

# The Slingshot

## BACKGROUND

The slingshot is an argument for the conclusion that any two sentences that have the same truth-value have the same denotation. The original idea for the argument is often credited to Frege, although it is hard to uncover it in his writings. But he was certainly committed to its conclusion, which probably helped to lead him to the further conclusion that sentences denote truth-values. It is but a short step from the idea that all true sentences have the same denotation to the conclusion that all true sentences denote the truth-value True.

The argument occurs explicitly in the writings of Church, Gödel, Quine, and Davidson, among others. The name ‘slingshot’ comes from Barwise and Perry (see “Semantic Innocence and Uncompromising Situations,” ch. 31 in Martinich). The idea is that, like David’s slingshot, it is a simple weapon that is able to bring down a formidable foe (Goliath).

“The argument is so small, seldom encompassing more than half a page, and employs such a minimum of ammunition — a theory of descriptions and a popular notion of logical equivalence — that we dub it *the slingshot*.”

## PRELIMINARIES

### Descriptions

Given any predicate,  $F$ , you can always form a definite description (a kind of singular term) out of it as follows: form the open sentence  $Fx$  and prefix the description-operator  $\iota x$  (to be read: “the  $x$  such that ...”), yielding  $\iota x Fx$  (to be read: “the  $x$  such that  $x$  is  $F$ ”).

### Examples

$\iota x (x = 5 + 3)$	“the sum of 5 and 3”
$\iota x (x = \text{Bill})$	“the $x$ such that $x$ is Bill”
$\iota x (x = \text{Bill} \wedge x \text{ is a Democrat})$	“the $x$ such that $x$ is Bill and a Democrat”

Note that ‘ $\iota x (x = \text{Bill})$ ’ and ‘Bill’ have the same denotation, viz., Bill. Note further that if Bill is a Democrat, ‘ $\iota x (x = \text{Bill} \wedge x \text{ is a Democrat})$ ’ also denotes Bill. (If Bill is not a Democrat, then the description has either no denotation or some other denotation.)

## Bridge sentences

For any two sentences  $Fa$  and  $Gb$  that agree in truth-value and contain distinct singular terms  $a$  and  $b$ , there is some sentence  $Rab$  that has the same truth-value. (This is trivial: if ' $a$ ' and ' $b$ ' have the same denotation and the sentences are true,  $Rab$  can be ' $a = b$ ', etc.)

## Some equivalences

$$Fa \Leftrightarrow a = \lambda x (x = a \wedge Fx)$$

'Bill is a Democrat' is equivalent to 'Bill is the Democrat who is Bill'.

$$Rab \Leftrightarrow a = \lambda x (x = a \wedge Rxb)$$

'Bill preceded George' is equivalent to 'Bill is the Bill who preceded George'.

$$Rab \Leftrightarrow b = \lambda x (x = b \wedge Rax)$$

'Bill preceded George' is equivalent to 'George is the George who was preceded by Bill'.

## THE ARGUMENT

### Premises

#### 1. Logically equivalent sentences have the same denotation.

That is: If  $\phi$  is logically equivalent to  $\psi$ , then  $D(\phi) = D(\psi)$ .

#### 2. The denotation of a sentence is not changed if you replace a singular term occurring in it with another singular term that has the same denotation.

That is: If  $D(\alpha) = D(\beta)$ , then  $D(\dots\alpha\dots) = D(\dots\beta\dots)$ .

Call these 'the equivalence premise' and 'the replacement premise', respectively. Note that the replacement premise is a direct consequence of Frege's compositionality principle.

### Conclusion :

**Any two sentences that have the same truth-value have the same denotation.**

## A STANDARD VERSION OF THE ARGUMENT

Let us suppose that ' $Fa$ ' and ' $Gb$ ' are two sentences that are both true, but (in an intuitive sense) have nothing to do with one another in terms of what they are about.

E.g., ' $Fa$ ' might be 'Clinton is a Democrat' and ' $Gb$ ' might be '5 is a prime number'.

Let us further assume that there is some true, relational sentence, ' $Rab$ ' in which both ' $a$ ' and ' $b$ ' occur.

E.g., ' $Rab$ ' might be 'Clinton has five toes on his left foot'. (If this does not seem to be a relational sentence, try recasting it as expressing a relation between Clinton and the number 5: 'Clinton is related to the number 5 by the relation *has that many toes on his left foot*'.)

Now consider the following sequence of sentences:

1.  $Fa$
2.  $a = \lambda x (x = a \wedge Fx)$
3.  $a = \lambda x (x = a \wedge Rxb)$
4.  $b = \lambda x (x = b \wedge Rax)$
5.  $b = \lambda x (x = b \wedge Gx)$
6.  $Gb$

It follows from our two premises (Equivalence and Replacement) and the assumption that ' $Fa$ ', ' $Gb$ ', and ' $Rab$ ' are all true, that **all six of these sentences have the same denotation**. (It is **not** being claimed that they are all equivalent sentences!) Here's why:

(1) and (2) are logically equivalent, so they have the same denotation.

Explanation: this move takes advantage of a clever way of producing, for any arbitrary subject-predicate sentence, an **identity sentence** that is equivalent to it.

(3) results from (2) by replacing one expression with another that has the same denotation, so (2) and (3) have the same denotation.

Explanation: Given that  $Fa$  and  $Rab$  agree in truth-value, the descriptions ' $\lambda x (x = a \wedge Fx)$ ' and ' $\lambda x (x = a \wedge Rxb)$ ' both denote  $a$ .

(3) and (4) are logically equivalent (each is equivalent to ' $Rab$ '), so they have the same denotation.

Explanation: (3) and (4) seem to be non-equivalent, because they are about different things: (3) is about  $a$  and (4) is about  $b$ . But, from a logical point of view, each sentence is about **both**  $a$  and  $b$ .

(3) says, about Clinton, that he's the unique thing that is Clinton and has 5 toes on his left foot; (4) says, about the number 5, that it's the unique thing that equals 5 and numbers the toes on Clinton's left foot. That is, both (3) and (4) are equivalent to ' $Rab$ '.

(5) results from (4) by replacing one expression with another that has the same denotation, so (4) and (5) have the same denotation.

Explanation: ' $\lambda x (x = b \wedge Gx)$ ' and ' $\lambda x (x = b \wedge Rax)$ ' both refer to  $b$ .

(5) and (6) are logically equivalent, so they have the same denotation.

**Therefore, (1) and (6) have the same denotation.**

Explanation: '...has the same denotation as ...' is a transitive relation.

This is basically Church's version of the Slingshot (*Introduction to Mathematical Logic*, pp. 23-25). Church's ' $Fa$ ' and ' $Gb$ ' were 'Scott wrote *Waverly*' and 'There are 29 counties in Utah'. His "bridge sentence", ' $Rab$ ', was '**Scott** wrote **29** *Waverly* novels altogether'.

## A STREAMLINED VERSION OF THE ARGUMENT

John Perry has produced a more streamlined version of the Slingshot, which omits the "bridge sentence". Instead, this version takes advantage of a clever way of producing, for any arbitrary sentence, an equivalent sentence that is an **identity sentence about a number**.

Let  $P$  and  $Q$  be any two sentences that are alike in truth-value.

Let ' $t_P$ ' abbreviate the following description:  $\lambda x ((x = 1 \wedge P) \vee (x = 0 \wedge \neg P))$ , and let ' $t_Q$ ' abbreviate the following description:  $\lambda x ((x = 1 \wedge Q) \vee (x = 0 \wedge \neg Q))$

That is: the description amounts to:

- 'the number which either equals 1 and  $P$ , or equals 0 and  $\neg P$ '; alternatively,
- 'the number which equals 1 if  $P$ , and equals 0 if  $\neg P$ '

We note the following two crucial facts:

- The denotation of ' $t_P$ ' and ' $t_Q$ ' depends entirely on the truth-values of  $P$  and  $Q$ : if  $P$  is true,  $t_P = 1$ ; if  $P$  is false,  $t_P = 0$ ; if  $Q$  is true,  $t_Q = 1$ ; if  $Q$  is false,  $t_Q = 0$ . So, **if  $P$  and  $Q$  are alike in truth-value, ' $t_P$ ' and ' $t_Q$ ' have the same denotation.**
- **The sentences  $P$  and ' $t_P = 1$ ' are logically equivalent.**

They are logically equivalent because they logically imply one another. From  $P$ , it follows that 1 is, indeed, the number,  $x$ , which satisfies the condition ( $x = 1 \wedge P$ ); from  $\neg P$ , it follows that 1 is **not** the number,  $x$ , which satisfies the condition ( $x = 0 \wedge \neg P$ ).

In effect, ' $t_P = 1$ ' says that it is true that  $P$  ("the truth-value of ' $P$ ' = 1"), but without ascending into the metalanguage. So it is clear that ' $t_P = 1$ ' is equivalent to ' $P$ '.

Now consider the following sentences:

1.  $P$
2.  $t_P = 1$
3.  $t_Q = 1$
4.  $Q$

We are assuming only that  $P$  and  $Q$  are alike in truth-value, not that they are logically equivalent. We can now prove that  $P$  and  $Q$  have the same denotation.

(1) and (2) have the same denotation.

Reason: (1) and (2) are logically equivalent.

(2) and (3) have the same denotation.

Reason: (2) and (3) differ only with respect to two singular terms with the same denotation — either ' $t_P$ ' and ' $t_Q$ ' both denote the number 1 (if  $P$  and  $Q$  are both true), or they both denote the number 0 (if  $P$  and  $Q$  are both false).

(3) and (4) have the same denotation.

Reason: (3) and (4) are logically equivalent.

Therefore, (1) and (4) have the same denotation.

Reason: ‘...has the same denotation as ...’ is a transitive relation.

It follows that all true sentences denote the same thing (Truth? The True? Reality?)  
and all false sentences denote the same thing (Falsehood? The False? Unreality?).