Critical Temperature for Deconfinement in the (2+1)-d Georgi-Glashow Model

Joaquín E. Drut
University of Washington

In collaboration with Amy Nicholson.
Outline

- Definition of the theory
- Motivation
- The simple case: SU(2)
- What happens for SU(N)?
- Summary
The theory:
Pure gauge SU(N) + adjoint scalars

◆ Lagrangian

\[ \mathcal{L} = \frac{1}{2} (D_\mu \phi^a)^2 + \lambda (\phi^a \phi^a - v^2)^2 + \frac{1}{4} F_{\mu \nu}^a F_{\mu \nu}^a \]

Adjoint scalars (N²-1)

\[ \phi^a T^a \in ASU(N) \]

Covariant derivative

\[ D^a_\mu = \partial_\mu + ad(A_\mu)^a \]

Field strength tensor

\[ F_{\mu \nu} = F_{\mu \nu}^a T^a = [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \]
The theory: Pure gauge SU(N) + adjoint scalars

- Lagrangian

\[ \mathcal{L} = \frac{1}{2} (D_\mu \phi^a)^2 + \lambda (\phi^a \phi^a - v^2)^2 + \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} \]

- Perturbatively...

<table>
<thead>
<tr>
<th>SU(2)</th>
<th>SU(N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>massive bosons</td>
<td>$W^\pm = A^1 \pm iA^2$</td>
</tr>
<tr>
<td>massless photon</td>
<td>$A^3$</td>
</tr>
</tbody>
</table>

In the Unitary gauge: \[ \phi^a = \delta^{a3}(v + \sigma(x)) \]

- Weak coupling...

\[ m_W \sim m_H \gg g^2 \]
Motivation: why is this interesting?

- These theories have topologically non-trivial classical solutions of finite action (a.k.a. solitons/instantons).

- In (2+1)-d, instantons provide a mechanism for confinement of charge, and this happens at weak coupling.

  - One can prove, e.g.:
    - Confinement
    - $T_c$ for Deconfinement

  ... analytically!
Confinement in SU(2)

I. Polyakov’s proof:

‘**Instantons generate a non-perturbative mass for the photon**’

Classical solutions (photon part)

\[
F_{\mu\nu}^3 = \frac{e}{4\pi} \epsilon_{\mu\nu\lambda} \frac{x_\lambda}{x^2} \quad e = \frac{4\pi}{g}
\]

Write the partition function in a saddle-point approximation as a sum over multi-instanton configurations.

→ Coulomb gas (of instantons) representation.
Confinement in SU(2)

Map Coulomb gas onto Sine-Gordon model.

\[ L_{\text{eff}} \propto (\partial_\mu \varphi)^2 - M^2 \cos \varphi \]

\[ S_0 = \frac{4\pi v}{g} \epsilon \left( \frac{m_H}{m_W} \right) \]

\[ M^2 \propto \frac{m_w^{7/2}}{g^3} e^{-S_0} \]

\[ m_\gamma \sim e^{-S_0/2} \]

Compute Wilson loop

\[ F(C) = e^{-E(R)T} = e^{-\gamma A} \]

\[ E(R) = \gamma R \]

\[ \gamma \propto g^2 M \]

Confinement!
Deconfinement in SU(2)

2. Deconfinement transition
(Dunne et al.; Kovchegov & Son).

Again map onto Sine-Gordon model...
(plus W bosons, in dimensional reduction)

\[
H = \int dx \left[ \frac{1}{2} (\partial_x \Phi)^2 + \frac{1}{2} (\partial_x \Theta)^2 + 2\zeta_0 \cos \beta \Phi + 2\tilde{\zeta}_0 \cos \tilde{\beta} \Theta \right]
\]

\[
\zeta_0 \sim T^2 e^{-S_0}, \quad \tilde{\zeta}_0 \sim T^2 e^{-m_W/T}
\]

\[
\beta = e \sqrt{T}, \quad \tilde{\beta} = \frac{g}{\sqrt{T}}
\]

\[
S_0 = \frac{4\pi v}{g} e \left( \frac{m_H}{m_W} \right)
\]

\[
e = \frac{4\pi}{g}
\]
Deconfinement in SU(2)

Study RG flow...

\[ T_c = \frac{g^2 \cdot \epsilon + 2}{4\pi \cdot 2\epsilon + 1} \]

\[ S_0 = \frac{4\pi v}{g} \epsilon \left( \frac{m_H}{m_W} \right) \]

Universality class

\( \overline{\text{BKT}} \)

\( Z_2 \)

Critical exponents are as in the 2D Ising model
Numerical results for SU(2)

\[ S = \beta \sum_{x, \mu > \nu} \left( 1 - \frac{1}{2} \text{Tr} U_{\mu \nu}(x) \right) + 2 \sum_x \text{Tr} (\Phi(x) \Phi(x)) - 2\kappa \sum_{x, \mu} \text{Tr} (\Phi(x) U_{\mu}(x) \Phi(x + \mu \sigma) U_{\mu}^{\dagger}(x)) + \lambda \sum_x (2\text{Tr} (\Phi(x) \Phi(x)) - 1)^2 \]

\[ \bar{L}_F = \frac{1}{N_s^2} \sum_{\vec{x}} \left( \text{Tr}_F \left( \prod_{t=1}^{N_t} U_t (\vec{x}, t) \right) \right) \]

\[ \chi_{L_F} = N_s^2 \left( \langle \bar{L}_F \rangle^2 - \langle \bar{L}_F \rangle^2 \right) \]

\[ Z_2 \] Universality class confirmed by finite size scaling
What happens for SU(N)?

Mapping onto Sine-Gordon...

\[
\mathcal{H}_{\text{eff}} = \frac{1}{2} \left[ (\partial_x \Phi)^2 + (\partial_x \Theta)^2 \right] - \sum_{\alpha \in \Delta_+} \left[ \zeta_\alpha \cos \left( \frac{4 \pi \sqrt{T}}{g} \alpha \cdot \Phi \right) + \tilde{\zeta}_\alpha \cos \left( \frac{g}{\sqrt{T}} \alpha \cdot \Theta \right) \right]
\]

Nature of the phase transition \( \mathbb{Z}_N \)
(Svetitsky & Yaffe; Lecheminant)
What happens for SU(N)?

• Put the theory on the lattice.
• Study Polyakov loops and their correlations
• Do finite size scaling analysis
  Critical exponents?
• Quantum phase transitions?
  Dynamical exponents?
In the (2+1)-d SU(N) Georgi-Glashow model confinement of charge happens at weak coupling and can be proven analytically ('t Hooft & Polyakov).

The nature of the finite temperature deconfinement transition is believed to be understood for all N (Svetitsky & Yaffe; Kogan et al.; Lecheminant). Needs numerical check.

The critical temperature has only been computed analytically in the simplest case of SU(2) (Son & Kovchegov).

The critical temperature for SU(N) remains unknown and is hard to compute analytically.
To be continued...

Thanks!