measurements
  - Do all CP-violating processes in K and B decays agree with the SM?
  - Do precision measurements such as g_μ-2 agree with SM predictions?



Searching for new physics

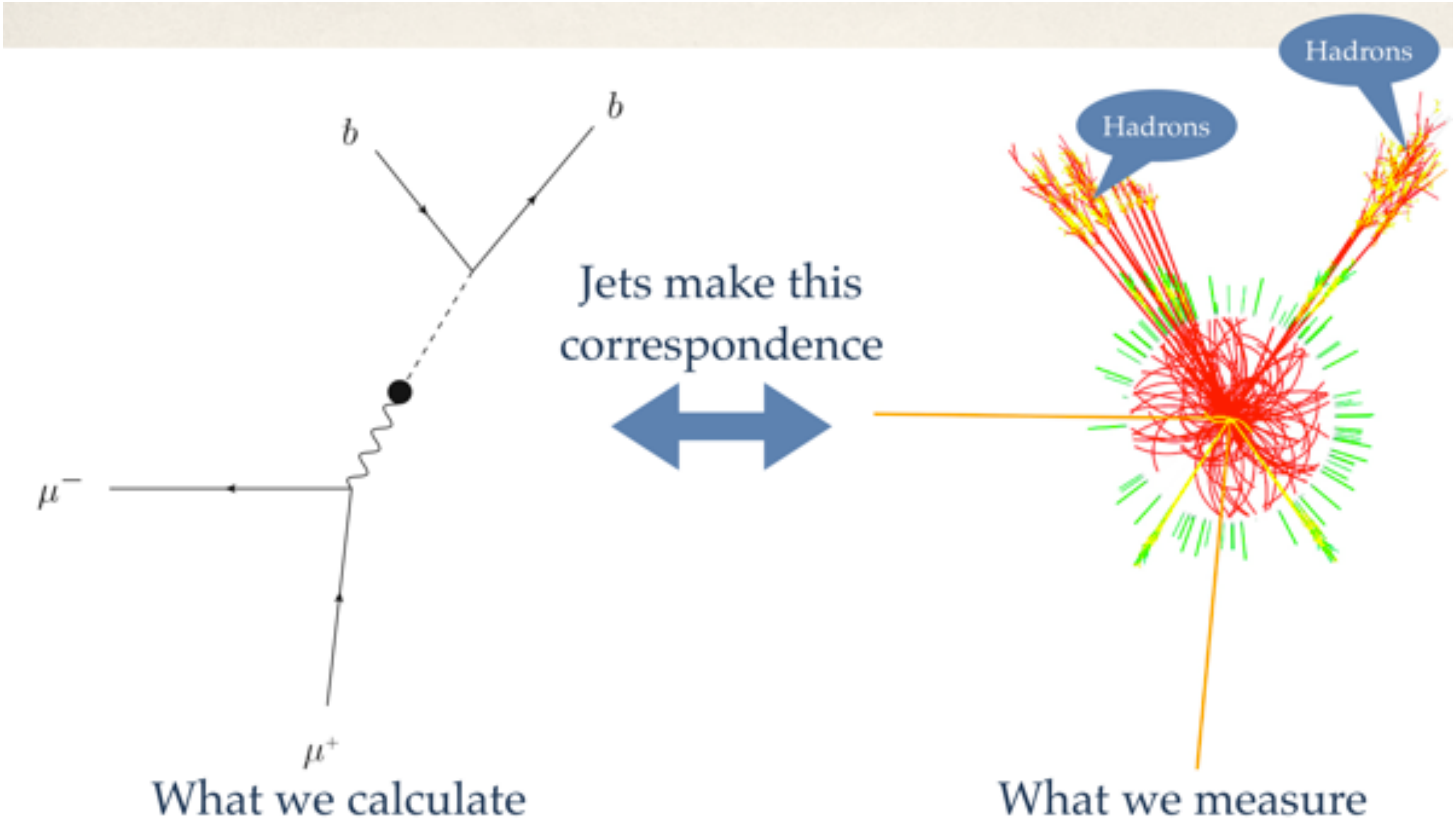
- Dark matter searches



CDMS—graphic from SLAC

QCD as a background

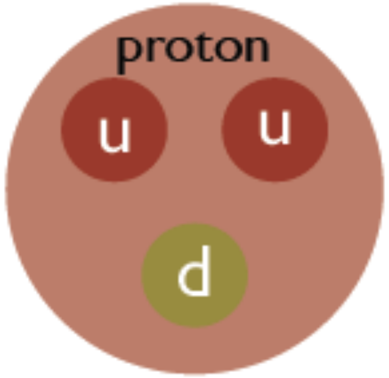
- Quarks become jets at the LHC



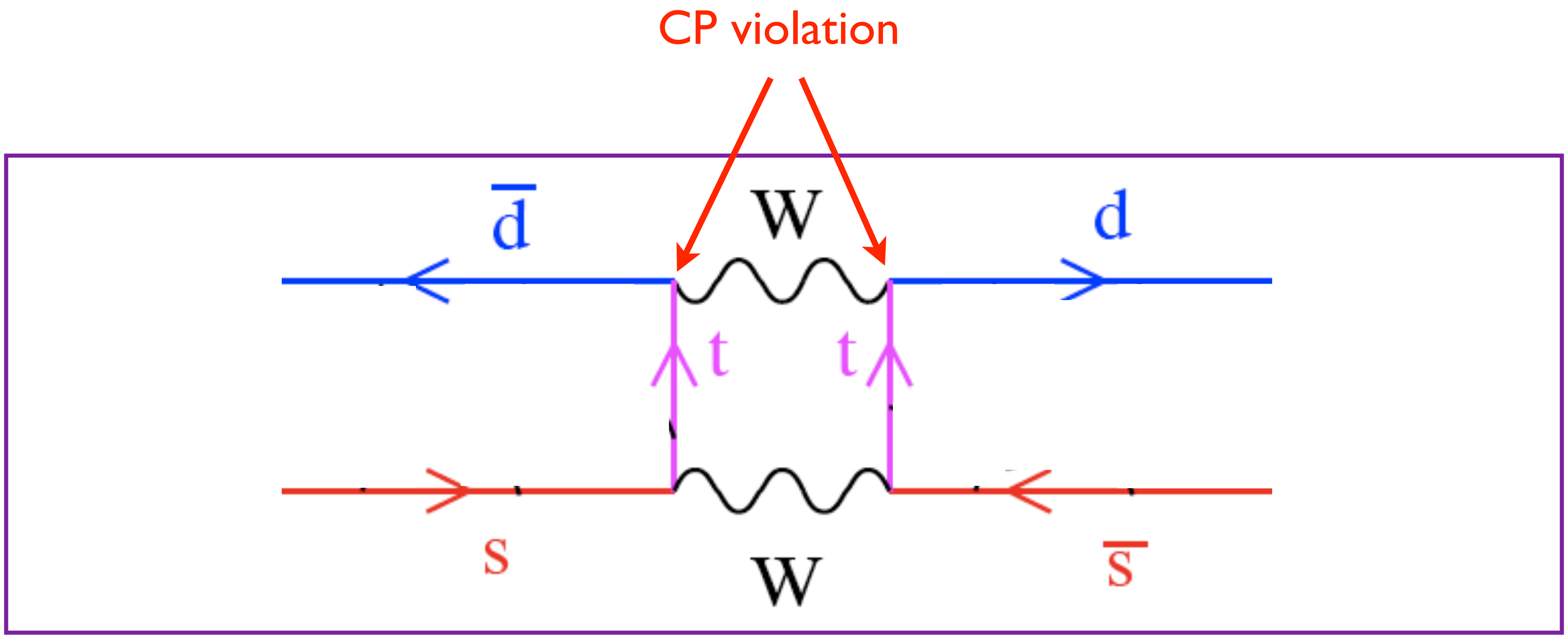
quantumdiaries.org

Disentangling requires perturbative QCD and modeling of non-perturbative confinement physics

QCD as a background



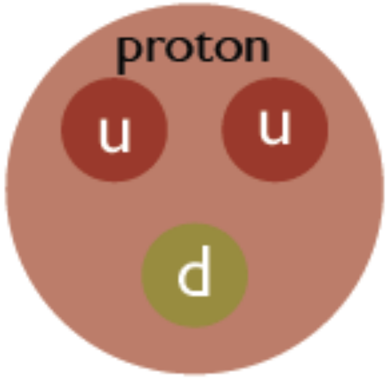
- "Brown muck" distorts hadronic decays
- E.g. CP-violation in kaon-antikaon mixing



Quark level process that might hope to calculate in perturbation theory

is really a hadronic process that involves non-perturbative QCD

QCD as a background



- “Brown muck” distorts hadronic decays
- Distortions can be huge, e.g. ΔI=½ rule

$$\frac{\Gamma(K_S^0 \to \pi\pi)}{\Gamma(K^+ \to \pi\pi)} \approx 330$$

Same underlying quark weak decay: $s \longrightarrow \bar{u}ud$

- Must be able to calculate these “distortions” to interpret many rare decay experiments

Outline

- Standard model & searching for physics beyond
- QCD & Lattice QCD
- High precision lattice QCD
- Constraining the Standard Model with LQCD
- Extending the LQCD frontier
- Future outlook

What is QCD?

$$\mathcal{L}_{QCD} = \bar{q}_{a,i}\left[\left(i\gamma^{\mu}\partial_{\mu} - m_i\right)\delta_{ab}\delta_{ij}\right]q_{b,j} - g\ G^a_{\mu}\ \bar{q}_{i,b}\gamma^{\mu}T^a_{bc}\ q_{i,c} - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$

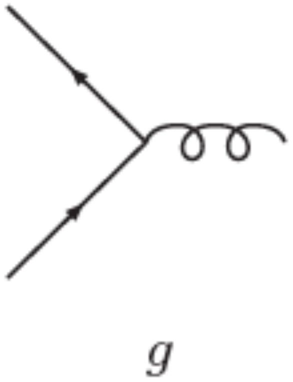
$$G^a_{\mu\nu} = \partial_{\mu}G^a_{\nu} - \partial_{\nu}G^a_{\mu} - g\ f^{abc}G^b_{\mu}G^c_{\nu}$$

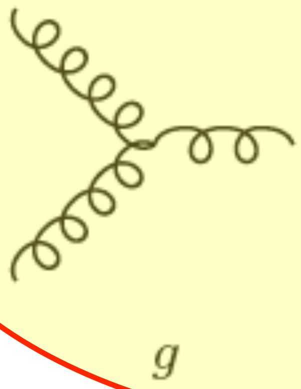
$q_{a,i} =$ quark, 3 colors “a” and 6 flavors “i”

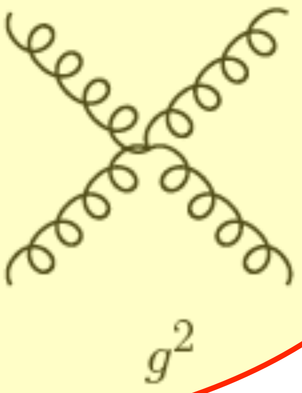
$G^a_{\mu} =$ gluon, 8 colors “a”

$g =$ QCD coupling

These make QCD challenging!

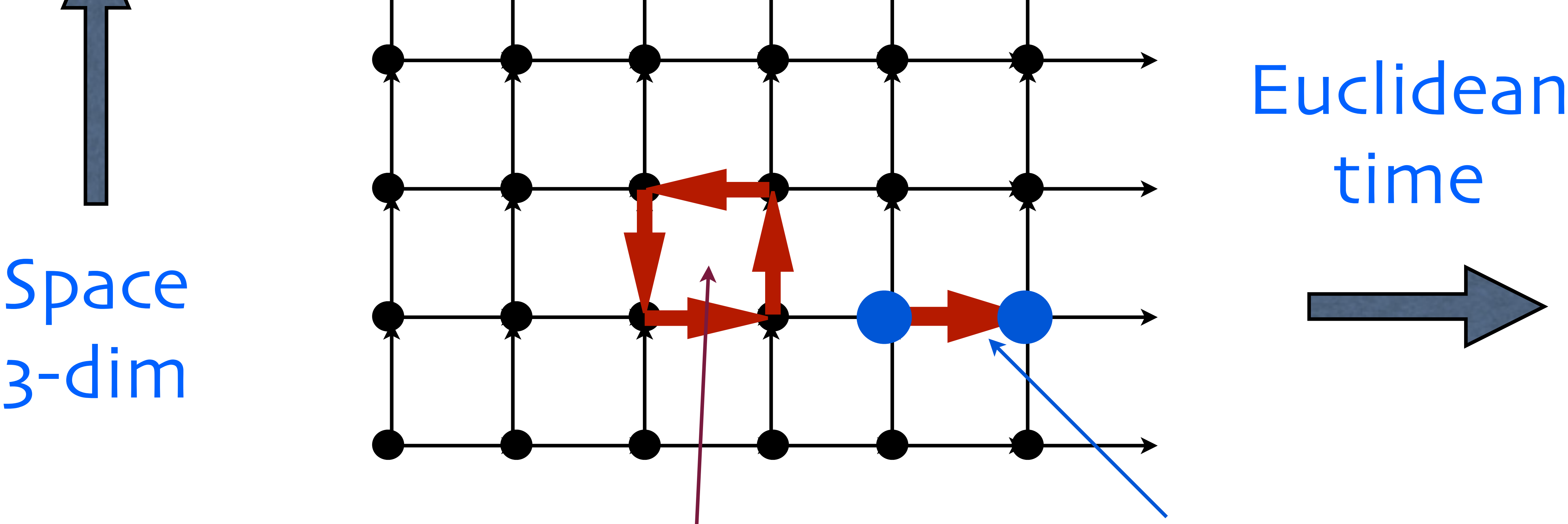






Lattice QCD

Ken Wilson, 1974



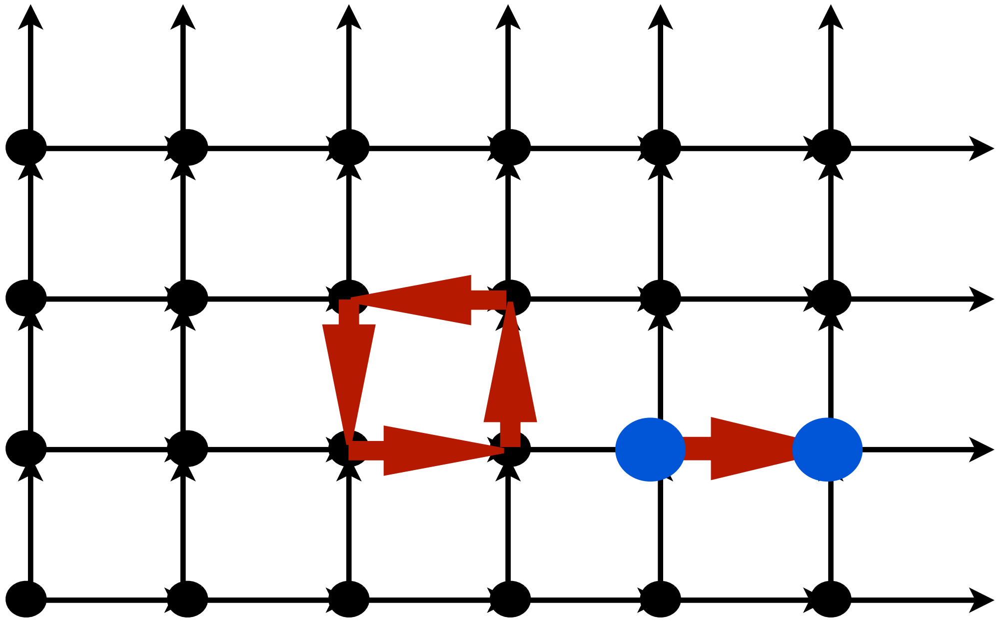
Space
3-dim

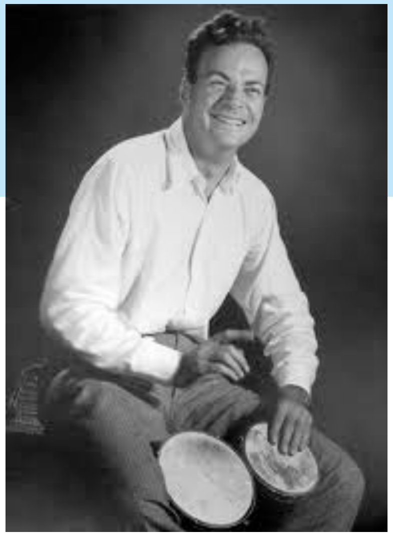
Euclidean
time

$$S_E^{\text{latt}} = -\sum_{\square} \frac{6}{g^2} \,\text{Re tr}_N (U_{\square,\mu\nu}) - \sum_q \overline{q} (D_\mu^{\text{lat}} \gamma_\mu + a m_q) q$$

Wilson gauge action   Lattice fermion action

Lattice QCD

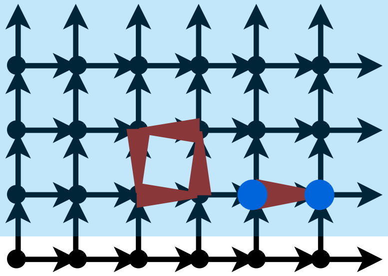




Use Feynman path integral definition of QM

$$Z_E = \int \prod dU\, d\bar{q}\, dq\, e^{-S_E^{\mathrm{lat}}}$$

- Non-perturbative regularization of QFT
- Provides rigorous definition of QCD
  - Take a→0 by sending g(a)→0
- Amenable to numerical simulation using Monte Carlo methods



Simulating fermions is hard

$$Z_{\mathrm{QCD}} = \int \prod dU\, d\bar{q}\, dq\, e^{-S_E^{\mathrm{lat}}}$$

$$= \int dU\, e^{-S_{\mathrm{glue}}^{\mathrm{lat}}} \prod_q \det\left(D_\mu^{\mathrm{lat}} \gamma_\mu + m\right)$$

gluon loops

fermion loops

- Fermion determinant leads to non-local effective gauge action
- Orders of magnitude more difficult to simulate than the "pure gluon" theory

Timeline

- 1974, invention of lattice QCD (K.Wilson)
- 1980, simulations of pure gluon theory demonstrate confinement (M. Creutz)
- 1980's: "quenched era" (no quark l
- 1987, Hybrid Monte
- 1990' … loops)
- … (Kaplan)
- … ermions (Neuberger)
- 2000's: fully unquenched era (light quark loops)
- 2009-10: simulations with physical up, down and strange quark masses
- Present: inclusion of electromagnetism; studies of light nuclei; …

CPU speedup, theoretical & algorithmic advances have allowed lattice QCD to become a precision tool

Cray 1, 1MFlop/s

Cray 2, 1GFlop/s

1TFlop/s

Blue gene P, 1 PFlop/s

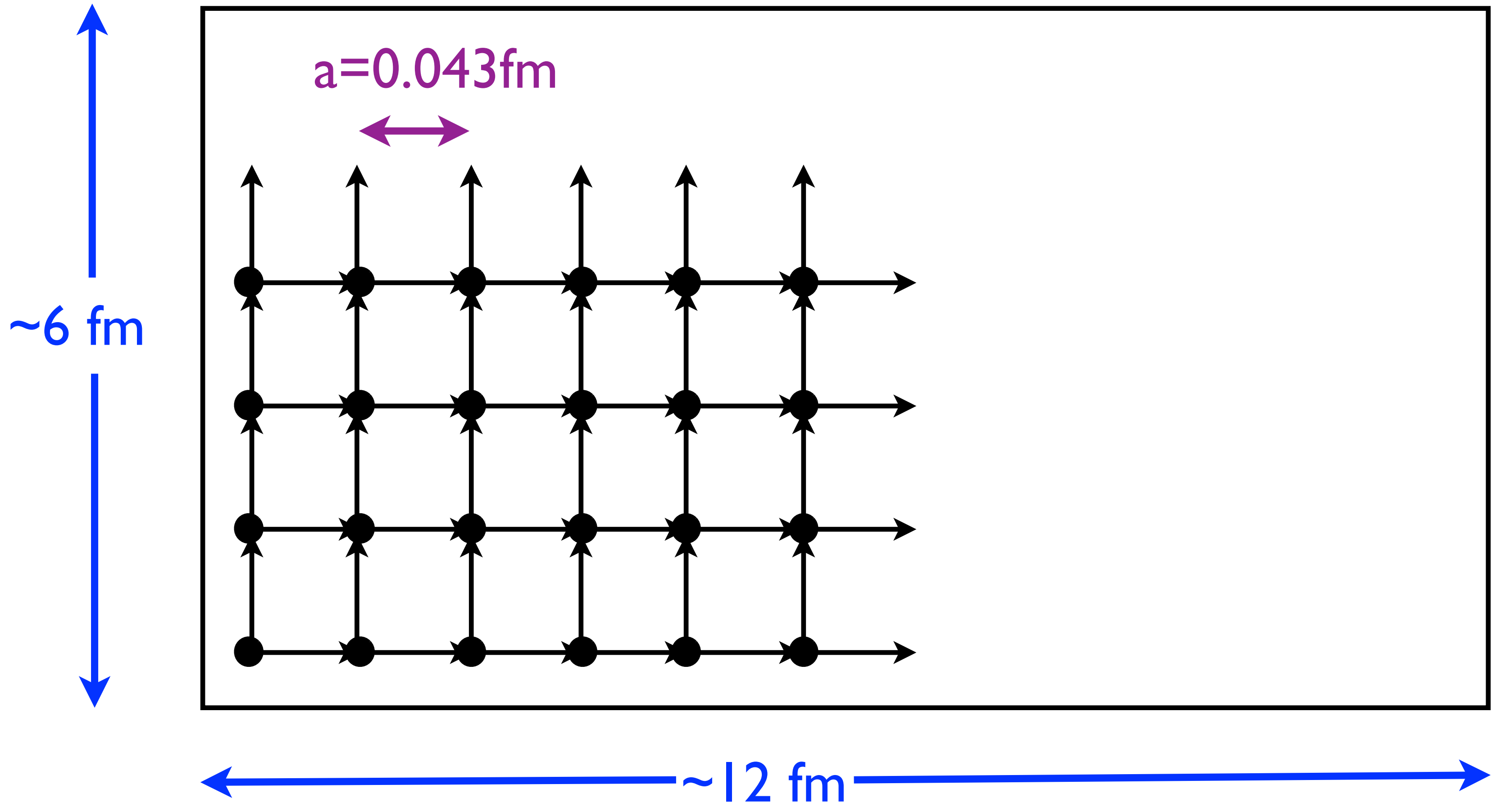
Blue gene Q, 10 PFlop/s

Summit (IBM-Nvidia), 100 PFlop/s

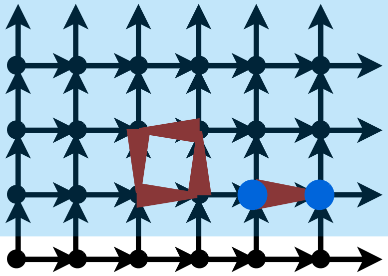
State of the art

144x144x144x288 lattice [MILC collaboration]

Highly Improved Staggered (HISQ) fermions
Physical quark masses (in isospin limit: $m_u = m_d$)



Need to invert matrices
of size
~ $(3\text{x}10^9)$ x $(3\text{x}10^9)$

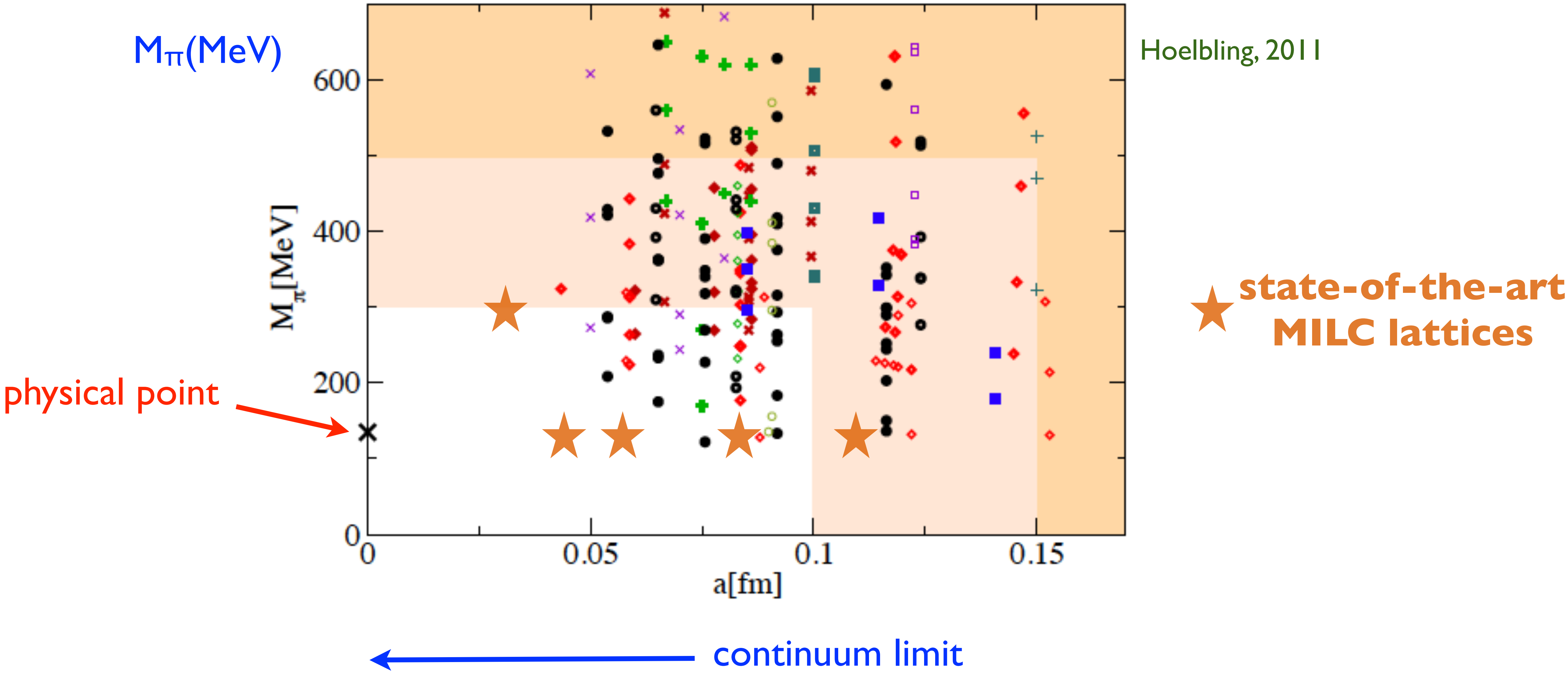


State of the art

Extrapolating to the physical point

M_π(MeV)

Hoelbling, 2011



physical point

★ **state-of-the-art MILC lattices**

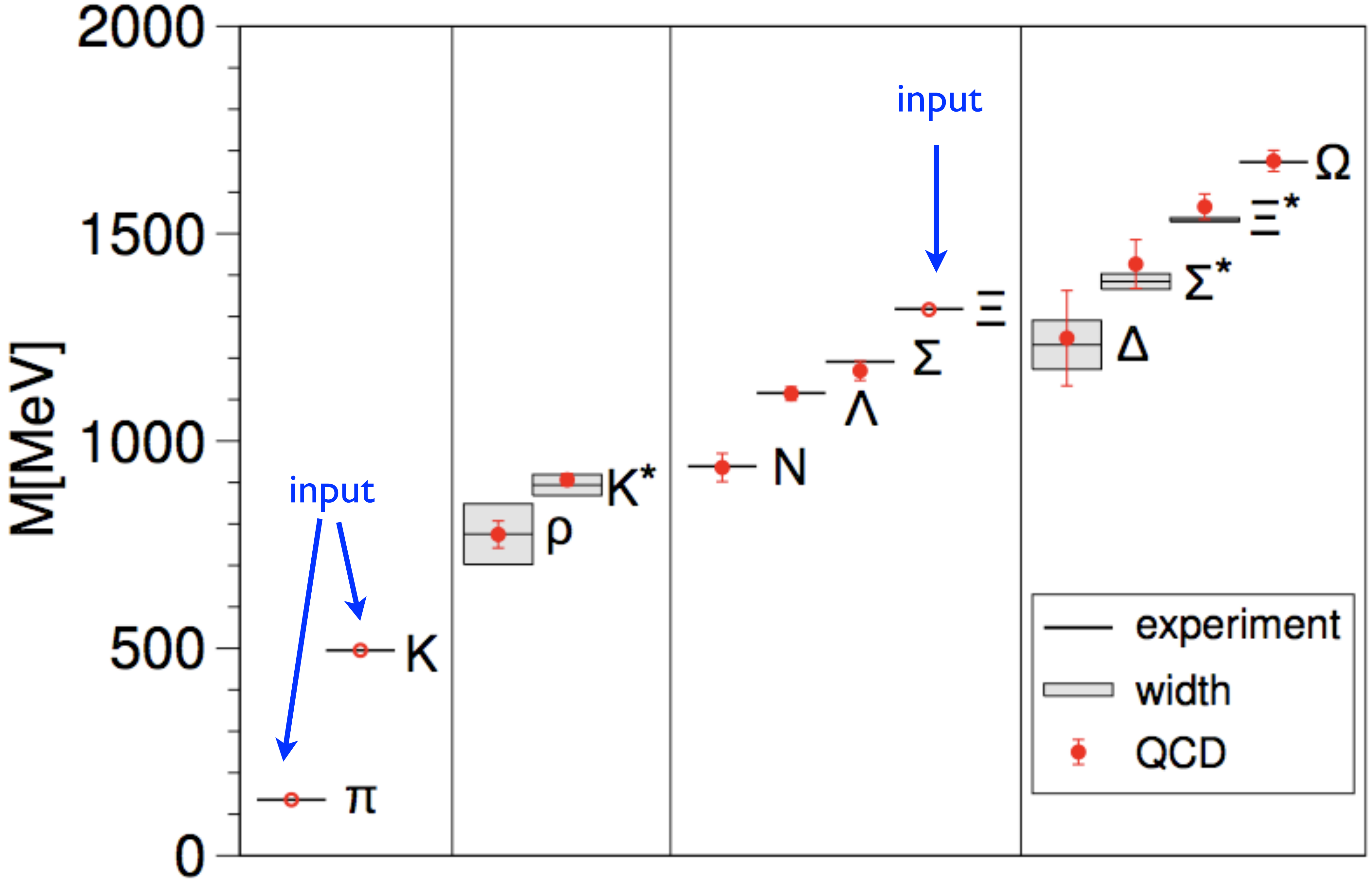
continuum limit

Outline

- Standard model & searching for physics beyond
- QCD & Lattice QCD
- High precision lattice QCD
- Constraining the Standard Model with LQCD
- Extending the LQCD frontier
- Future outlook

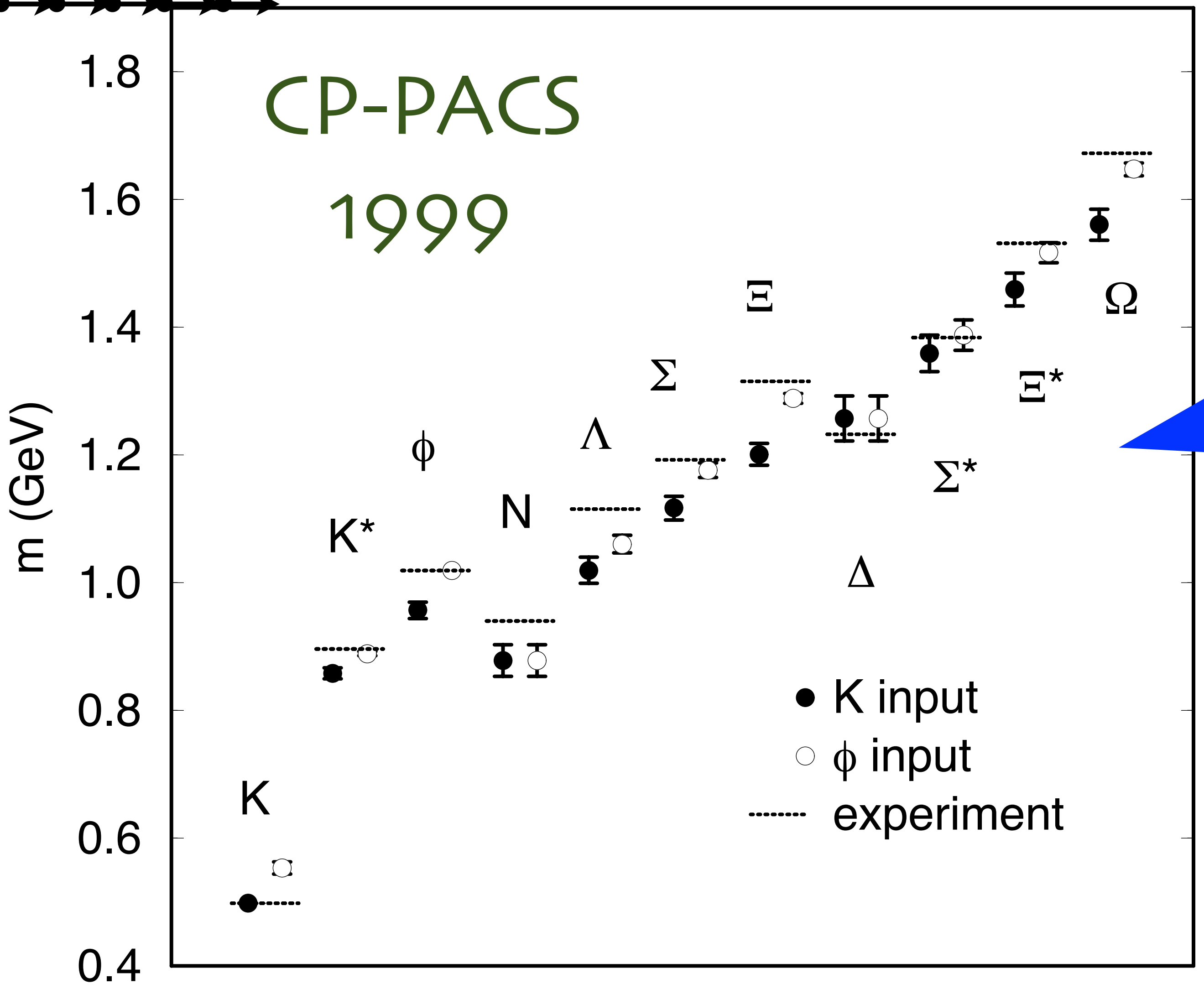
Postdiction: spectrum

BMW Collaboration, 2008

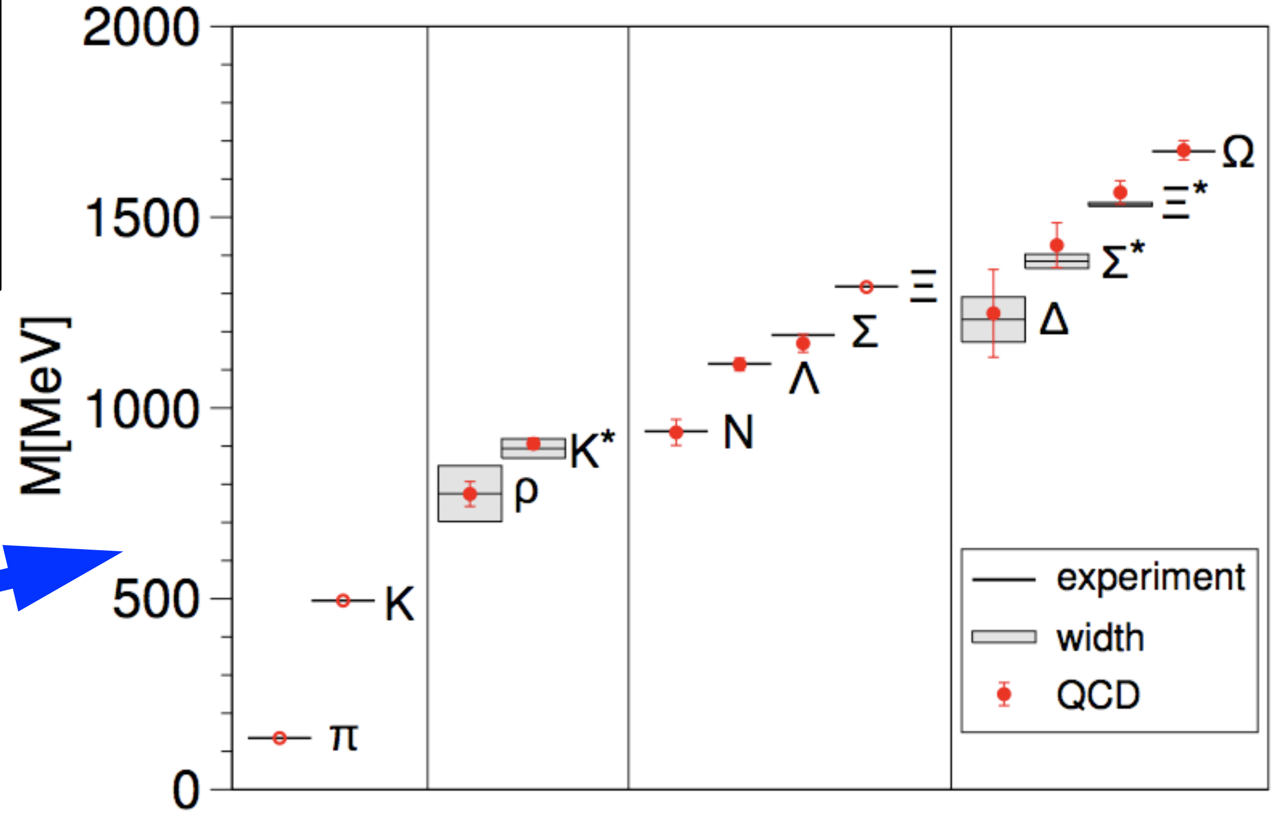


Few percent accuracy, and complete consistency

Fermion loops are needed!



No fermion
loops

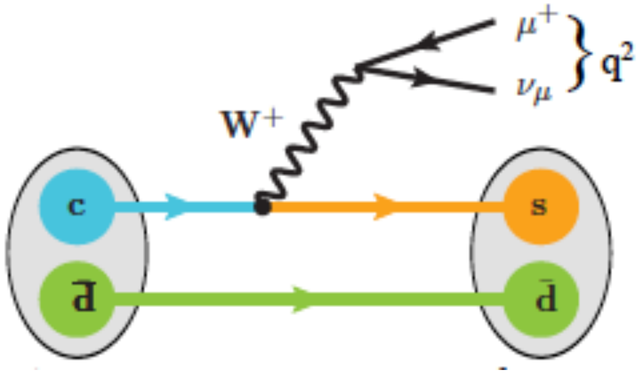


BMW collab. 2008

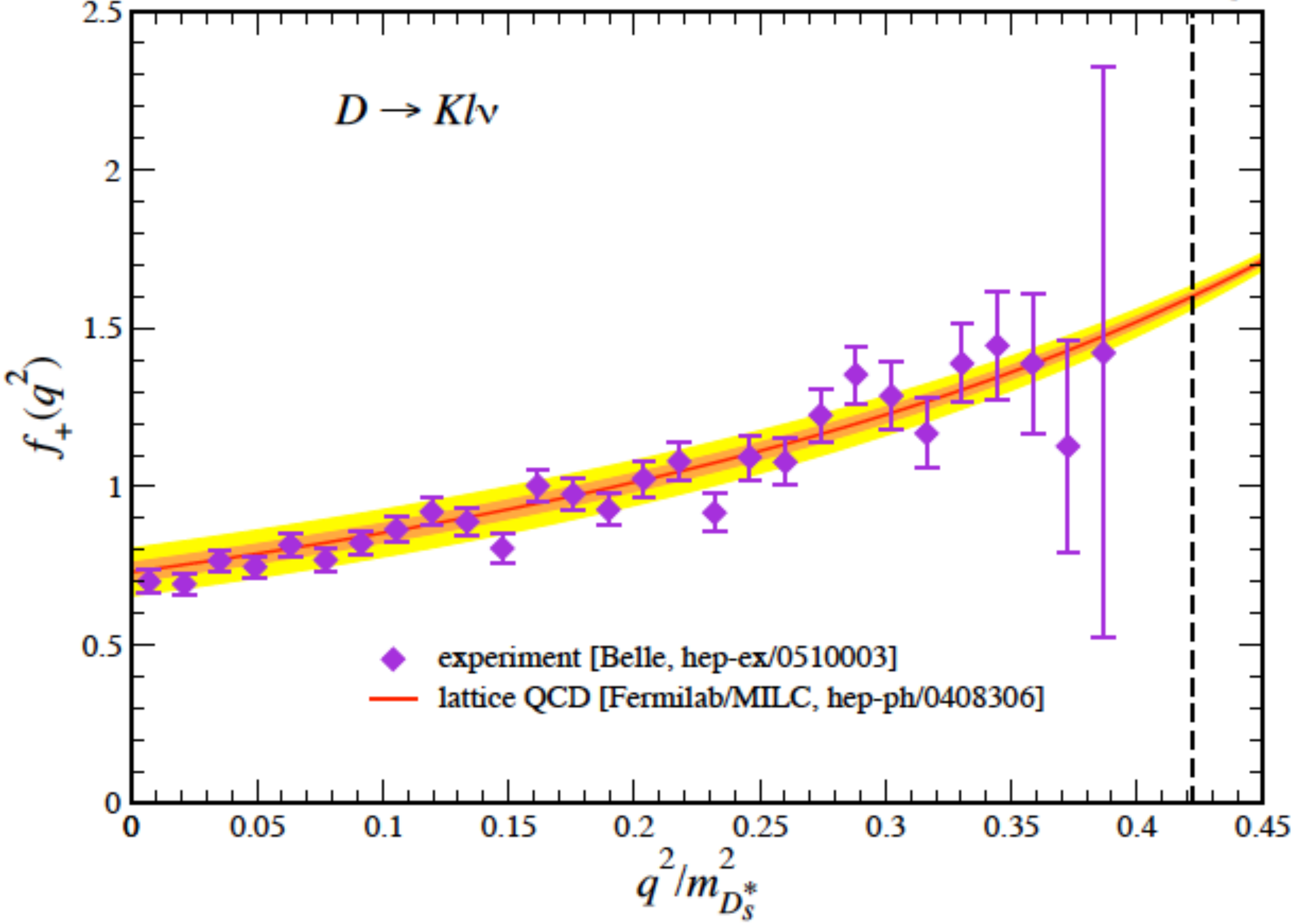
Fermion
loops
included

Several predictions

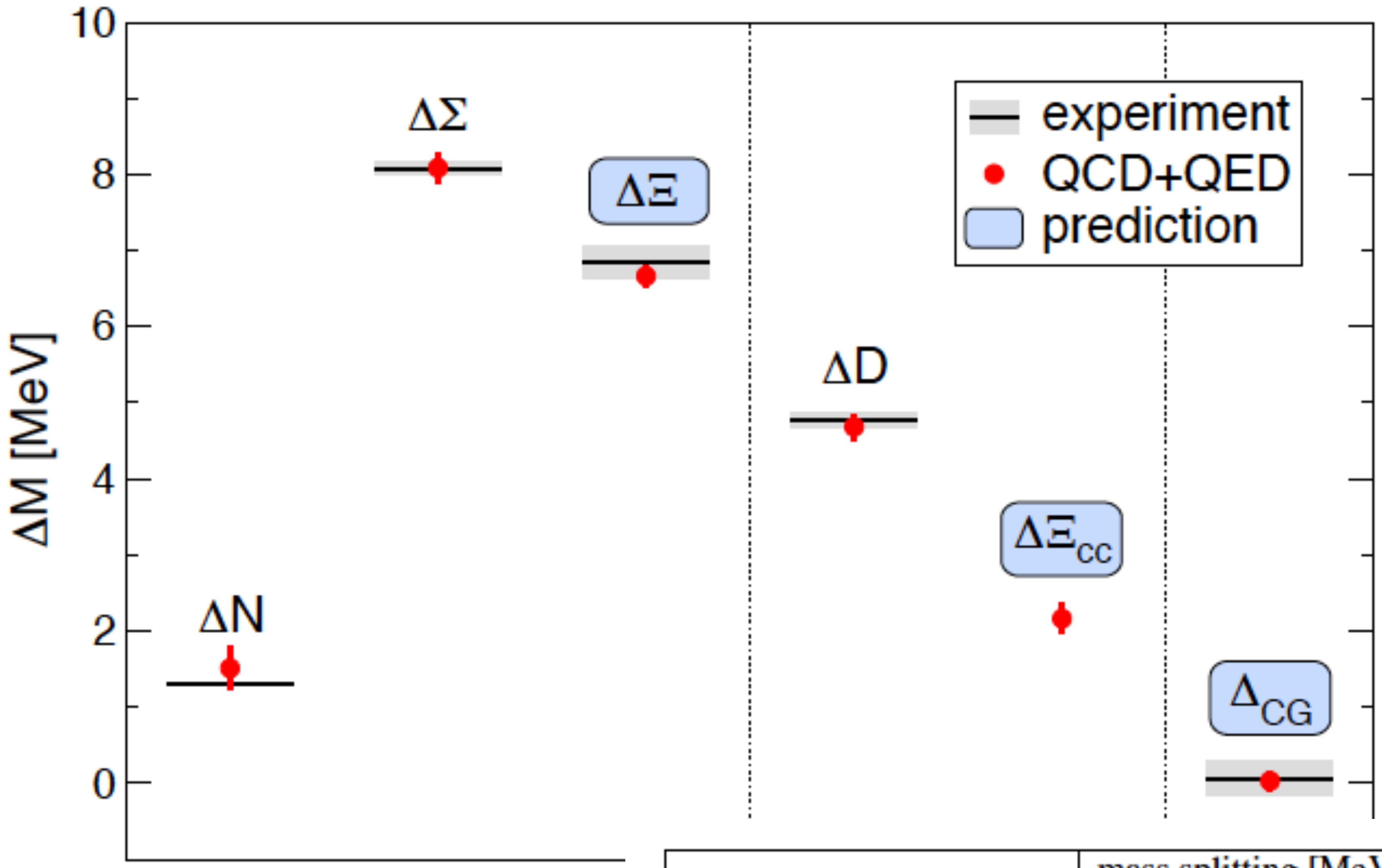
- D→Klν form factor



Ruth Van de Water



Isospin splittings



BMW Collaboration
2014

u, d, s & c in loops

$m_u \neq m_d$

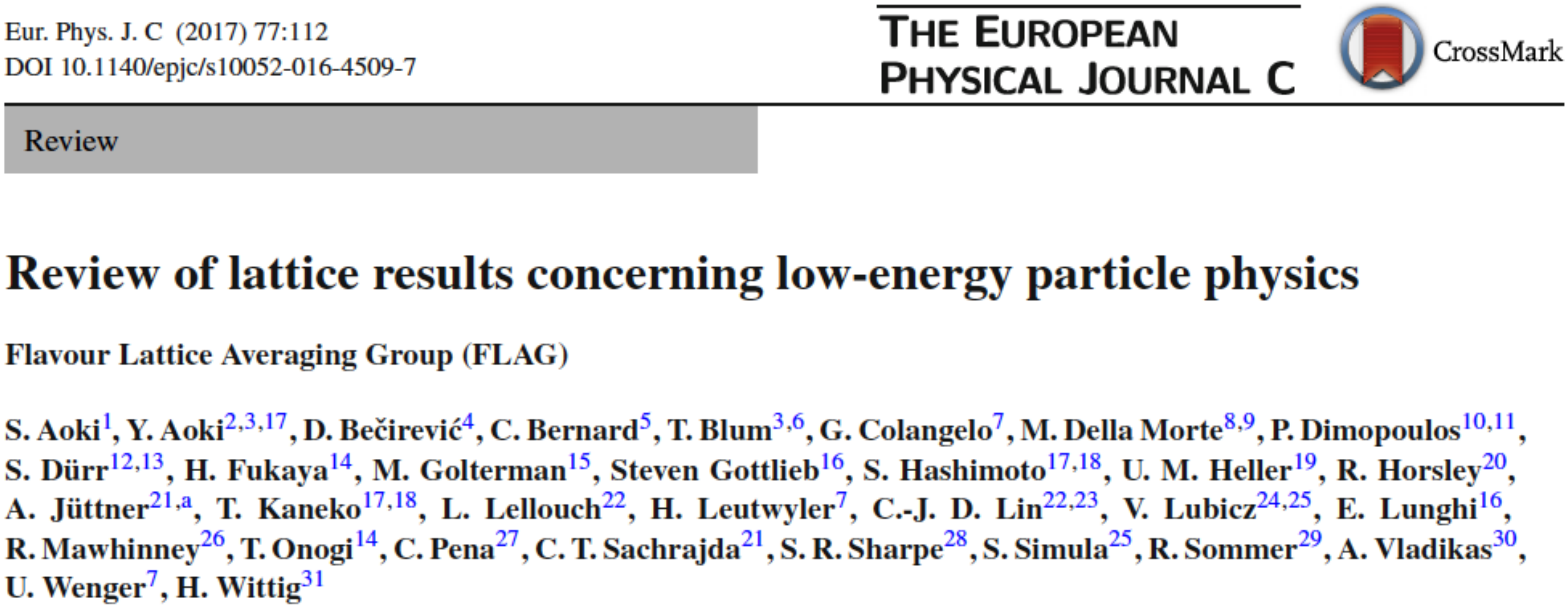
QED included

quark masses & scale
determined using
π^+, K^+, K^0, D^0, Ω

Errors ~ 0.2 MeV !

	mass splitting [MeV]	QCD [MeV]	QED [MeV]
$\Delta N = n - p$	1.51(16)(23)	2.52(17)(24)	-1.00(07)(14)
$\Delta\Sigma = \Sigma^- - \Sigma^+$	8.09(16)(11)	8.09(16)(11)	0
$\Delta\Xi = \Xi^- - \Xi^0$	6.66(11)(09)	5.53(17)(17)	1.14(16)(09)
$\Delta D = D^\pm - D^0$	4.68(10)(13)	2.54(08)(10)	2.14(11)(07)
$\Delta\Xi_{cc} = \Xi_{cc}^{++} - \Xi_{cc}^{+}$	2.16(11)(17)	-2.53(11)(06)	4.69(10)(17)
$\Delta_{CG} = \Delta N - \Delta\Sigma + \Delta\Xi$	0.00(11)(06)	-0.00(13)(05)	0.00(06)(02)

Flavo(u)r Lattice Averaging Group



Eur. Phys. J. C (2017) 77:112
DOI 10.1140/epjc/s10052-016-4509-7

THE EUROPEAN
PHYSICAL JOURNAL C

CrossMark

Review

Review of lattice results concerning low-energy particle physics

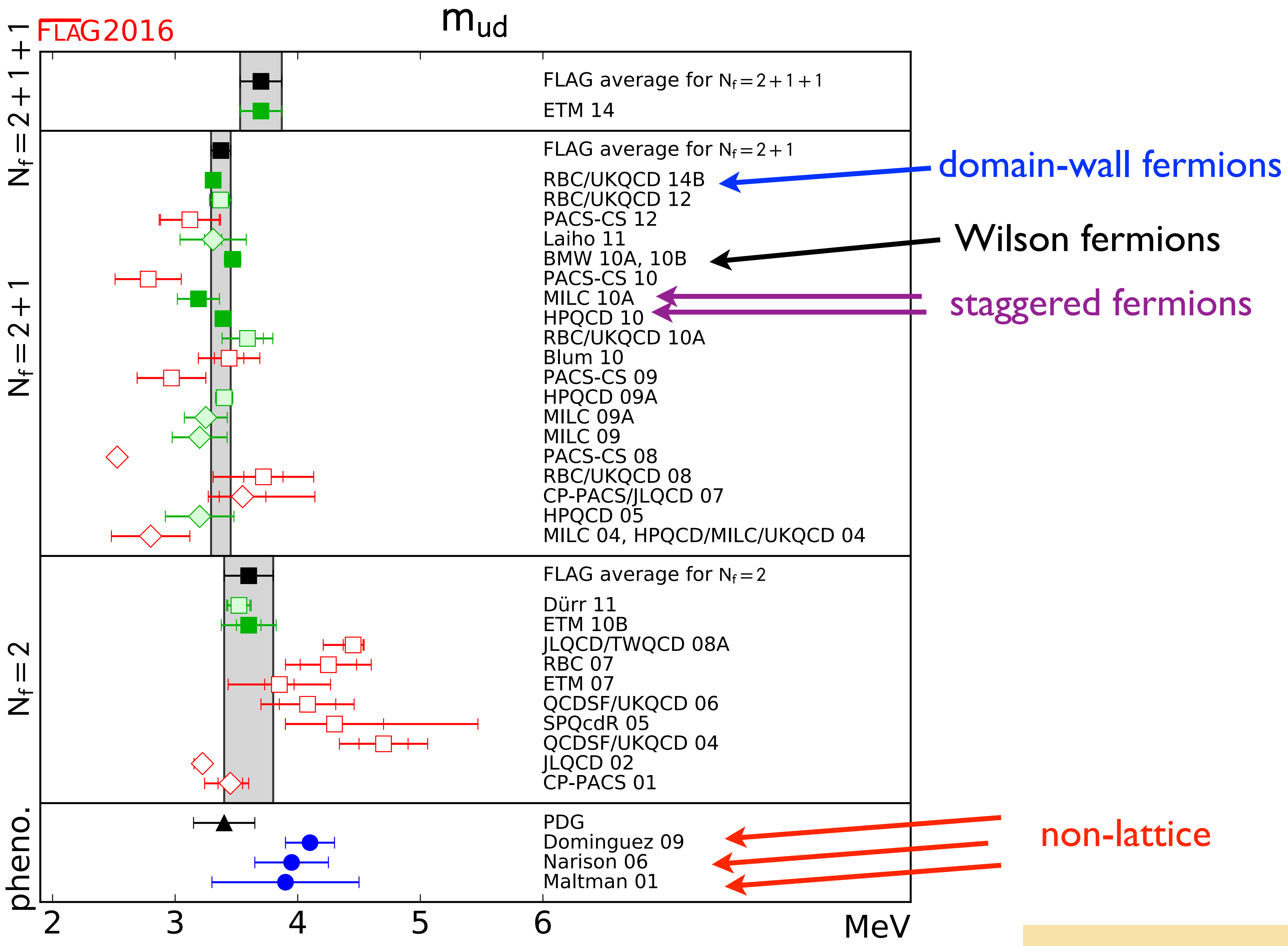
Flavour Lattice Averaging Group (FLAG)

S. Aoki[1], Y. Aoki[2,3,17], D. Bečirević[4], C. Bernard[5], T. Blum[3,6], G. Colangelo[7], M. Della Morte[8,9], P. Dimopoulos[10,11], S. Dürr[12,13], H. Fukaya[14], M. Golterman[15], Steven Gottlieb[16], S. Hashimoto[17,18], U. M. Heller[19], R. Horsley[20], A. Jüttner[21,a], T. Kaneko[17,18], L. Lellouch[22], H. Leutwyler[7], C.-J. D. Lin[22,23], V. Lubicz[24,25], E. Lunghi[16], R. Mawhinney[26], T. Onogi[14], C. Pena[27], C. T. Sachrajda[21], S. R. Sharpe[28], S. Simula[25], R. Sommer[29], A. Vladikas[30], U. Wenger[7], H. Wittig[31]

- Reviews every ~3 years: provide "vetted" averages
- "PDG or HFLAV for Lattice QCD"

Quark masses

$(m_u + m_d)/2 = 3.37(8)$ MeV [N_f=2+1]



domain-wall fermions

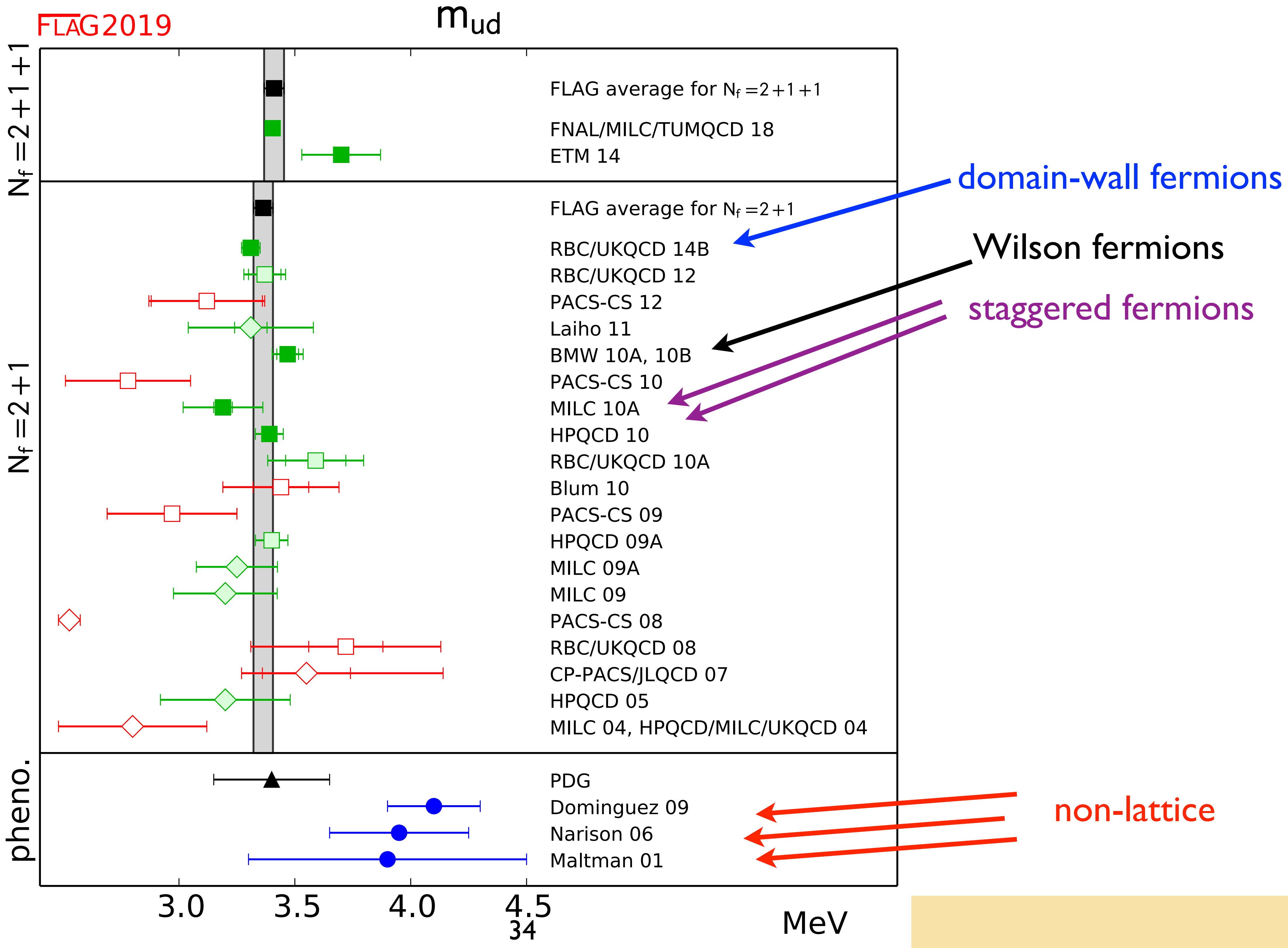
Wilson fermions

staggered fermions

non-lattice

Quark masses

$(m_u + m_d)/2 = 3.36(4)$ MeV $[N_f=2+1]$



domain-wall fermions

Wilson fermions

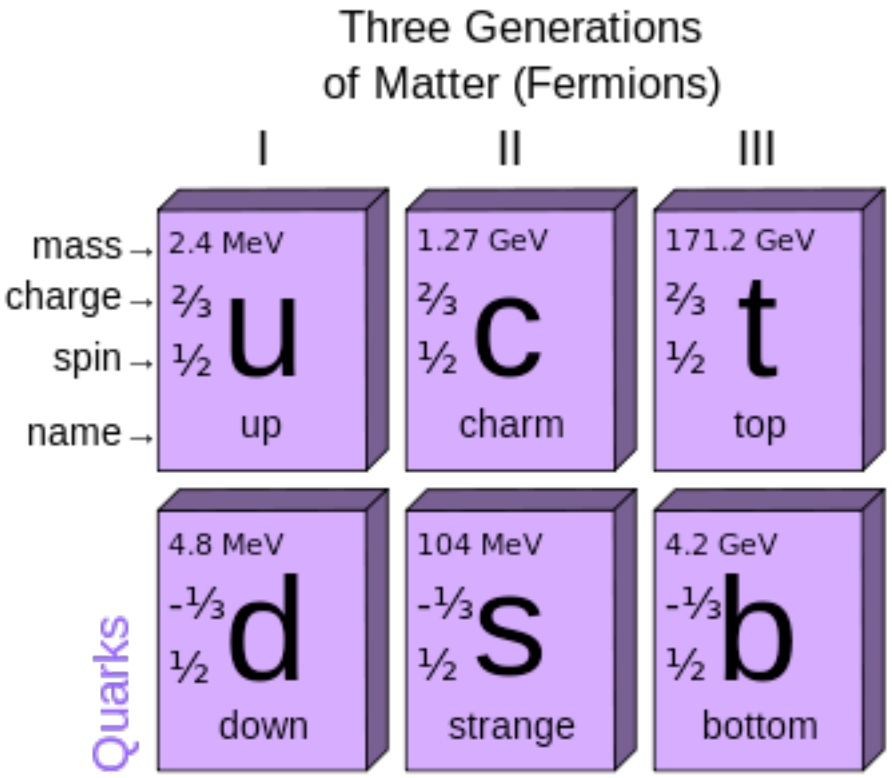
staggered fermions

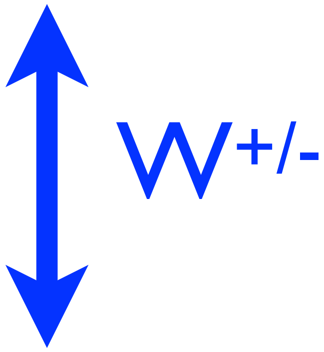
non-lattice

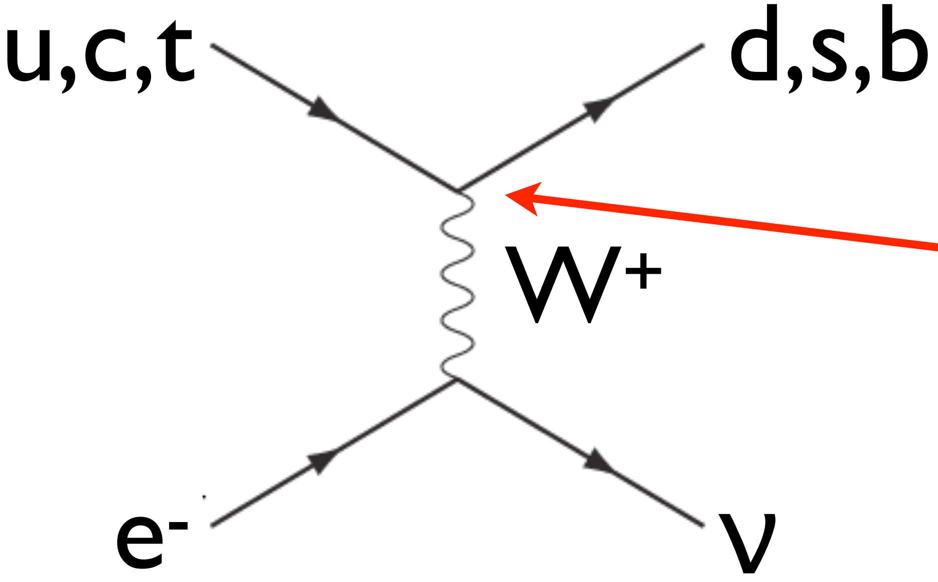
Outline

- Standard model & searching for physics beyond
- QCD & Lattice QCD
- High precision lattice QCD
- **Constraining the Standard Model with LQCD**
- Extending the LQCD frontier
- Future outlook

CKM matrix & CP violation







$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Fundamental parameters of the SM

CKM matrix & CP violation

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-\boxed{i\eta}) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-\boxed{i\eta}) & -A\lambda^2 & 1 \end{pmatrix}.$$

Unitary matrix

CP violation!

- Each element can be measured in several ways
- Consistency of SM requires all measurements to agree, and that V_{CKM} be unitary
- CP violating parameter η must explain observed CP violation in Kaon and B meson systems
- New physics would shows up as inconsistencies

Need for non-perturbative QCD

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ \hline \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \hline \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s & \\ K_0 \leftrightarrow \overline{K}_0 & K_0 \leftrightarrow \overline{K}_0 & \end{pmatrix}$$

$$\text{Experiment} = \text{known factors} \times V_{CKM} \times \text{Hadronic matrix element}$$

Perturbative SM

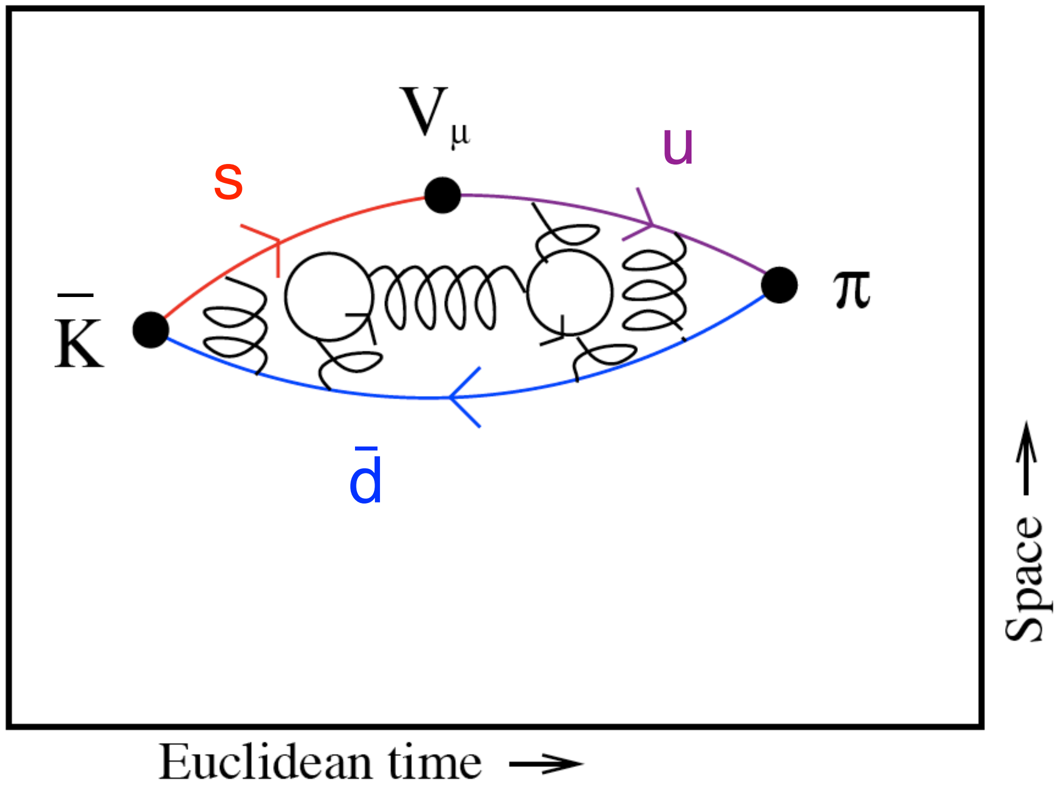
Lattice QCD

Measure

to determine this

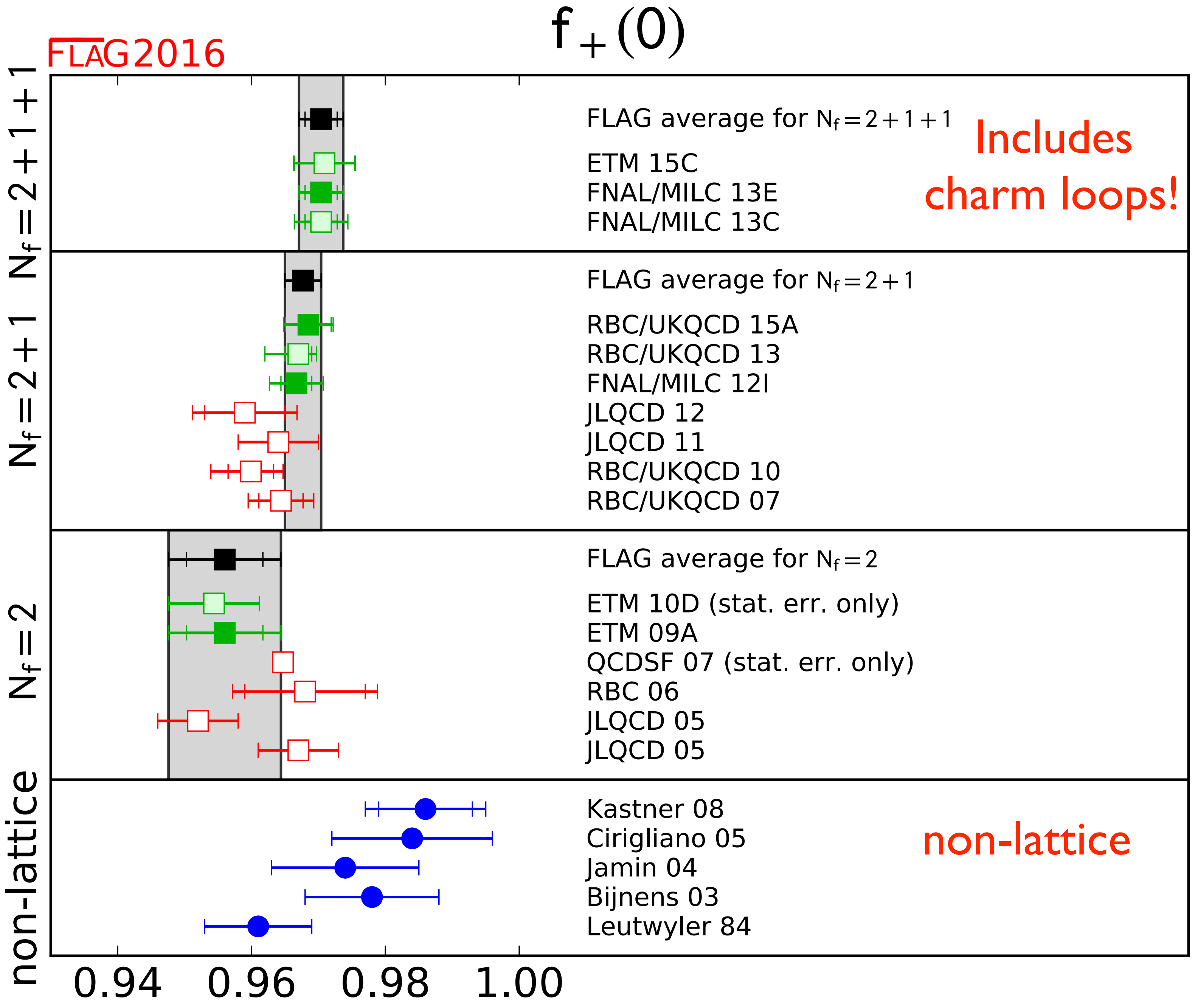
Example: K→π form factor

- Experimental measurement: $\overline{K}^0 \longrightarrow \pi^+ e^- \overline{\nu}$
- Underlying process: $s \longrightarrow u e^- \overline{\nu}$
- CKM element: V_{us}
- Required matrix element: $\langle \pi(p_2) | V_\mu | K(p_1) \rangle \sim f_+(0)$
- Lattice QCD calculation:



Results for K→π form factor

$f_+(0) = 0.9704(33)$ [N_f=2+1+1]

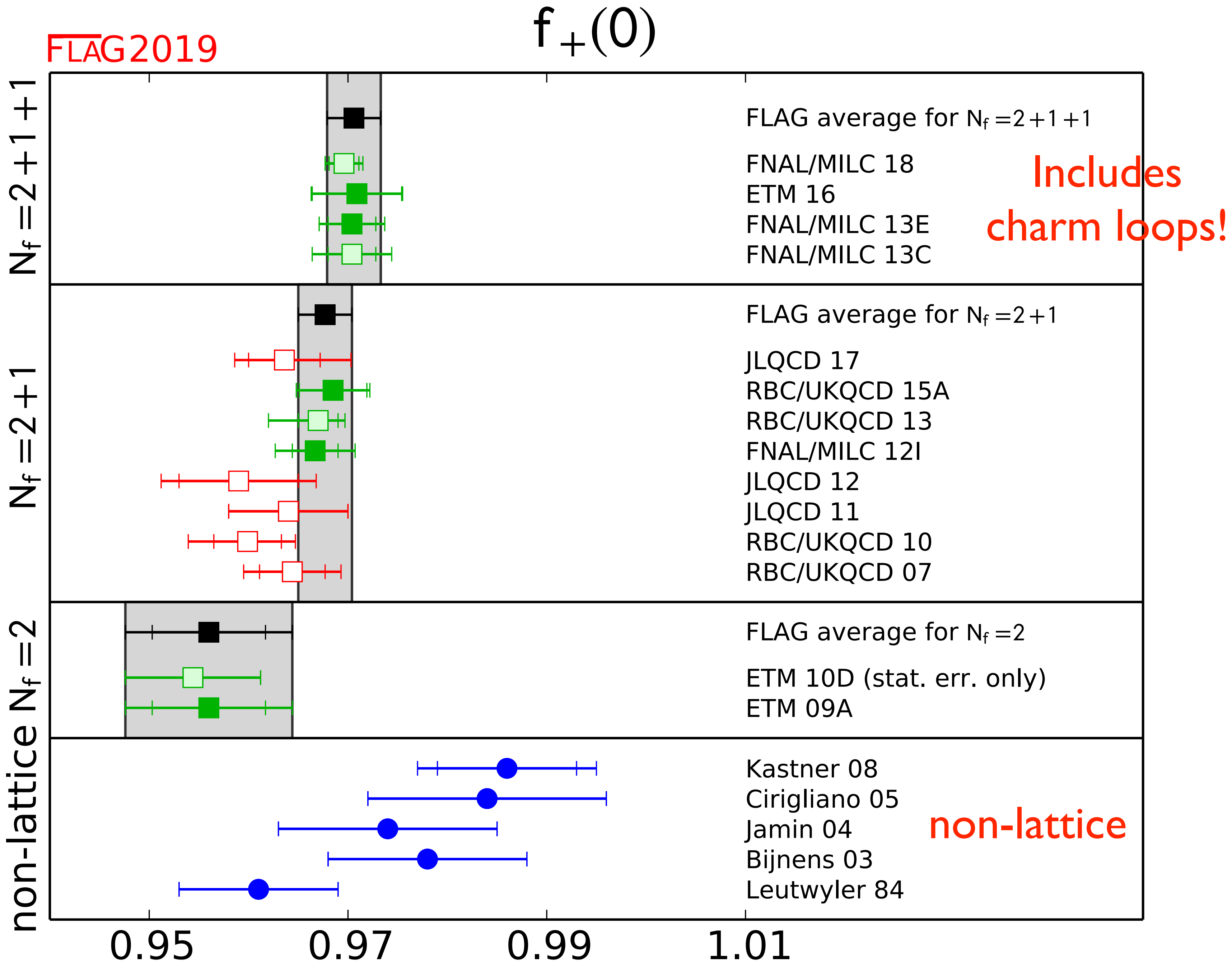


Includes charm loops!

non-lattice

Results for K→π form factor

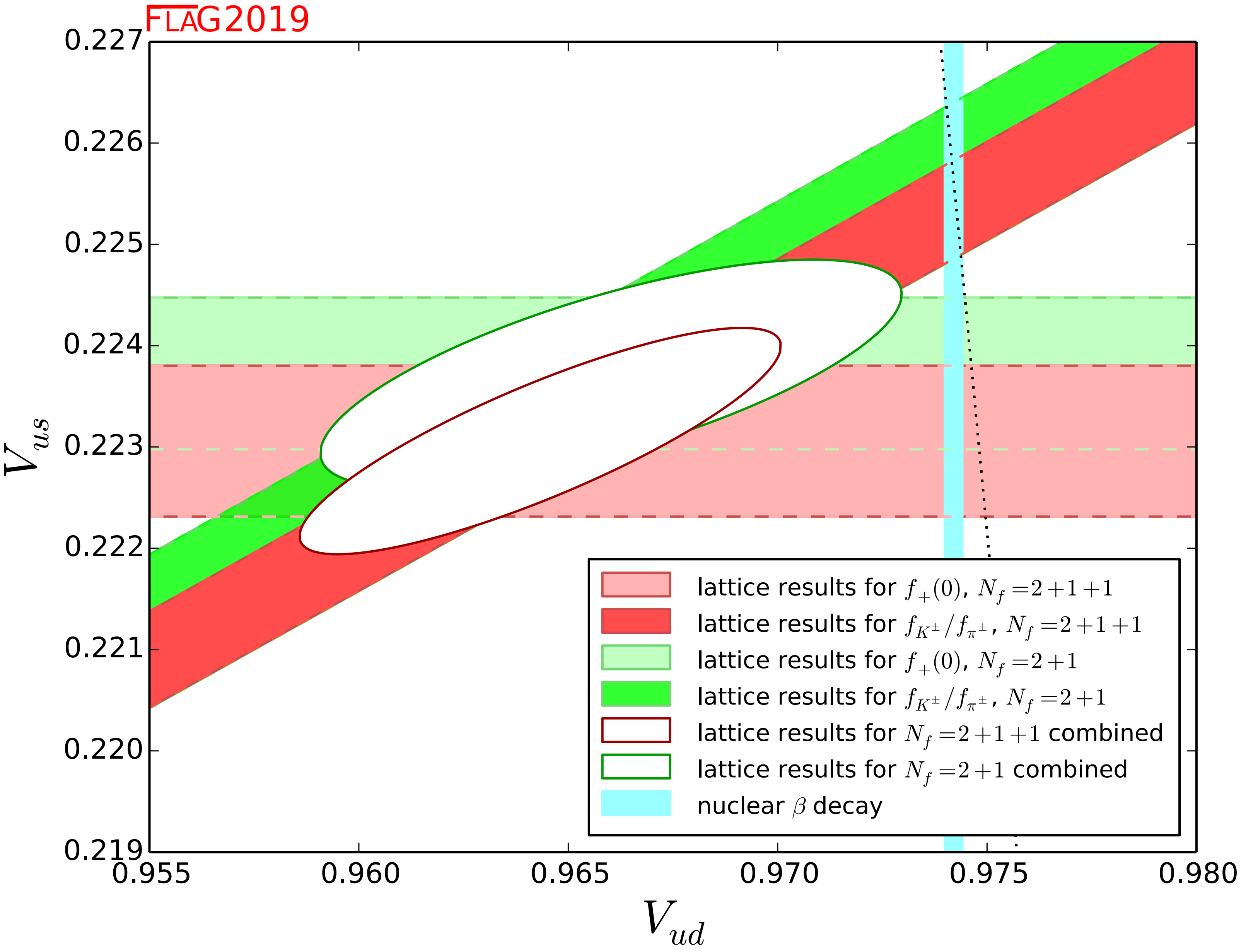
$$f_+(0) = 0.9706(27)\ [N_f=2+1+1]$$



Includes charm loops!

non-lattice

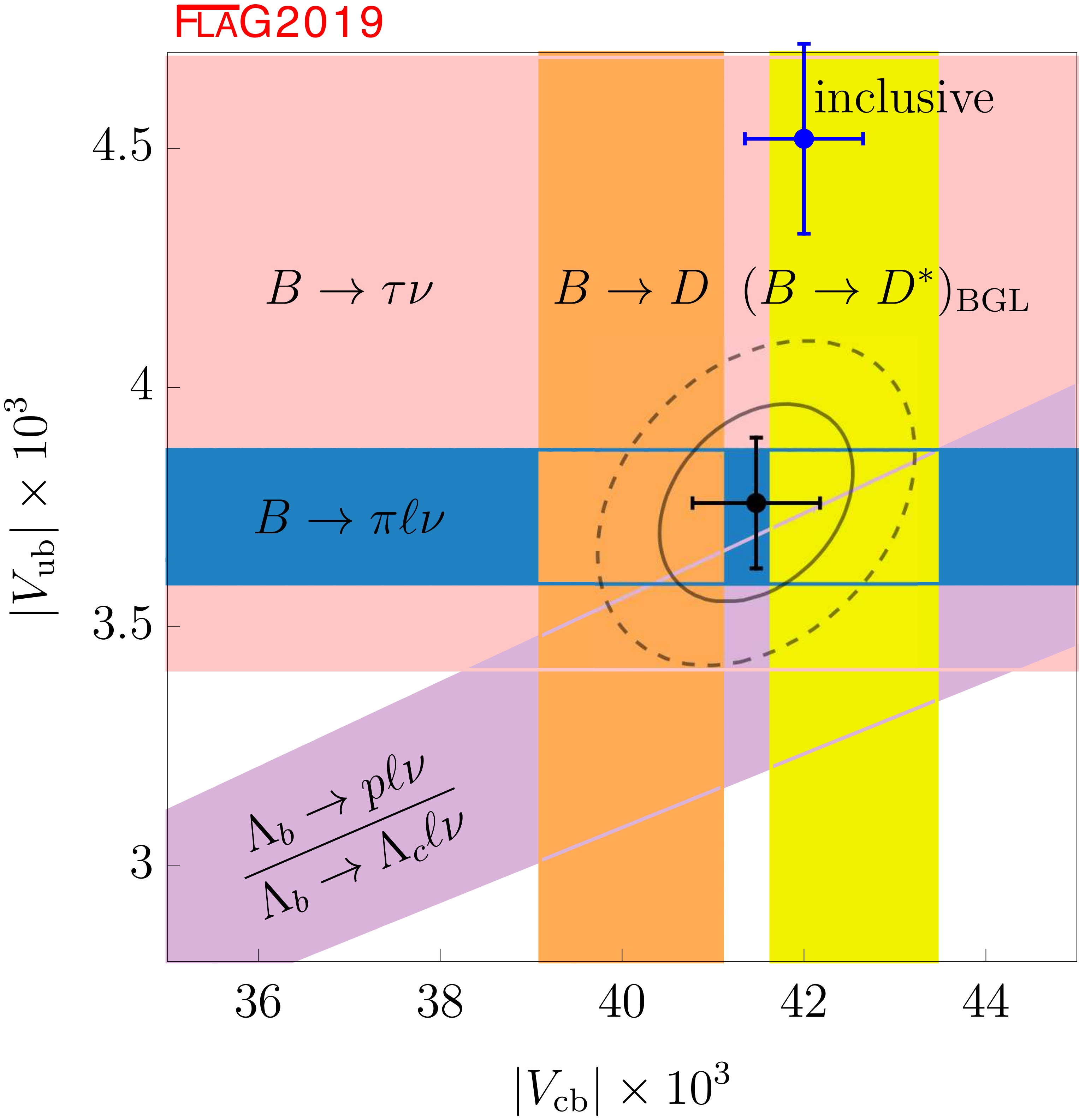
Results for V_{ud} & V_{us}



Values determined using f_K / f_π & $f_+(0)$

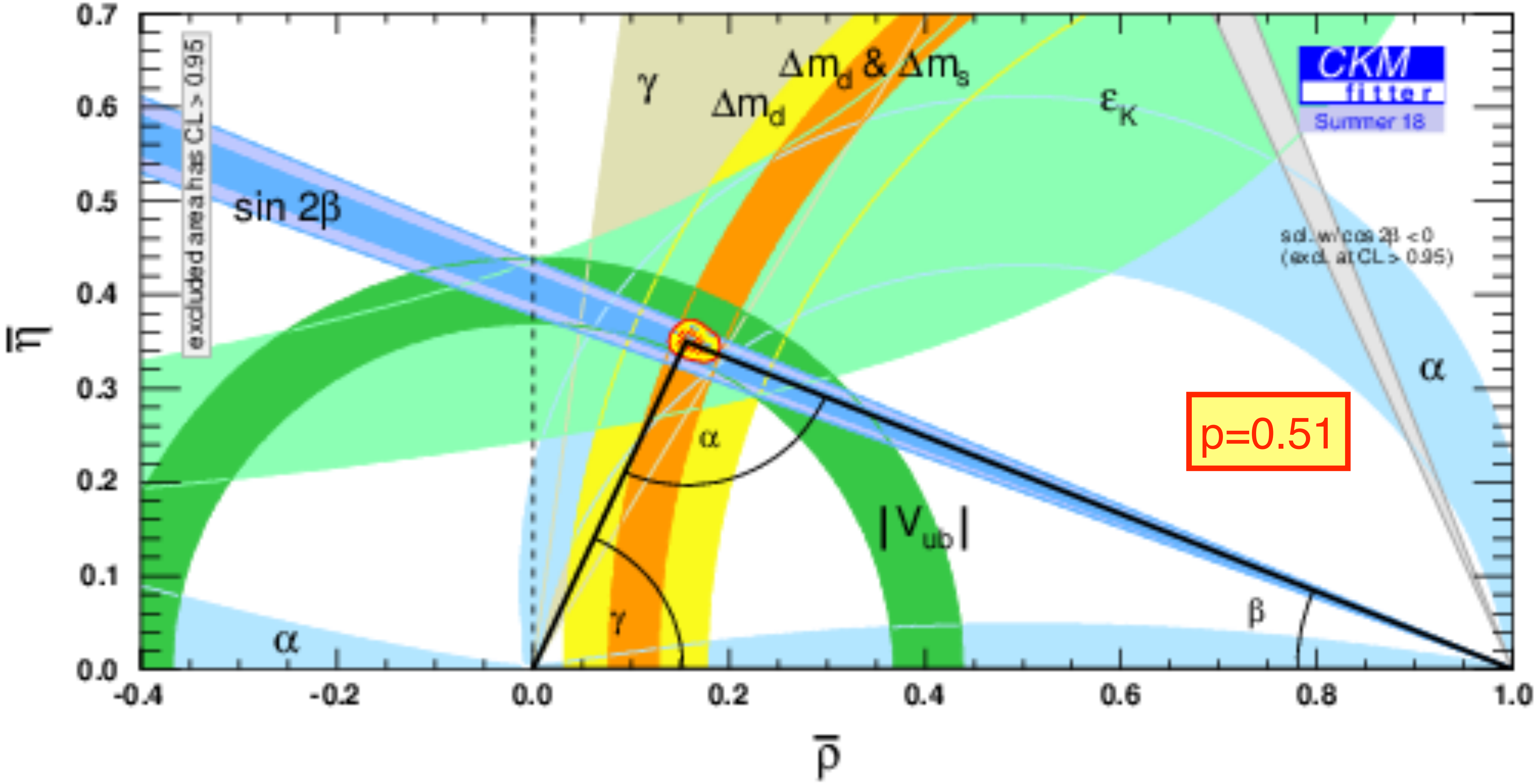
Consistent with unitarity: $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9797(74)$

Some tensions

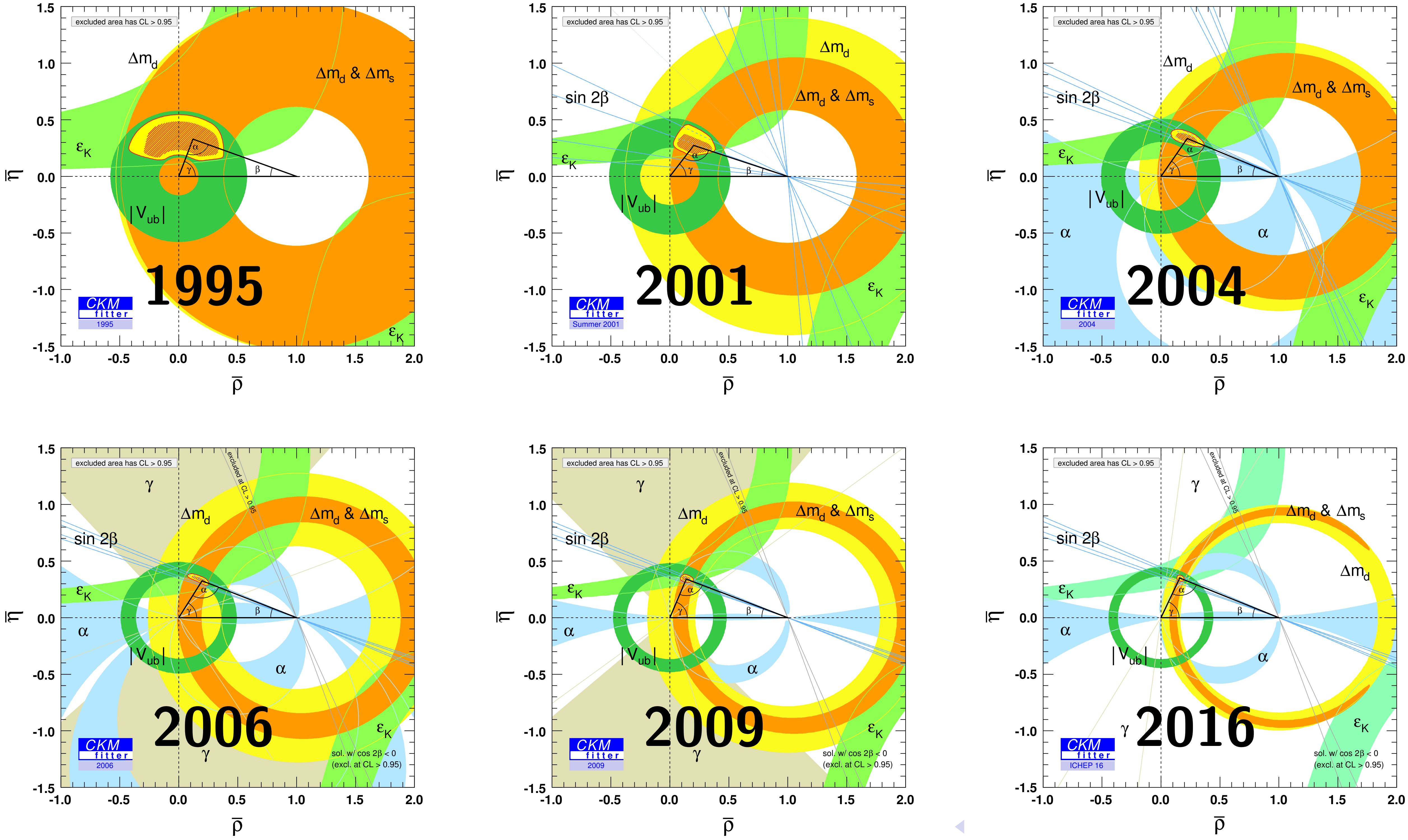


…but overall consistency

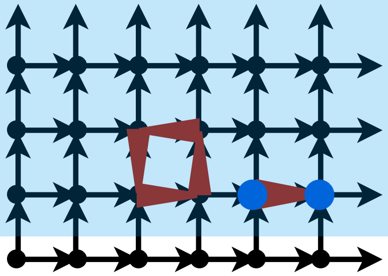
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$



History of steady progress



CKMfitter (ckmfitter.in2p3.fr): talk by Luiz Silva, 9/2018

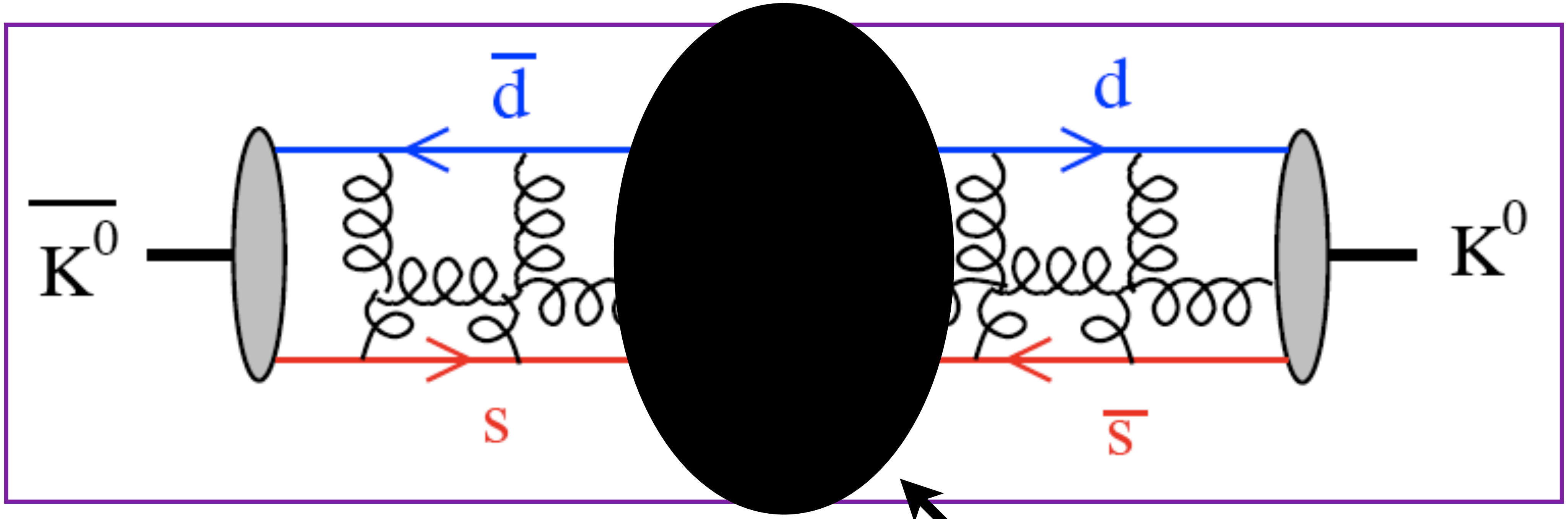


Outline

- Standard model & searching for physics beyond
- QCD & Lattice QCD
- High precision lattice QCD
- Constraining the Standard Model with LQCD
- Extending the LQCD frontier
- Future outlook

“Gold plated”

- Processes involving single hadrons are (by now) straightforward to calculate using LQCD
  - Hadron masses, decay constants, form factors, quark masses, α_s, K, D & B mixing matrix elements
  - E.g. $\hat{B}_K = 0.76 \pm 0.01$ [FLAG16]



space

Euclidean time

Known local four-fermion operator

Beyond "gold plated"

- Processes involving two hadrons are now beginning to be accessible
  - E.g. K→ππ decays
  - Does the SM reproduce the ΔI=1/2 rule?

$$\frac{\Gamma(K_S^0 \to \pi\pi)}{\Gamma(K^+ \to \pi\pi)} \approx 330$$

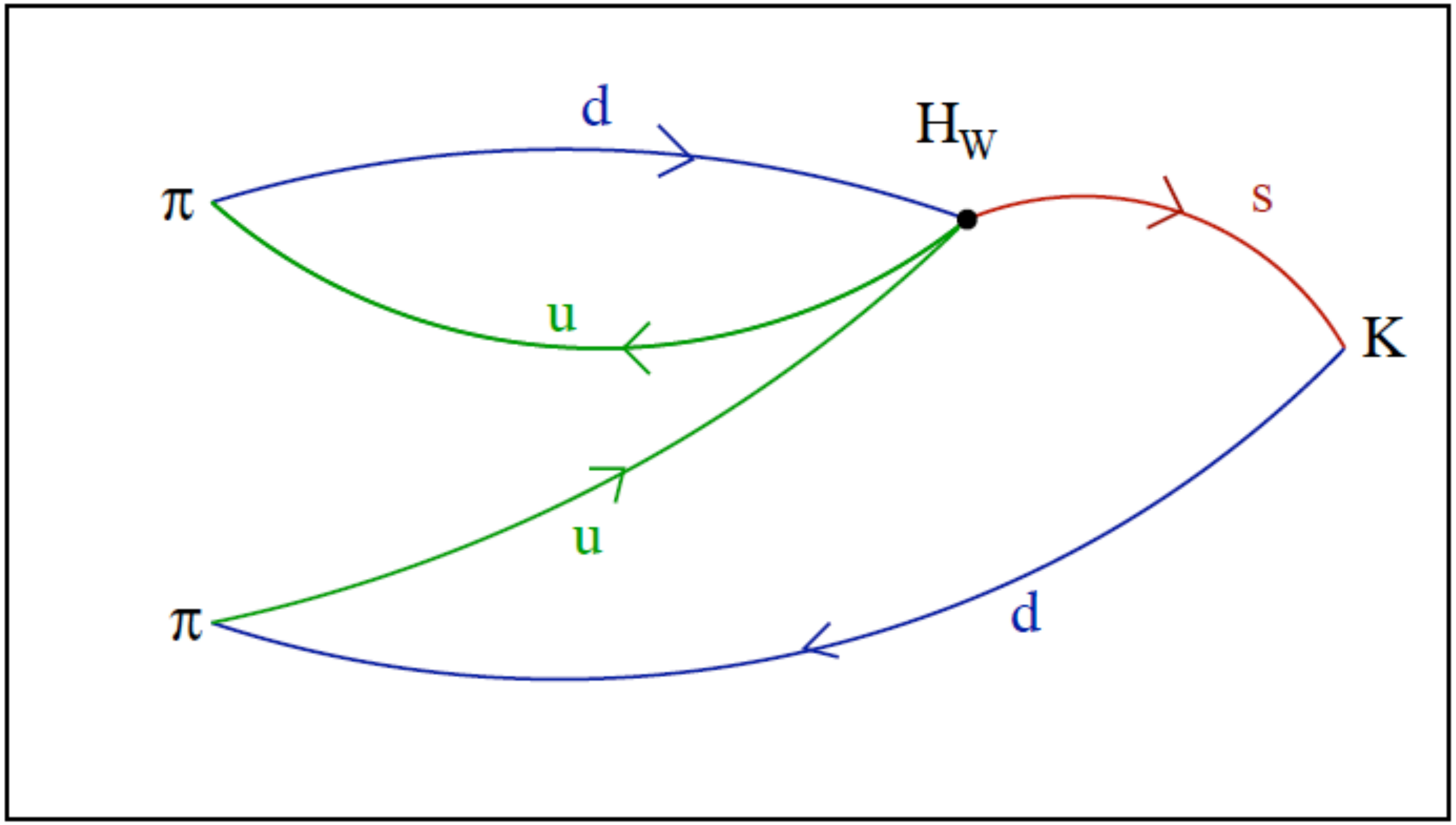
  - Does the SM reproduce direct CP-violation in K→ππ?

$$\frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)}\frac{\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^+\pi^-)} \approx 1 - 6\mathrm{Re}(\epsilon'/\epsilon)$$

$\varepsilon'/\varepsilon = 1.63(0.26)\ 10^{-3}$

K→ππ amplitude

gluons & sea-quark loops now implicit



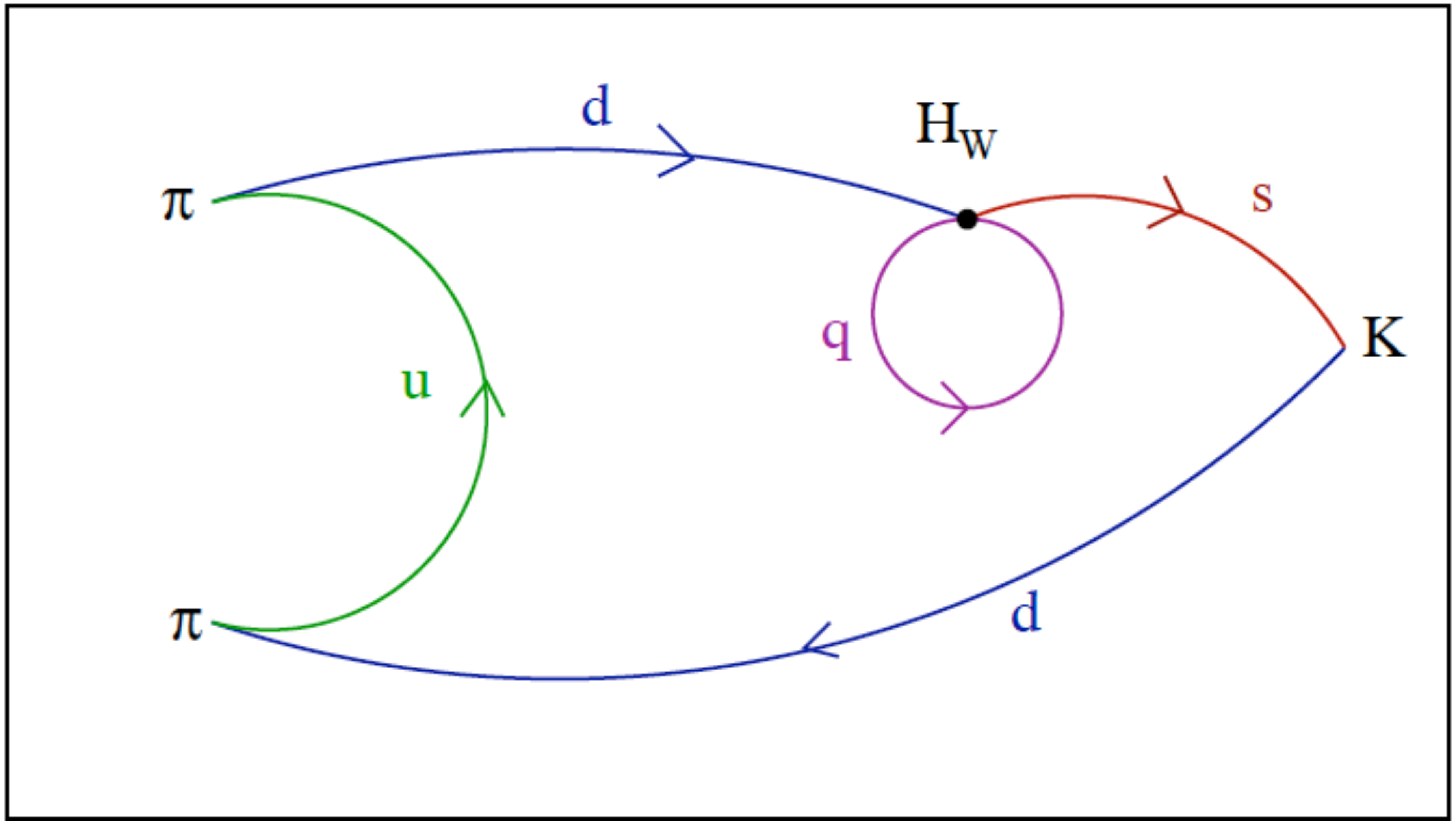
space

Euclidean time

Many Wick contractions—some with poor signal/noise

K→ππ amplitude

gluons & sea-quark loops now implicit



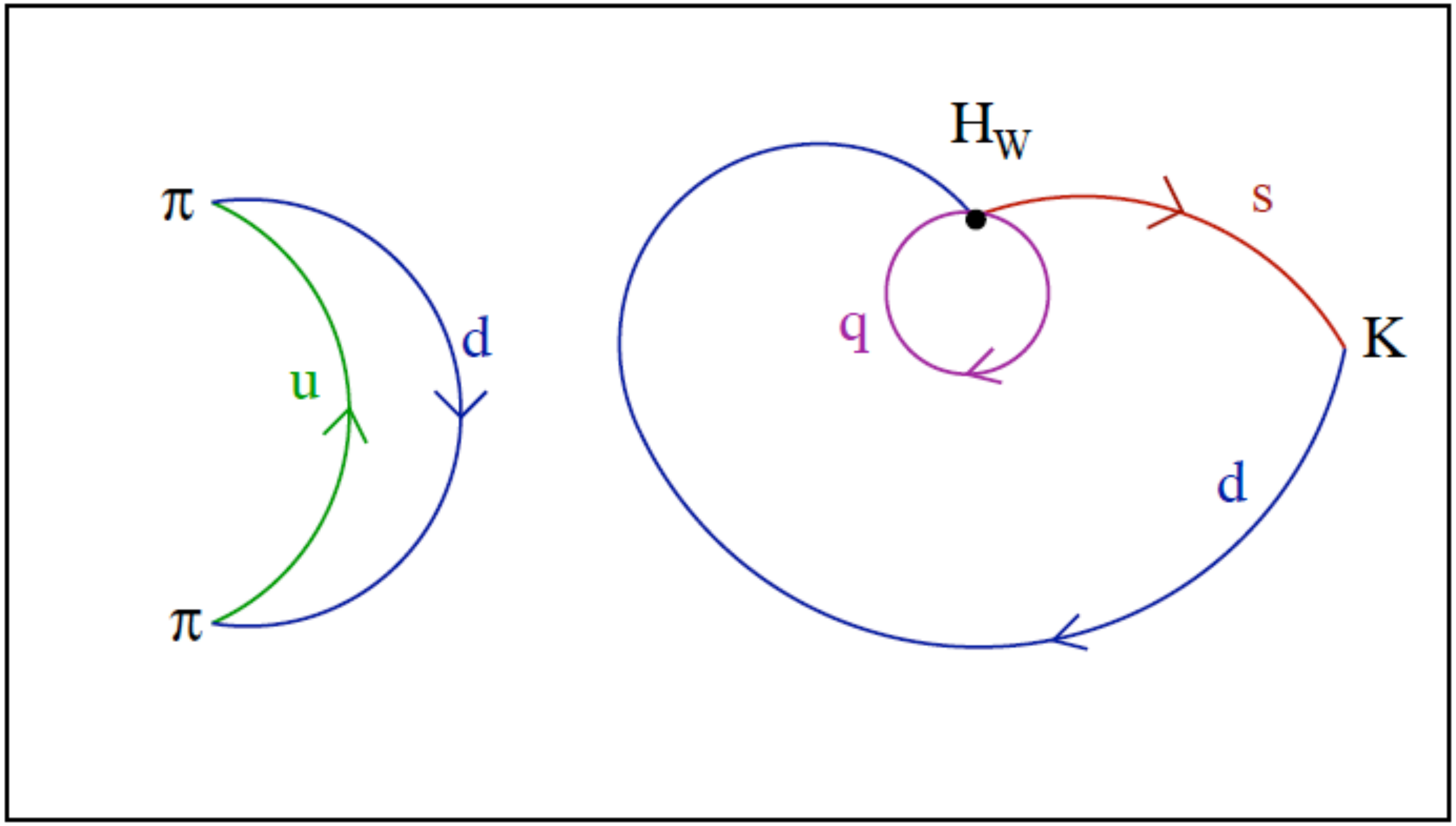
space

Euclidean time

Many Wick contractions—some with poor signal/noise

K→ππ amplitude

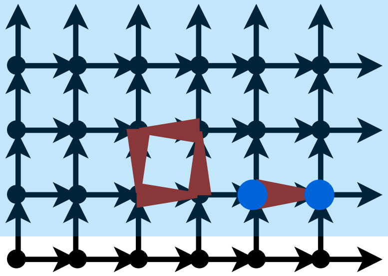
gluons & sea-quark loops now implicit



space

Euclidean time

Many Wick contractions—some with poor signal/noise



Calculational challenges

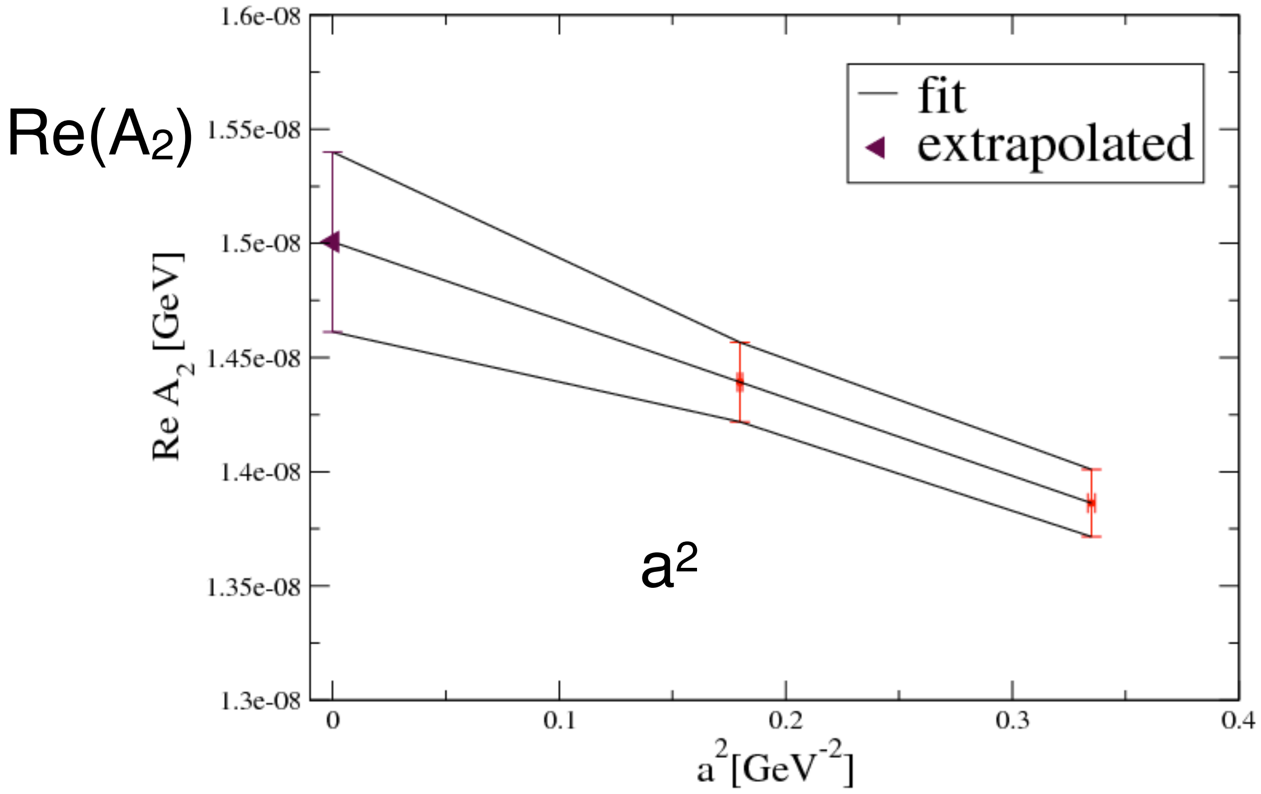
- Quark-disconnected Wick contractions
    - Solved using new noise-reduction methods
- Breaking of chiral symmetry by lattice action
    - Solved using Domain-wall fermions
- Connecting finite-volume matrix element to measured infinite-volume one

$$ {}_L\langle \pi\pi | \mathcal{H}_W | K \rangle_L \xrightarrow{?} \langle \pi\pi | \mathcal{H}_W | K \rangle $$

- Solved by large box (~6 fm) & using QFT to relate finite & infinite volume two-pion states [Lüscher, …]

Pioneering K→ππ results

- Amplitude for $K^+ \to \pi\pi$ (isospin 2 final state) <u>at physical quark masses</u> in fully controlled calculation



[RBC/UKQCD 1502.00263]

- Result: $\mathrm{Re}(A_2)$ = 1.50 (15) GeV
- Consistent with experiment! $\mathrm{Re}(A_2)=1.479(3)\ 10^{-8}$ GeV (K^+ decays)

Pioneering $K\to\pi\pi$ results

- Amplitude for isospin 0 $\pi\pi$ final state at physical point (but so far with only a single lattice spacing $a\approx0.15$fm)

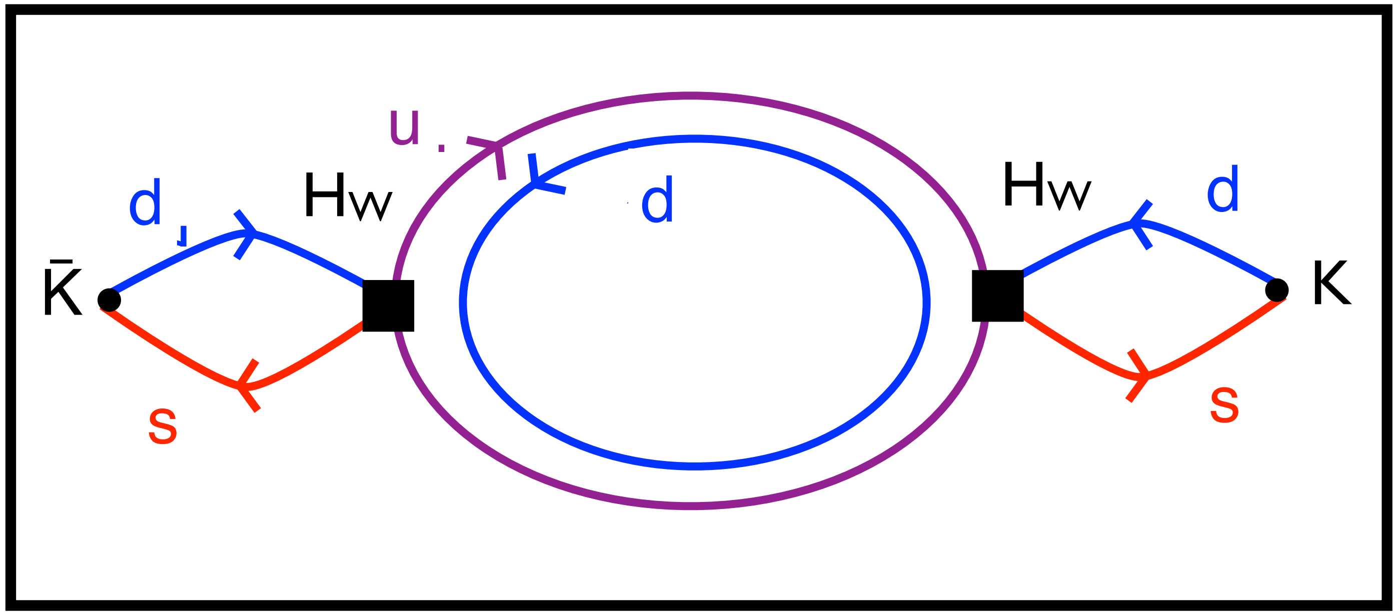
  [RBC/UKQCD 1505.07863]

  - Result (without all errors controlled): $\mathrm{Re}(A_0) = 4.7(1.6)\ 10^{-7}$ GeV
  - Consistent with experiment: $\mathrm{Re}(A_2)=3.3(2)\ 10^{-7}$ GeV
  - Reproduces the $\Delta I=½$ rule from first principles!

- Fully controlled results in next few years

Future K→ππ results

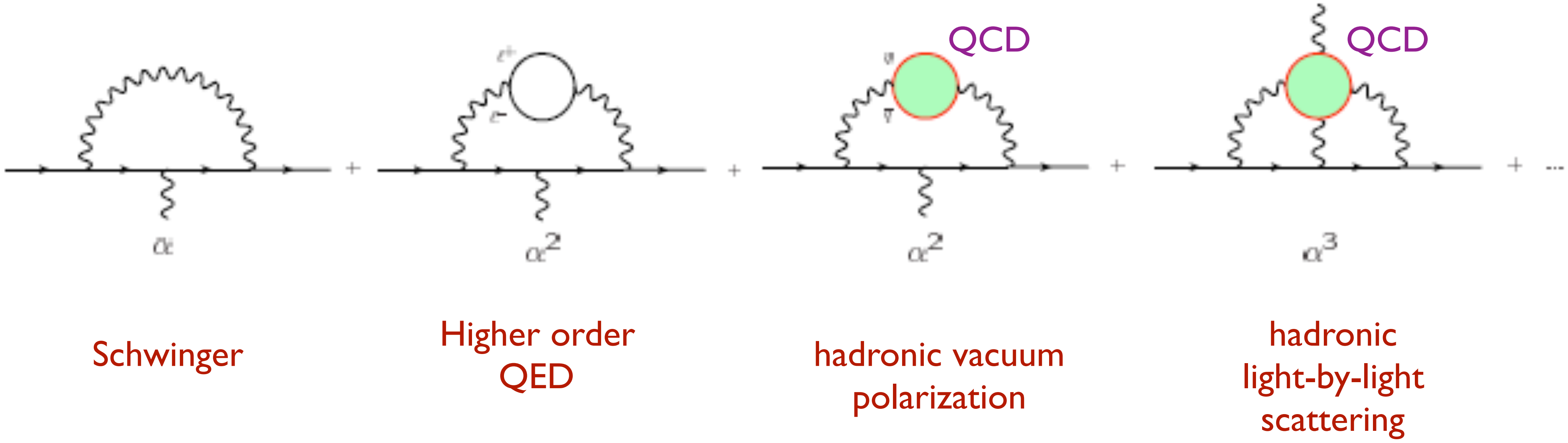
- CP violation in K→ππ
  - First result obtained: $\varepsilon'/\varepsilon = 0.1(7)\ 10^{-3}$ [RBC/UKQCD 1505.07863]
  - Will know in a few years if SM explains $\varepsilon'/\varepsilon = 1.63(26)\ 10^{-3}$
- Calculation of K_L—K_S mass difference ΔM_K
  - Method developed and tested: fully controlled result in 3-5 years?



Muon g-2

- Magnetic moment of muon is proportional to its spin

$$\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{S} \qquad g = 2 + \frac{a_\mu}{2} \qquad a_\mu = \frac{\alpha_{EM}}{2\pi} + \ldots$$



- Dominant theory error is from QCD!

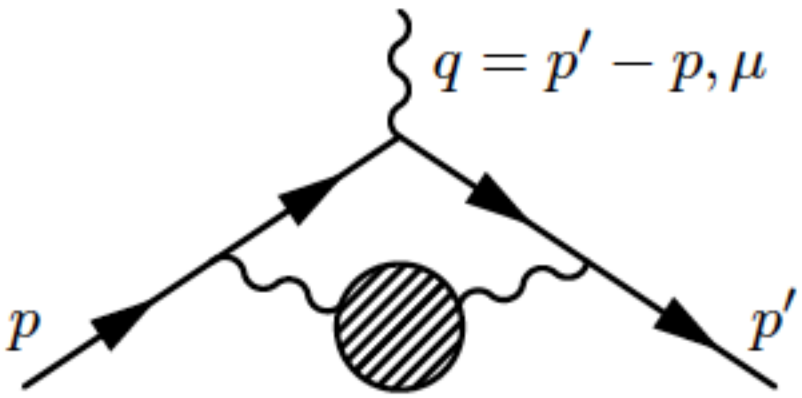
Muon g-2

Dominant theory errors

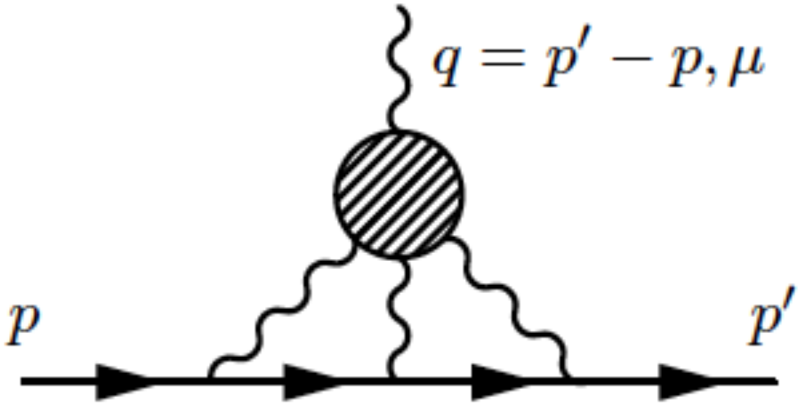
	Value ± Error	Reference
Experiment (0.54 ppm)	116592089 ± 63	E821, The $g-2$ Collab. 2006
Standard Model	116591828 ± 50	arXiv:1311.2198
Difference (Exp – SM)	261 ± 78	
HVP LO	6949 ± 43	Hagiwara et al. 2011
Hadronic Light by Light	105 ± 26	Glasgow Consensus, 2007

Table 1. Standard model theory and experiment comparison [in units 10^{-11}]

Educated guess!



Hadronic Vacuum Polarization (HVP)



Hadronic Light by Light (HLbL)

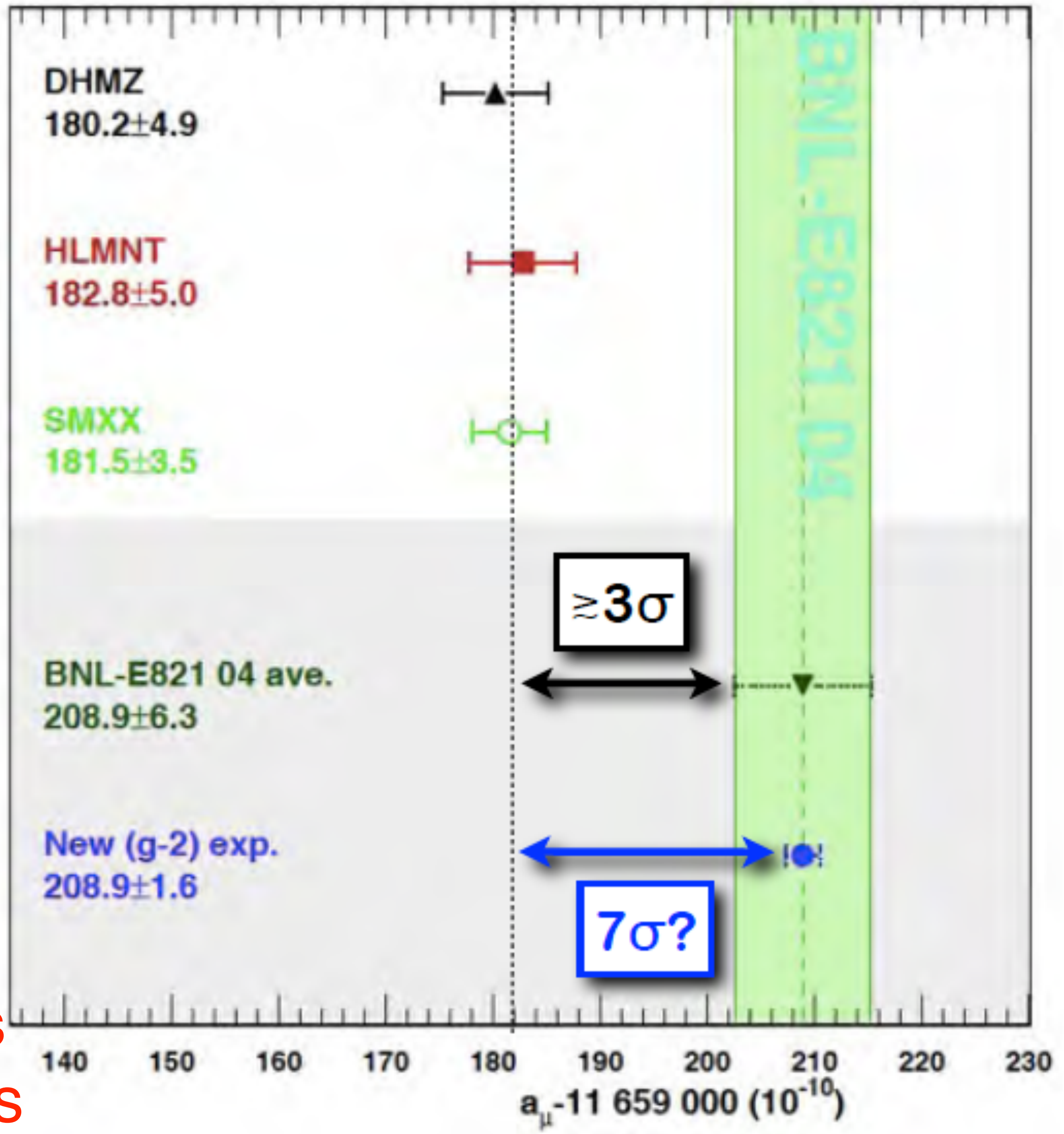
Muon g-2

Estimates for Standard Model prediction

In tension with present experiment

New experiment can discover new physics!

But to do so requires precise LQCD results

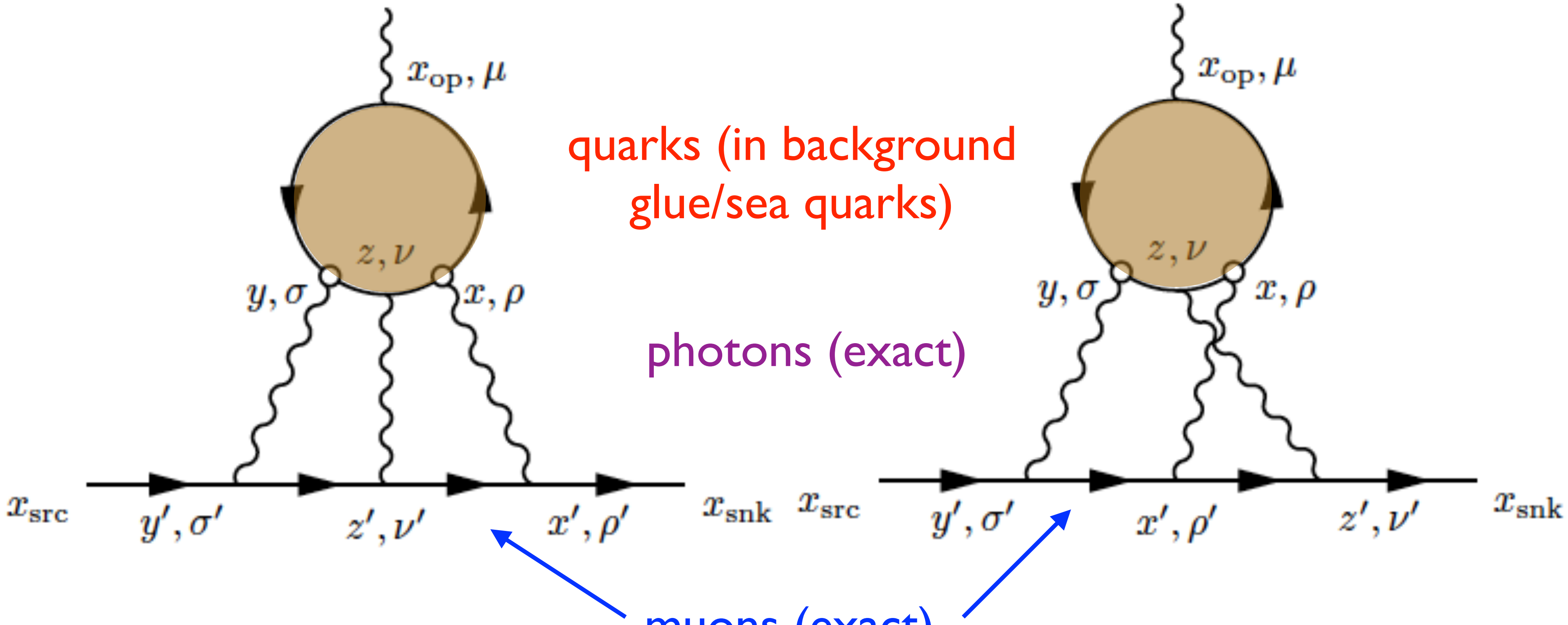


[Blum *et al.*,arXiv:1311.2198]

Hadronic light-by-light from LQCD

- Lattice result from direct calculation at physical masses in few yrs

[RBC Collab., Blum et al. and other groups]

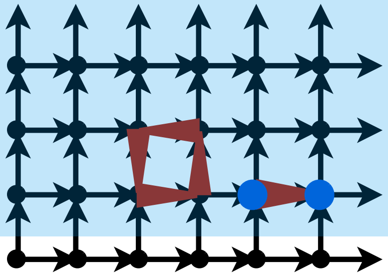


quarks (in background glue/sea quarks)

photons (exact)

muons (exact)

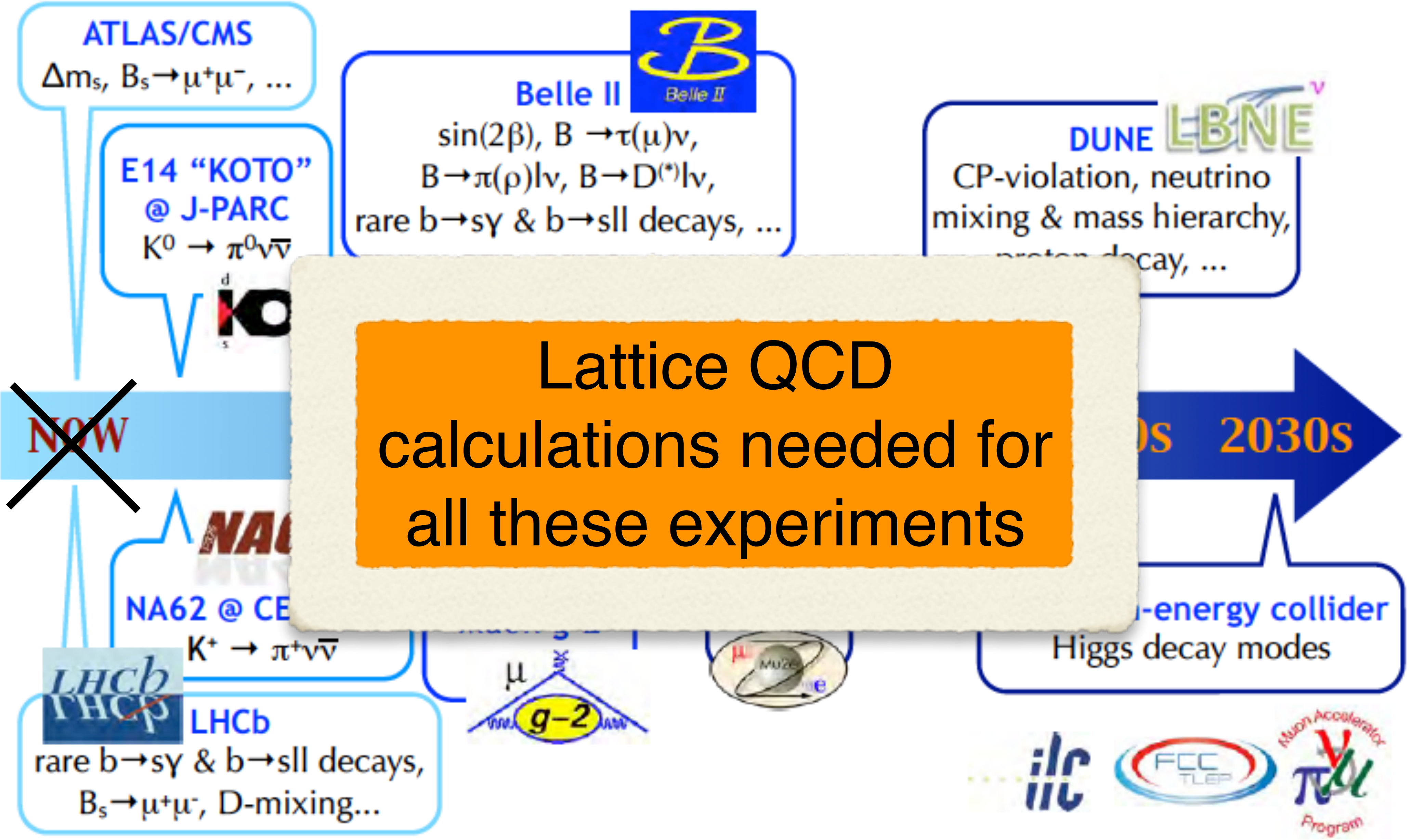
- Methods now have 20% statistical errors; systematics under study



Outline

- Standard model & searching for physics beyond
- QCD & Lattice QCD
- High precision lattice QCD
- Constraining the Standard Model with LQCD
- Extending the LQCD frontier
- Future outlook

Experimental frontier



Lattice QCD calculations needed for all these experiments

Ruth Van de Water

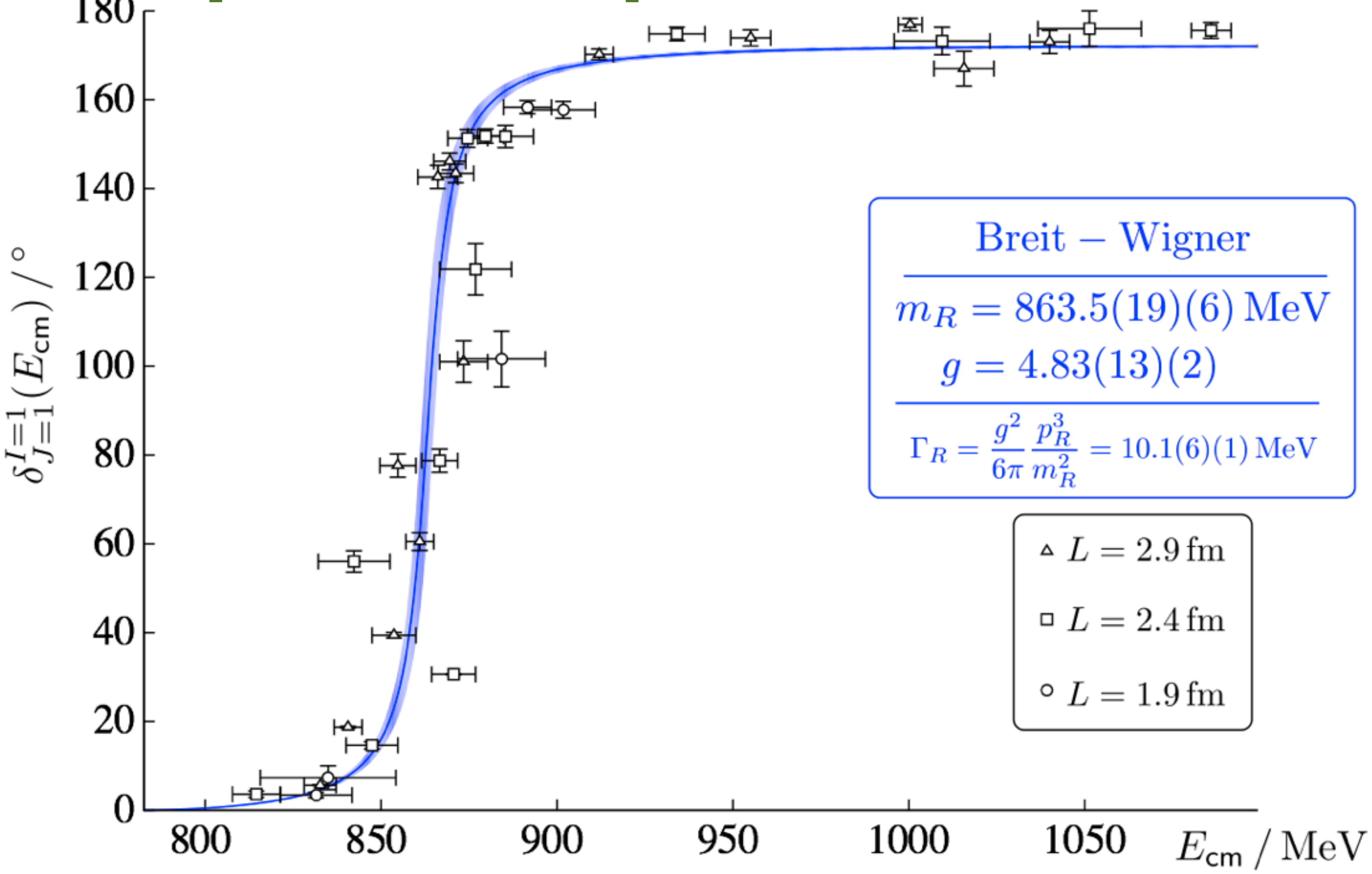
Coming in the near future

- 3 or more particles, e.g. K→πππ
- Resonance properties from Lattice QCD

ρ resonance in ππ phase shift

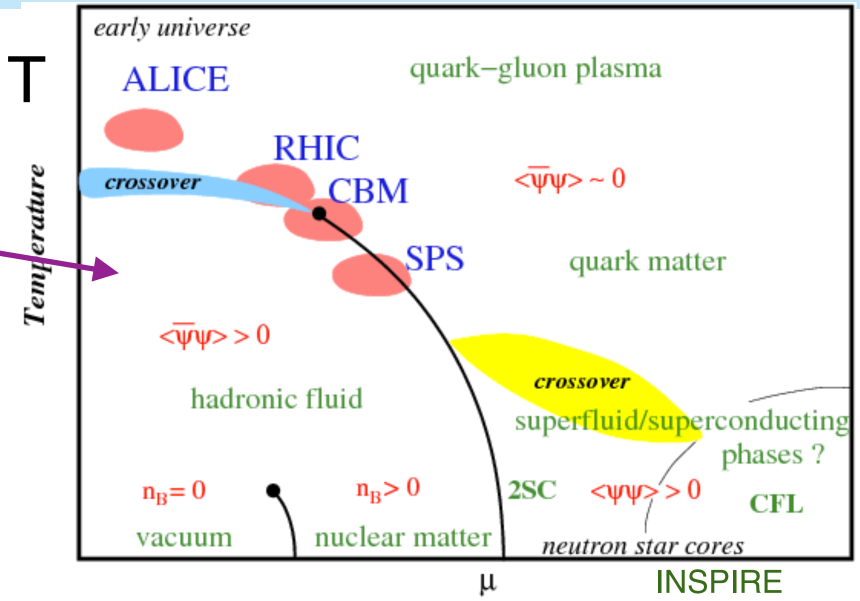
$$m_\pi = 391\,\text{MeV}$$

[Dudek et al., 2013]

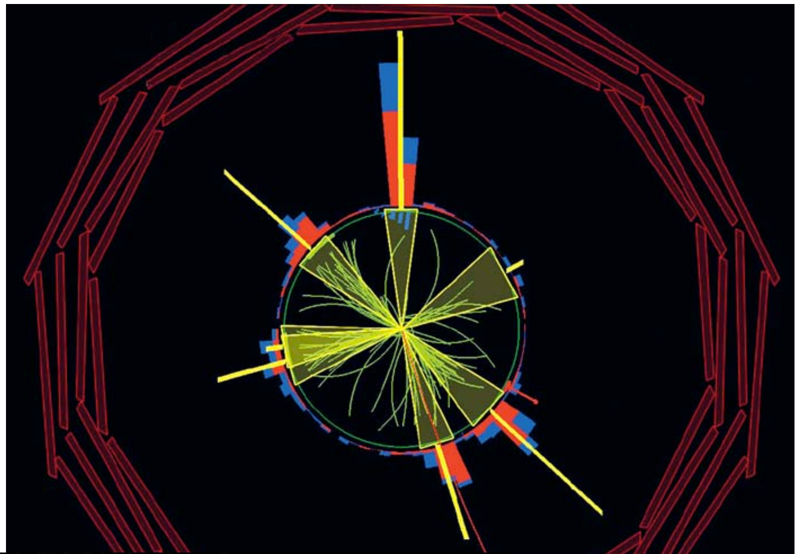


Open problems

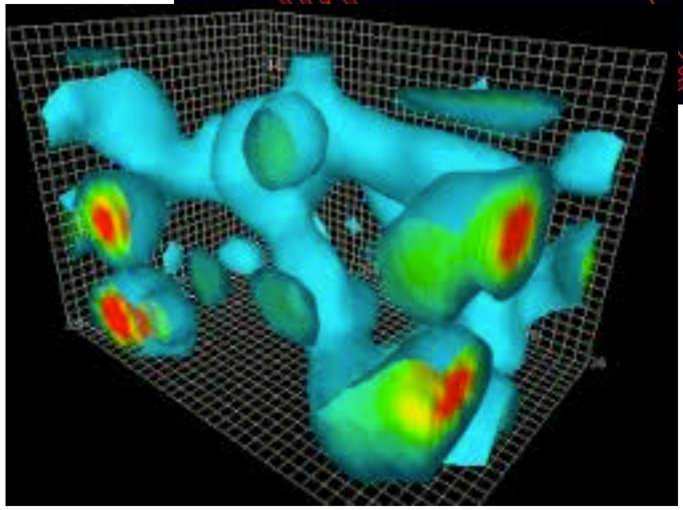
- Non-zero density (e.g. center of neutron stars)
- Real-time processes (e.g hadronization of jets)
- Decays with many open channels, e.g. B→ππ
- Qualitative understanding of confinement/vacuum
- Lattice formulation of chiral gauge theories



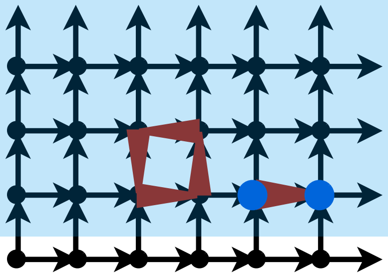
INSPIRE



Cern Courier



physics.adelaide.edu.au



Thank you!
Questions?