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# Meson spectroscopy with derivative quark sources

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## Introduction

Lattice QCD has the possibility of providing a model independent *ab initio* calculation of the QCD mass spectrum. Extracting signals from excited states has however proven to be a formidable task. A previous study of the BGR-Collaboration [1] employed the variational technique [2, 3] to study excited mesons on the lattice using standard meson interpolators with Gaussian-smeared sources and sinks.

For an accurate description of excited states it is crucial to use a basis with good overlap with those states. We therefore construct additional meson-interpolators with derivative sources obtained by a covariant derivative acting on the Jacobi smeared sources and explore their effect in the variational approach.

For our analysis we used 90 (100 for pseudovectors) quenched gauge configurations with Chirally Improved (CI) quarks and the Lüscher-Weisz gauge action on a $16^3 \times 32$ lattice with $a = 0.148$ fm.

## Details of the calculation

### The variational method

We use a basis of interpolators $O_i$, $i = 1, \ldots, N$ (projected to zero momentum) and compute the matrix of cross correlations:

$$C(t)_{ij} = \left\langle O_i(t)\overline{O_j}(0)\right\rangle \ .$$

It can be shown [3] that the solutions to the generalized eigenvalue problem

$$C(t)\vec{v}_i = \lambda_i(t)C(t_0)\vec{v}_i \ ,$$

behave as

$$\lambda_i(t) \propto e^{(-tM_i)}\left(1 + \mathcal{O}\left(e^{(-t\Delta M_i)}\right)\right) \ ,$$

where $\Delta M_i$ is the mass difference between the state $i$ and the closest lying state.

For an accurate description, the interpolators should be linearly independent, as orthogonal as possible and possess a strong overlap with physical states.

### Smeared sources and sinks

The first step for all sources is Jacobi smearing [4, 5] of point sources $s_0$:

$$s_0^{(\alpha,a)}(\vec{y},0)_{\rho,c} = \delta(\vec{y},0)\delta_{\rho\alpha}\delta_{ca} \ ,$$
$$s^{(\alpha,a)} = \sum_{n=0}^{N} \kappa^n H^n \ ,$$
$$H(\vec{x},\vec{y}) = \sum_{j=1}^{3}\left(U_j(\vec{x},0)\delta(\vec{x}+\hat{j},\vec{y}) + U_j(\vec{x}-\hat{j},0)^\dagger\delta(\vec{x}-\hat{j},\vec{y})\right) \ .$$

In [1] two different parameter values for $\kappa$ and $n$, giving rise to wide and narrow sources ($S_w$,$S_n$), have been used. In addition, we now construct covariant derivatives which act upon a wide smeared source to form our derivative quark sources $W_{d_j}$:

$$P_j(\vec{x},\vec{y}) = U_j(\vec{x},0)\delta(\vec{x}+\hat{j},\vec{y}) - U_j(\vec{x}-\hat{j},0)^\dagger\delta(\vec{x}-\hat{j},\vec{y}) \ ,$$
$$W_{d_j} = P_j S_w \ .$$

With these sources, meson interpolators of definite quantum numbers are constructed.

## Interpolators used

Table 1 shows the interpolators used for different meson channels. In some cases, an (anti-)symmetrization of the interpolators is necessary to obtain the correct behavior under charge conjugation. Therefore, interpolators of the type $\bar{u}_{d_i}\Gamma d_{n/w}$ should be read as $\bar{u}_{d_j}\Gamma d_{n/w} - \bar{u}_{n/w}\Gamma d_{d_j}$. We restrict ourselves to light, isovector ($I = 1$) mesons with degenerate quark masses $m_u = m_d$.

| | $J^{PC}$ | *old interpolators* | | *new interpolators* | | | ♯ |
|---|---|---|---|---|---|---|---|
| pseudoscalar | $0^{-+}$ | $\bar{u}_{n/w}\gamma_5 d_{n/w}$ | $\bar{u}_{n/w}\gamma_4\gamma_5 d_{n/w}$ | $\bar{u}_{d_j}\gamma_j\gamma_4\gamma_5 d_{n/w}$ | $\bar{u}_{d_j}\gamma_5 d_{d_j}$ | $\bar{u}_{d_j}\gamma_4\gamma_5 d_{d_j}$ | 10 |
| scalar | $0^{++}$ | $\bar{u}_{n/w} d_{n/w}$ | | $\bar{u}_{d_j}\gamma_j d_{n/w}$ | $\bar{u}_{d_j}\gamma_j\gamma_4 d_{n/w}$ | $\bar{u}_{d_j} d_{d_j}$ | 8 |
| vector | $1^{--}$ | $\bar{u}_{n/w}\gamma_j d_{n/w}$ | $\bar{u}_n\gamma_j\gamma_4 d_{n/w}$ | $\bar{u}_{d_j} d_{n/w}$ | $\bar{u}_{d_j}\gamma_k d_{d_j}$ | | 9 |
| pseudovector | $1^{++}$ | $\bar{u}_{n/w}\gamma_j\gamma_5 d_{n/w}$ | | $\bar{u}_{d_j}\gamma_k\gamma_5 d_{d_j}$ | | | 4 |

**Table 1:** *Meson interpolators; $n/w$ denote narrow and wide Jacobi smearing and $d_j$ stands for a derivative source in the $j$th direction. The last column shows the total number of different interpolators.*

## Results

To get an idea about the properties of the correlators, we first plot the normalized diagonal entries of the correlation matrices for the pseudoscalar and vector mesons. In Fig. 1 the simple interpolators have been colored blue, while the new correlators from derivative sources appear in red. As can be seen from the behavior at small times, the new correlators have stronger contributions from excited states.

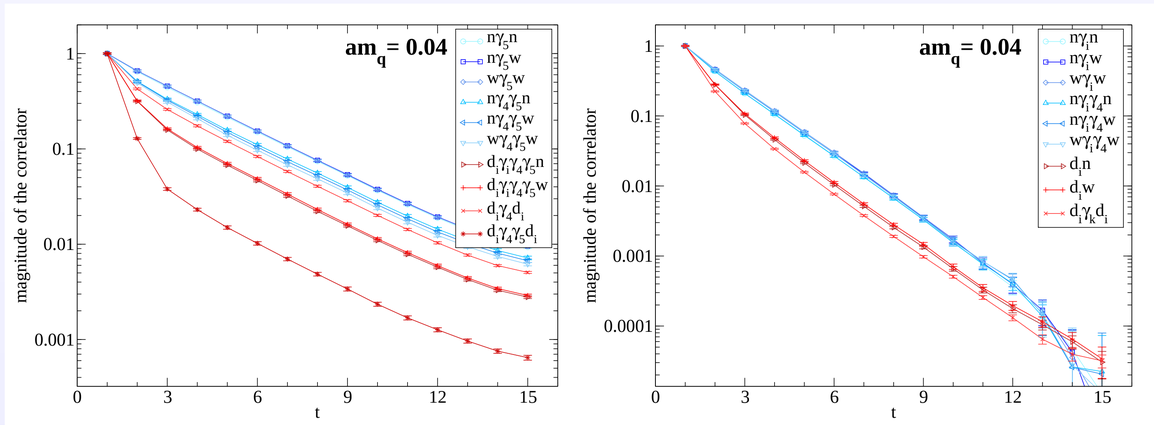


**Fig. 1:** *Diagonal correlators for the pseudoscalar (left) and vector mesons (right) at $am_q = 0.04$. $n/w$ denote narrow and wide Jacobi smearing and $d_i$ stands for a derivative source in the $i$th direction.*

Fig. 2 shows a comparison of selected effective mass plateaus from the old set of interpolators compared to the new results at different values of the quark mass. Plateaus are also found in the components of the corresponding eigenvectors. For the pseudoscalar and vector mesons, we obtain improved results for the excited states at all quark masses. For the scalar and pseudovector mesons, there is a noticeable improvement of the ground state plateaus.

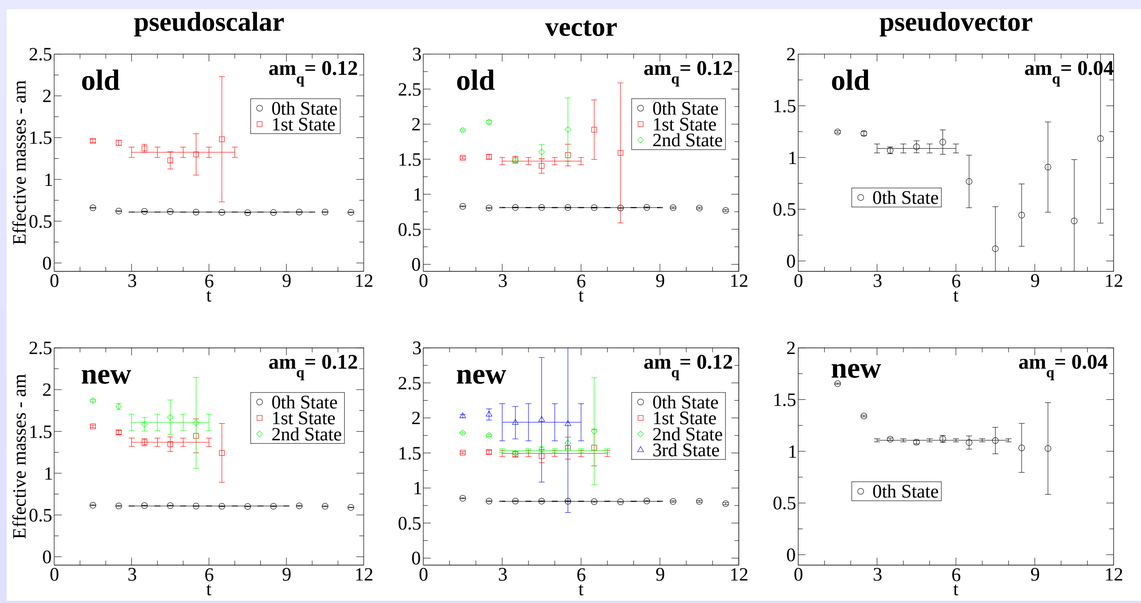


**Fig. 2:** *Effective mass plots for the pseudoscalar (left), vector (middle) and pseudovector (right) mesons. All errors are computed using the jackknife method. We compare the results from the the best selection of simple interpolators (top) to the case where the new derivative interpolators are added (bottom).*

## Conclusions and upcoming investigations

It has been demonstrated that interpolators constructed with derivative quark sources lead to an enhanced signal for a variety of different meson channels. For pseudoscalar and vector mesons, the correlators constructed from such interpolators display a significantly better overlap with excited states. For the scalar and pseudovector mesons, overlap with the ground states can be improved.

We are currently extending our calculations to dynamical configurations obtained in BGRs dynamical CI project. Furthermore, we investigate the construction of baryon interpolators from derivative quark sources.

## References

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