Addendum to lecture 11

Elitzur's theorem & the Higgs mechanism:

We learned last time that gauge symmetries cannot be broken spontaneously. How is that consistent with the standard description of the Higgs phenomenon? After all, the VEV of the Higgs is gauge variant & so should vanish by Elitzur.

To study this, consider a stripped down version of the standard model: SU(2) gauge + Higgs (complex doublet).

In the standard perturbative, unitary gauge description, the spectrum consists of a degenerate triplet of massive gauge bosons: $M_{W^\pm}^2 = M_{W^0}^2 \sim g^2 v^2$ (v = Higgs VEV). and a Higgs with $m_H^2 \sim \lambda v^2$.

Now put this theory on the lattice

$$S = S_{gauge} + S_\phi$$

$$S_{gauge} = -\beta \sum_{n,\mu<\nu} \frac{\mathrm{re\,tr}\, P_{n,\mu\nu}}{2} \qquad \beta = \frac{4}{g^2}$$

$$S_\phi = \sum_n \varphi_n^\dagger \cdot \varphi_n + \lambda \left(\varphi_n^\dagger \cdot \varphi_n - 1 \right)^2$$

SU(2) doublet

$$- k \sum_{n,\mu} \left(\varphi_{n+\mu}^\dagger U_{n,\mu}^\dagger \varphi_n + \varphi_n^\dagger U_{n,\mu} \varphi_{n+\mu} \right)$$

Gauge symmetry: $\mathcal{U}_{n,\mu} \to V_n \mathcal{U}_{n,\mu} V^{+}_{n+\mu}$

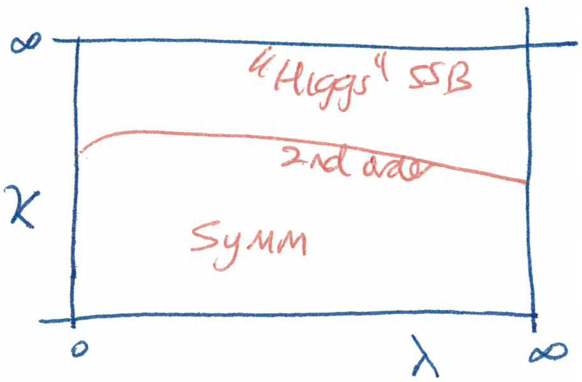
(as before).

$$\varphi_n \to V_n \varphi_n$$

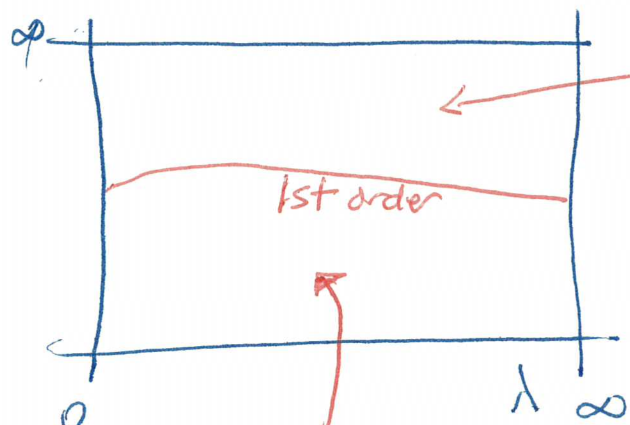
Phase diagram? Now have κ, λ & β.

If $\beta \to \infty$ ($g \to 0$) gauge fields decouple (all $\mathcal{U}$'s gauge equivalent to unity) & return to the ϕ^4 theory we discussed earlier, except now the scalar field has 4 components. Story is still the same qualitatively

$\boxed{\beta = \infty}$



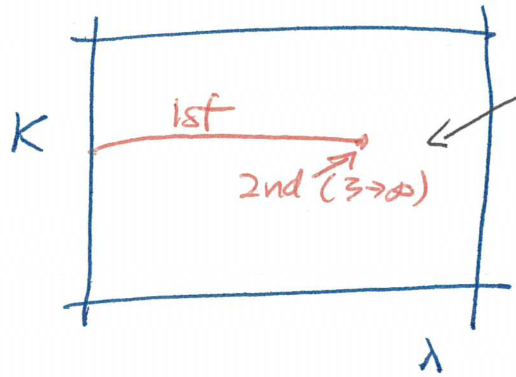
For finite $\beta \gtrsim 1$ numerical simulations find



? Elitzur tells us that VEV makes no sense – unless fix gauge.
Can we avoid?

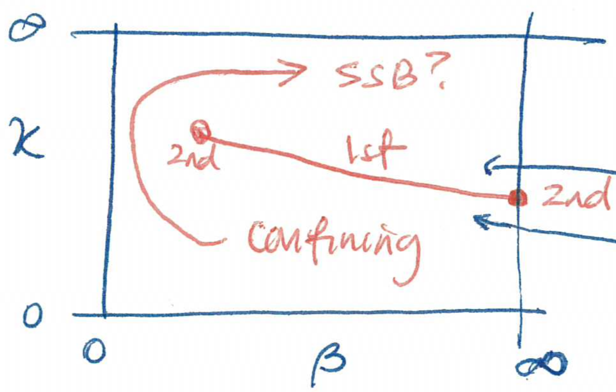
If $\kappa < \kappa_c$ then scalar is heavy, so have confining gauge theory w/ spectrum of glueballs, plus heavy scalar–antiscalar bound states.

The situation is clarified by looking at β near 0 ($0 < \beta \ll 1$)



The two "phases" are connected!

Another view is to hold $\lambda = \infty$ & look in K, β plane



Proof that two "phases" connected given by 't Hooft (1979)

detailed spectrum differs but no order param which distinguishes

This means that ~~there~~ must be a gauge-invariant description of the Higgs phenomenon.
(Called "complementarity".)

Indeed there is:

$$W^\pm \;\&\; W^0 \text{ created by } \varphi_n^\dagger\left[U_{n,\mu}\,\varepsilon\left(\varphi_{n+\mu}^\dagger\right)^{tr} - \varepsilon\left(\varphi_n^\dagger\right)^{tr}\right],$$

$$\varphi_n^{tr}\,\varepsilon\left[U_{n,\mu}\varphi_{n+\mu} - \varphi_n\right] \;\&\; \varphi_n^\dagger\left[U_{n,\mu}\varphi_{n+\mu} - \varphi_n\right]$$

where $\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ & we are using the property of $SU(2)$ [psuedoreality] that $-\varepsilon V_n^{tr}\varepsilon = V_n^\dagger$.

In fact, can show – see e.g. L.F. Abott & E. Farhi, Phys. Lett. 101B (1981) 69 – that there is an SU(2) global symmetry ("custodial symmetry") that makes these particles degenerate.

The "Higgs" is created by $\varphi_n^\dagger \varphi_n$.

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In the confining "phase" there is a much more complicated spectrum, including glueballs, many "hadrons" composed of $\varphi^\dagger\varphi$, $\varphi\varphi$ & $\varphi^\dagger\varphi^\dagger$, many of which mix & most of which are unstable.

As one moves around to the "Higgs" phase, the spectrum becomes closer & closer to that familiar from PT.
(except that $m_H \to 0$ in cont. limit due to triviality)

One can ask whether we know we are in the Higgs region or the confinement one – that was Abott & Farhi's point.

By now, the detailed agreement of data w/ PT for electroweak processes presumably rules out the "confining phase" option

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