A Preliminary Calculation of the B Mixing Hadronic Matrix Elements

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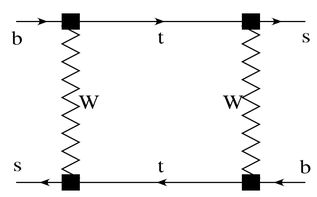
ABSTRACT

We present a preliminary calculation of the hadronic matrix elements relevant to the neutral B and B_s meson systems' mass differences. The ratio ξ is also calculated. The calculation is done on MILC lattices with 2+1 sea quarks. We use the Asqtad action for the light valence quarks and the Fermilab action for the b quark.



1 Introduction

1.1 The mass difference in the B meson system

Experimental measurements of the mass differences in the B and B_s meson systems have achieved errors of less than 1%,

$$\Delta M_d|_{exp.} = 0.508 \pm 0.004 ps^{-1} \quad (1)$$

$$\Delta M_s|_{exp.} = 17.77 \pm 0.10(stat) \pm 0.07(syst) ps^{-1}[1]. \quad (2)$$

The B mesons and antiparticles are able to mix via the diagram above, and the theoretical expression for the mass difference that arises from this diagram can be reduced to a product of CKM matrix elements, known perturbative factors, and a poorly known non-perturbative hadronic matrix element

$$\Delta M_q = \frac{G_F^2}{6\pi^2} m_W^2 \eta_B S_0(m_t, m_W) |V_{ts} V_{tb}^*|^2 \langle \bar{B}_q|Q|B_q\rangle. \quad (3)$$

The hadronic matrix element is conventionally parametrized as

$$\langle \bar{B}_q|Q|B_q\rangle = \langle \bar{B}|\bar{b}\gamma_\mu(1-\gamma_5)q\bar{b}\gamma^\mu(1-\gamma_5)q|B_q\rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q}. \quad (4)$$

f_{B_q} is the decay constant of the B_q meson, and B_{B_q} is the bag parameter.
Comparison between the experimental measurement and theoretically calculated factors multiplying the CKM matrix elements should give a precise determination of the poorly constrained $|V_{ts}|$. A lattice calculation is needed to precisely determine the hadronic matrix element contributing to the theoretical input.
On the lattice at one loop the operator Q_S,

$$\langle \bar{B}_q|Q_S|B_q\rangle = \langle \bar{B}_q|\bar{b}(1-\gamma_5)q\bar{b}(1-\gamma_5)q|B_q\rangle \quad (5)$$

also contributes and is calculated.
A very precise determination of the ratio of CKM matrix elements

$$\left|\frac{V_{td}}{V_{ts}}\right| = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}\sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}} = \xi\sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}} \quad (6)$$

can also be made because many of the lattice calculation's errors are partially canceled in the ratio ξ.

1.2 Lattice, Actions, and Operators

We performed the following calculations on three MILC coarse lattices ($a = 0.12fm$) with 2+1 sea quarks. The sea and light valence quarks are simulated using the Asqtad action. The heavy b quark is simulated using the Fermilab action.
For tree level $O(a)$ improvement of the operators we found that a rotation of the b quark via [2] is all that is necessary. We calculated the operators using all combinations of the sea and valence masses listed in the table below.

Quarks	Masses	Action	Errors
Heavy (b)	$\kappa_b = 0.086$	Fermilab	$O(\alpha_s\Lambda_{QCD}/M), O((\Lambda_{QCD}/M)^2)$
Light	$q = 0.0415, 0.03, 0.02$ 0.01,0.007,0.005	Asqtad	$O(a^2\alpha_s), O(a^4)$
Sea	$l = 0.007, 0.01, 0.02$ s=0.05	Asqtad	$O(a^2\alpha_s), O(a^4)$

2 Correlation Function Fitting

2.1 Correlation functions

We fit the two-point functions

$$C_{A4}(t) = \sum_{\vec{x}} \langle \bar{b}(\vec{x},t)\gamma_5 q(\vec{x},t)\bar{q}(0)\gamma_0\gamma_5 b(0)\rangle \rightarrow_{t\rightarrow\infty} \frac{1}{2m_{B_q}}\langle \bar{s}\gamma_5 b|B\rangle\langle B|\bar{b}\gamma_0\gamma_5 s\rangle e^{-m_{B_q}t} \quad (7)$$

and

$$C_Z(t) = \sum_{\vec{x}} \langle \bar{b}(\vec{x},t)\gamma_5 q(\vec{x},t)\bar{q}(0)\gamma_5 b(0)\rangle \rightarrow_{t\rightarrow\infty} \frac{1}{2m_{B_q}}|\langle \bar{q}\gamma_5 b|B\rangle|^2 e^{-m_{B_q}t}, \quad (8)$$

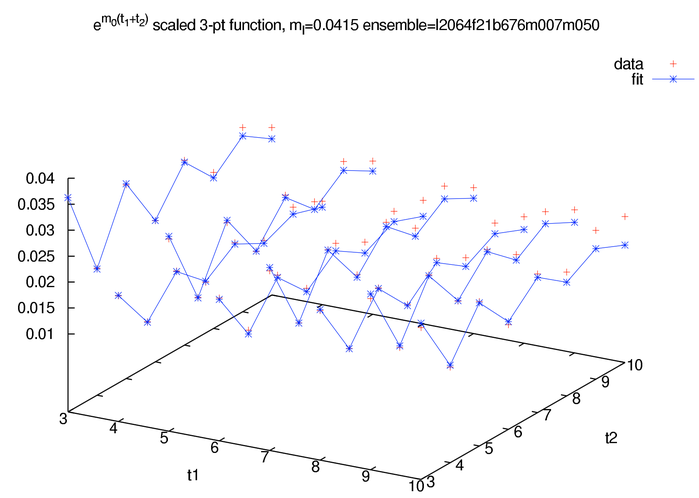
simultaneously with the three-point function

$$C_Q(t_1,t_2) = \sum_{\vec{x},\vec{y}} \langle \bar{b}(\vec{x},t_1)\gamma_5 q(\vec{x},t_1)[Q(0)]\bar{b}(\vec{y},t_2)\gamma_5 q(\vec{y},t_2)\rangle$$

$$\rightarrow_{-t_1,t_2\rightarrow\infty} \frac{1}{(2m_{B_q})^2}|\langle \bar{q}\gamma_5 b|B\rangle|^2\langle \bar{B}|Q|B\rangle e^{-m_{B_q}t_1}e^{-m_{B_q}t_2} \quad (9)$$

in order to extract the mixing matrix element.

Fig 1.

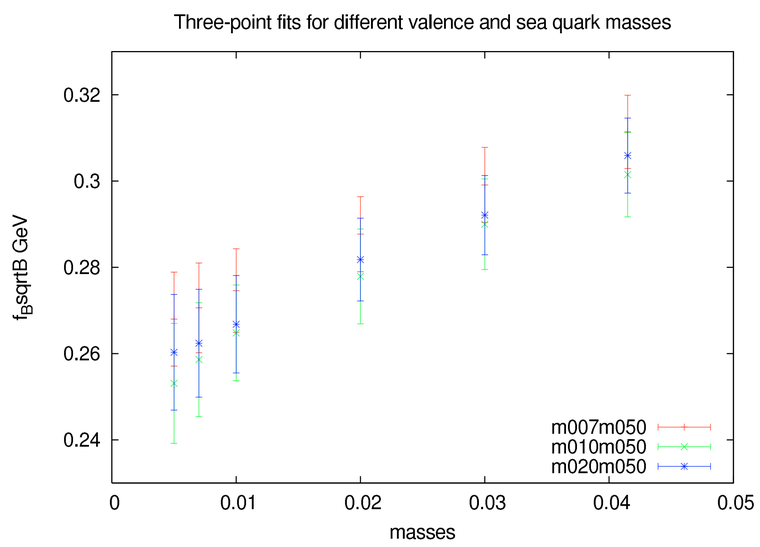


2.2 Fitting Details

The correlation functions have naive valence quarks which contain doublers that cause unphysical, higher energy 0^+ states to contribute, as can be seen in Fig 1. These states oscillate in euclidean time and are accounted for in the fits [3].
C_Z allows the overlap parameters in C_Q to be removed and the mixing matrix element to be isolated. C_{A_4} is used to determine f_{B_q} and can be used to isolate B_{B_q}. It should be noted that the parameter most directly of phenomenological interest, $f_{B_q}\sqrt{B_{B_q}}$, can be extracted by combining just C_Z and C_Q.
For our best fits we included the first 8 states (4 regular and 4 oscillating) with $t = 2-20$ for the two-point functions, and 16 exponentials with t_1 and t_2 taken over $t_{min} = 3, t_{max} = 12$ for the three-point functions. Starting guesses and priors with a generous width were used for all of the parameters. The priors and starting guesses had little effect on the fit results.
We looked at a variety of time slices and ranges along with including additional states in the model function. The fit results were typically consistent within 25% of the error bars of the best fit. We calculate Q_S in an identical way.

2.3 Fit Results

Fig 2.



Independent fits were done for each sea and valence quark mass combination. The sea quark mass dependence is extremely mild, while the valence quark mass dependence is less so. Errors on the fits range from $2-5\%$. The lattice spacing uncertainty has been included in Fig 2.'s data points' errors as well.

3 One-loop matching

The continuum $\bar{MS}$ value of $\langle Q\rangle$ is calculated from the lattice matrix elements up to $O((\alpha_s)^2), O(\alpha_s/aM)$ via

$$\frac{a^3}{2M_{B_q}}\langle Q\rangle^{\bar{MS}}(\mu) = [1+\alpha_s\rho_{LL}]\langle Q\rangle(a) + \alpha_s\rho_{LS}\langle Q_S\rangle(a) \quad (10)$$

where $\rho_{XY} = \rho_{XY}^{\bar{MS}}(\mu, m_b) - \rho_{XY}^{latt.}(am_b)$ and $\alpha_s = \alpha_V(q*)$, $q* = 2/a$. The q^* value used is close to what is used in heavy-light currents. For the finalized calculation an optimized value needs to be determined. While we have results for the matching coefficients they are still preliminary and an independent calculation needs to be completed. One-loop matching has not been included in Fig 2. but has been in the determination of ξ.
A powercounting estimate of the two-loop effects suggests a matching error of $\approx 9\%$, however in the ratio ξ almost all of this uncertainty is removed. It has also been shown that for Fermilab and Fermilab-Asqtad currents that the bulk of the renormalization coefficients can be calculated non-perturbatively, leaving a perturbative coefficient that gives $1-2\%$ corrections to the matching. Methods such as this should work for our coefficients and will greatly reduce the largest errors in our calculation.

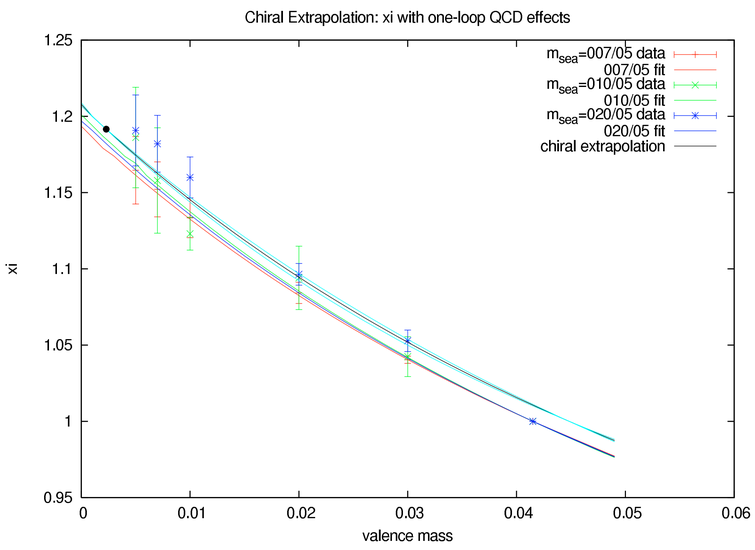
4 Chiral Fits and Extrapolations

The sea and valence quark masses are not simulated at their physical values and so staggered chiral perturbation theory is necessary. Taste changing interactions are accounted for and removed at the order we are working to obtain a physical result.
The complicated NLO chiral expression for $\langle \bar{B}_q|Q|B_q\rangle$ can be summarized as

$$\langle \bar{B}_q|Q|B_q\rangle = \beta(1 + w * L(b,q)) + c_0 m_q + c_1(m_U + m_D + m_S) + c_2 a^2, \quad (11)$$

where $L(b,q)$ is a sum of chiral logs including taste violating terms, m_q is the light valence quark mass, and the m_L's are the sea quark masses[8][9]. The four parameters, β, w, c_0, c_1, and c_2, are the LEC's allowed to vary in the fits to the matrix elements. There are only two free parameters left in the ratio ξ.
There are additional poorly determined parameters contained in $L(b,q)$ which are held fixed during the fits. The systematic errors associated with adjusting these values within their estimated uncertainty has been preliminarily examined and are generally comparable to the statistical errors. It should also be noted that c_2 cannot be determined because only one lattice spacing has been used. Conveniently this c_2 term cancels in the ratio ξ.

Fig 3.



5 Results and Errors

$$\xi = 1.191 \pm 0.004$$

ξ was determined using the chiral extrapolation shown in Fig 3 . Values for $f_{B_q}\sqrt{B_{B_q}}$ do not yet include one-loop matching and so no chiral extrapolation has been performed. Successful fits have been done on the unmatched values, however the one-loop effects reduce the values $\approx$10% so the perturbative matching must be finalized before reliable physical values can be reported. The one-loop effects on ξ are so small that we are confident any small discrepancies between the present and final matching procedure will not significantly affect the above value.

Expected Errors		
Source of Errors	$f_{B_q}\sqrt{B_{B_q}}$	ξ
statistics	2-5%	0.4%
scale	1.8%	0
Higher Order Matching	$\approx$7%	$< 1\%$
Heavy quark discret.	2-3%	$< 0.5\%$
Light quark discret. + χPT fits	Work in Progress	

6 Summary and Outlook

- The statistical uncertainties of this calculation are straightforward to reduce. Particularly, we have performed the same runs on three other time sources for each ensemble. Including these additional sources appears to reduce the statistical uncertainty about 50%.
- Using the nonperturbative matching techniques mentioned above will remove the largest uncertainty in the calculation of the matrix elements.
- We will have to complete the same calculation on multiple lattice spacings to extrapolate to the continuum limit.
- Although the error on ξ quoted above may change as systematic uncertainties are more thoroughly explored, it is likely an uncertainty of $< 1\%$ will be achieved.

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