Frege: "On Sense and Denotation"

TERMINOLOGY

- 'On Sense and Nominatum' is a quirky translation of '*Über Sinn und Bedeutung'*. 'On Sense and Denotation' is the usual translation.

- 'Sameness' is misleading in stating the initial puzzle. As Frege's n.1 makes clear, the puzzle is about **identity**.

THE PUZZLE

What is identity? Is it a relation? If it is a relation, what are the relata? Frege considers two possibilities:

1. A relation between objects.

2. A relation between names (signs).

(1) leads to a puzzle; (2) is the alternative Frege once preferred, but now rejects.

The puzzle is this: if identity is a relation between objects, it must be a relation between a thing and **itself**. But everything is identical to itself, and so it is **trivial** to assert that a thing is identical to itself. Hence, every statement of identity should be **analytic** and knowable **a priori**.

The "cognitive significance" of '$a = b$' would thus turn out to be the same as that of '$a = a$'. In both cases one is asserting, of a single object, that it is identical to itself. Yet, it seems intuitively clear that these statements have different cognitive significance:

> "… sentences of the form $a = b$ often contain very valuable extensions of our knowledge and cannot always be justified in an *a priori* manner" (p. 217).

METALINGUISTIC SOLUTION REJECTED

In his *Begriffschrift* (1879), Frege proposed a metalinguistic solution: identity is a relation between signs. '$a = b$' thus asserts a relation between the signs 'a' and 'b'. Presumably, the relation asserted would be *being co-referential*.

That is: '$a = b$' is taken to mean that 'a' and 'b' are co-referential.

Frege's objection:

Frege points out that the choice of a particular sign to stand for a particular object is **arbitrary**, or conventional. Hence, it is also arbitrary or conventional that two different signs have been used for the same object. Since this is so, the statement '*a* = *b*' would (on the metalinguistic solution) report an arbitrary decision about nomenclature, and not a substantive fact.

But statements of the form '*a* = *b*' can be used to report substantive facts, and not just arbitrary conventions.

> **Example**: The Roman orator and philosopher that we know as Cicero is known by a different name in Britain: Tully. But according to the metalinguistic treatment of identity statements, if I now say to you:
>
> (1) Cicero = Tully
>
> I have not conveyed any information to you beyond the linguistic facts (about Britain and the U.S.) that I just mentioned. The objection to the metalinguistic solution is that it treats all identity sentences as having this kind of "linguistic" informational content.
>
> Of course, (1) can be used in circumstances in which it **does** report a substantive (non-conventional) fact. Suppose you read the philosophical works of Cicero, and find them boring and unoriginal. Later, you run across the orations of Tully (in a British edition, of course), and find them dynamic and compelling. You are so impressed by Tully's writing that you compare him most favorably to Cicero. Then I utter (1), and you are amazed. I have informed you of a substantive fact, and not an arbitrary linguistic convention.

Sharpening Frege's objection: a use-mention confusion

According to the metalinguistic solution, we understand the following as equivalent:

1. The morning star = the evening star.

2. 'The morning star' and 'the evening star' denote the same object.

But it can be shown that this is very implausible. For someone might well know that (1) is true without knowing that (2) is true. We can see this most easily if we translate (1) and (2) into another language, say German:

3. Der Morgenstern = der Abendstern.

4. ‘The morning star’ und ‘the evening star’ bedeuten den gleichen Gegenstand.

Notice that in translating (2) we leave the quoted expressions untranslated. For (2) is about two **English** expressions, not two **German** expressions. To translate the quotation names into German would be to commit a use-mention confusion.

But (3) and (4) are not equivalent; they do not express the same fact. Consider Otto, a German speaker who knows no English, but knows that the morning star = the evening star. Otto certainly knows what (3) expresses, but he does not know what (4) expresses. For (4) says that the two English expressions, ‘the morning star’ and ‘the evening star’, denote the same object, and Otto knows nothing about those two English expressions.

A more general objection:

There is a still deeper problem with the metalinguistic solution. For it proposes to analyze identity statements in terms of the notion of *being co-referential*. But the notion of *being co-referential* needs to be explained, and it seems likely that that explanation will itself require the notion of identity.

Presumably, for signs *S* and *S′* **to be co-referential** is for them **to designate one and the same object**. Let’s write ‘*Des* (*S*)’ to abbreviate ‘the designation of *S*’ or ‘the object designated by *S*’. Then the claim:

‘*a*’ and ‘*b*’ are co-referential

amounts to the claim that:

Des (‘*a*’) and *Des* (‘*b*’) are one and the same thing.

But now we have to say what it is for these objects to be one and the same thing. There seem to be two options:

1. One option is to say that for *Des* (‘*a*’) and *Des* (‘*b*’) to be the same thing is for it to be the case that *Des* (‘*a*’) = *Des* (‘*b*’). If so, we end up with identity as a relation between things, after all.

2. The other option is to apply the metalinguistic solution to the statement ‘*Des* (‘*a*’) = *Des* (‘*b*’).’ That is, it is a relation between the signs ‘*Des* (‘*a*’)’ and ‘*Des* (‘*b*’)’. I.e., it amounts to the claim that:

   ‘*Des* (‘*a*’)’ and ‘*Des* (‘*b*’)’ are co-referential.

And now we must once again explain what this co-referentiality amounts to. Presumably, it means that *Des* (‘*Des* (‘*a*’)’) and *Des* (‘*Des* (‘*b*’)’) are one and the same thing. But now we are faced with our two options once again for our choice of what this claim amounts to.

1. *Des* (‘*Des* (‘*a*’)’) = *Des* (‘*Des* (‘*b*’)’).

2. ‘*Des* (‘*Des* (‘*a*’)’)’ and ‘*Des* (‘*Des* (‘*b*’)’)’ are co-referential.

The first option once again gives us objectual identity. The second, metalinguistic, option reinvites our question about what co-referentiality amounts to. We cannot avoid objectual identity without facing an infinite regress.

It is hard to see how we are going to get rid of objectual identity, for it seems to be presupposed by the notion of co-referentiality, in terms of which the metalinguistic solution is stated. Hence, the metalinguistic solution should be rejected.

SENSE AND DENOTATION

A Solution to the Identity Puzzle

Frege’s solution to the puzzle about identity-sentences requires him to find a difference in “cognitive significance” between ‘*a* = *a*’ and ‘*a* = *b*’. The difference cannot consist in a difference between the objects the signs ‘*a*’ and ‘*b*’ stand for, for there is no such difference. Nor can it consist in a difference between the signs, for the signs are different whether or not they refer to the same thing. Rather, the difference must consist in “a difference in the way in which the designated objects are given” (p. 217) — a difference in **mode of presentation**.

Hence, Frege distinguishes between **sense** (*Sinn*) and **denotation** (nominatum, *Bedeutung*). The denotation of a name is the object it picks out; the sense of the name is the **mode of presentation** of the object.

Since different names with different senses can have the same denotation, identity statements can be both true (for the names have the same denotation) and informative (for the names have different senses).

The basic idea of sense and denotation

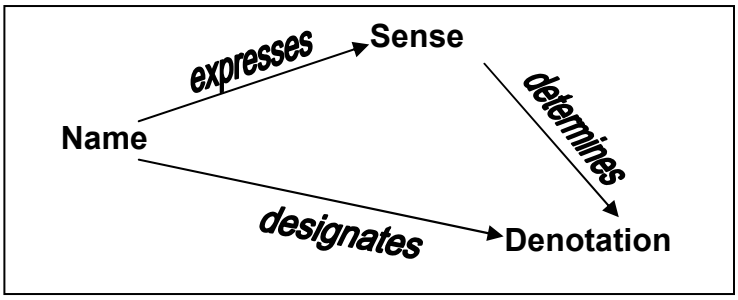
Frege used the term *name* more broadly than we normally do. He certainly meant it to apply to any definite singular noun phrase — including both proper names ('Cicero', 'Plato') and definite descriptions ('the most famous Roman orator', 'the teacher of Aristotle').

In fact, as we'll see, he effectively regards every linguistic expression as a 'name'.

So in holding that every name has both a sense and a denotation, he actually holds that **every expression has both a sense and a denotation**. A name **designates** or **denotes** its denotation, and **expresses** its sense:

> "A proper name (word, sign, sign-compound, expression) *expresses* its sense, and *designates* or *signifies* its denotation. We let a sign express its sense and designate its denotation" (p. 220).

There is also an important relation between the sense and the denotation of a given name. That is, the sense of a name **determines** its denotation. The relations between name, sense, and denotation thus look like this:



It is clear that Frege has more in mind by 'sense' than just conceptual content (= mode of presentation). The sense of a name, Frege tells us, is not just a mode of presentation, but a mode of presenting **a definite object**. That is, it is the job of the sense of a name to **fix on** a certain object as **the denotation** of the name.

So, for example, with the triangle example on pp. 217-218: the sense of the name 'the intersection of *a* and *b*' fixes the denotation of that name on the centroid of the triangle; the (different) sense of the name 'the intersection of *b* and *c*' fixes the denotation of that name on the same denotation — the centroid.

Of course, not every name with a sense will also have a denotation: e.g., 'the celestial body most distant from Earth', 'the least rapidly convergent series', 'the largest integer'. But it these cases it is the sense which determines that the names do not have a denotation. So the sense still (in some sense) fixes the denotation.

A sense is thus like a **route** — it is one way (among many possible ways) to get someplace. A sense is to its denotation as a route is to a destination.

Note that the sense-denotation relation is like the route-destination relation in several ways:

- Different routes (senses) can have the same destination (denotation).
- It is a **one-way** relation: a route (sense) determines a destination (denotation), but the converse does not hold. There is no way to recover a route (sense) from the destination (denotation) that it leads to.

Finally, the sense provides us with the **semantic content** of the name (roughly: what we **mean** by it). To summarize: a Fregean sense has these three aspects, simultaneously:

1. **Mode of presentation of an object**
2. **Fixing the denotation of a name**
3. **Semantic content of a name**

SENSE VS. IDEA

Fregean senses are not private objects — ideas in someone's mind. They are **objective**, not subjective. For a sign's sense "may be the common property of many and therefore is not a part or mode of the single person's mind" (p. 219).

Frege is here assuming that no two people can share the same idea — that ideas are essentially private objects. Since senses are shareable, they are not (private) ideas.

A DIGRESSION: "THE THOUGHT"

The contrast between sense and idea is developed more fully in "The Thought." A **thought**, Frege tells us, is the **sense of a sentence** (p. p. 38, left):

> "The thought, in itself immaterial, clothes itself in the material garment of a sentence and thereby becomes comprehensible to us."

This characterization makes thoughts sound like mental entities — ideas, but Frege argues against this. First, he distinguishes between the **outer world** of "trees, stones, and houses" and the **inner world** of "sense-impressions, feelings, moods, wishes, and decisions" (p. 41).

Correction to translation p. 41, right, lines 2-5:

Frege is raising two questions: (1) how are thoughts related to (external) objects? and (2) how are thoughts related to persons (minds)? Talking about external objects (trees, stones, and houses) at the end of the paragraph, he asks this question (in our translation):

> *Obviously, no thought belongs to these things. Now can he, nevertheless, stand in the same relation to a person as to a tree?*

This makes no sense in the context, and mistranslates the German (Blumberg, 1971). The correct translation reads:

> *Obviously, a thought is not one of these things. Now can it, nevertheless, stand in the same relation to a person as does a tree?*

FEATURES OF IDEAS

There are four characteristic feature of ideas (mental entities), which Frege summarizes on pp. 41-42:

1. **Immaterial:** ideas cannot be seen or touched.
2. **Graspable:** ideas are had.
3. **Dependent:** ideas need a bearer.
4. **Subjective:** every idea has only one bearer.

Armed with this characterization of ideas, Frege argues that thoughts are not ideas. Therefore thoughts, the senses of sentences, must belong to a **third world**, a world of sense.

The argument: thoughts are like ideas with respect to (1) and (2) — they are immaterial, and they are "had" (i.e., are grasped by the mind). But they differ with respect to (3) and (4).

Independence: Unlike ideas, thoughts do not need a bearer. That is, a thought does not depend for its existence on its being the content of someone's idea. Thoughts, like "outer" objects, can exist "unperceived."

Objectivity: Unlike ideas, thoughts are shareable. You and I can have the same thought, although we cannot have the same idea. That two people can have the same thought is a prerequisite for "a science common to many.". Without shared thoughts, "it would be idle to dispute about truth" (p. 43, left).

Hence:

> "the thought … expressed in the Pythagorean theorem is timelessly true, true independently of whether anyone takes it to be true. It needs no bearer. It is not true for the first time when it is discovered, but is like a planet which, already before anyone has seen it, has been in interaction with other planets" (p. 43, left).

So construed, the Fregean world of sense is much like the Platonic realm of Forms.

Back to "On Sense and Denotation."

THE SENSE OF A SENTENCE

Frege holds that the sense of a sentence is (what he calls) a **thought** (*Gedanke*: what we would nowadays call a **proposition**). Why is this?

The argument is on p. 220: it is clear that a sentence "contains a proposition" — but is this proposition the sense or the denotation of the sentence? Frege assumes it must be one or the other. But it cannot be the denotation, because of Frege's **principle of compositionality**, which we will discuss in detail in a moment:

> If we replace a name occurring in a sentence with another name having the same denotation, but a different sense, the denotation remains the same; but "we realize that in such cases the proposition is changed" (p. 220).

His example: these two sentences must have the same denotation, but clearly express different propositions (since a person might believe one, but not the other):

> "The morning star is a body illuminated by the sun."
>
> "The evening star is a body illuminated by the sun."

Since these two sentences do not differ with respect to the denotation of any of their parts, their denotation (if any) will be the same. Therefore, the different propositions they contain must be the respective **senses** of the two sentences.

THE DENOTATION OF A SENTENCE

Does a sentence as a whole even have a denotation? Frege does not assume that it does, but offers an argument that it must. The argument:

> If all we are interested in is the sense of a sentence, we have no concern about the denotations of its parts, but only about their senses. E.g., we can understand the sentence:
>
> "Odysseus deeply asleep was disembarked at Ithaca."
>
> without knowing whether the name 'Odysseus' has a denotation. It is sufficient to know its sense (along with the senses of the other words in the sentence). But that is just to say that we can understand the sentence without knowing whether or not it is true.
>
> As soon as we want to know whether the sentence is true, though, we must inquire into whether 'Odysseus' has a denotation. Once all the parts get a denotation, the sentence as a whole gets a truth-value. Since the denotation (if any) of a complex expression is a function of the denotations of its parts, and a truth-value for a sentence is what gets determined once all of the parts of the sentence have a denotation, Frege concludes that **the denotation of a sentence is a truth-value**.
>
> > "Why is the proposition alone not sufficient? We answer: because what matters to us is the truth value. … It is the striving for truth which urges us to penetrate beyond the sense to the denotation" (p. 221).

Since there are only two truth-values (Frege calls them The True and The False), it follows that all true sentences have the same denotation (The True) and all false sentences have the same truth-value (The False).

There is also a famous argument, inspired by Frege, but only clearly articulated by some of his successors, to try to establish this result (which is at the very heart of Fregean semantics) more conclusively. We will look at this argument ("The Slingshot") if time permits. But first, let's look at how Fregean semantics works. Its central thesis is the **principle of compositionality**.

COMPOSITIONALITY

The **denotation (sense) of a complex expression** (including a sentence) **is a function of (i.e., is determined by) the denotations (senses) of its constituent expressions**.

Denotation

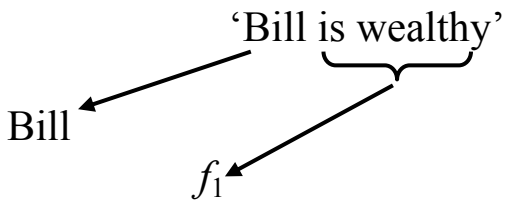
All linguistic expressions (proper names, predicates, sentences) denote **objects**.

- The denotation of a proper name is an **individual**.
- The denotation of a predicate is a **function** (which maps one object as *argument* to another object as *value*).
- The denotation of a sentence is a **truth-value**.
- A **concept** is a function whose values are truth-values.

A simple example: 'Bill is wealthy'

'Bill' denotes Bill. Bill = **D**('Bill')

'is wealthy' denotes a function, f_1. f_1 = **D**('is wealthy')



f_1 maps individuals onto truth-values: f_1 (Bill) = The True
f_1 (Marc) = The False

That is, f_1 maps Bill onto The True, maps Marc onto the False, etc. f_1 maps every wealthy individual onto The True, and everything else onto The False.

D('Bill is wealthy') is a function of **D**('Bill') and **D**('is wealthy').

D('Bill is wealthy') = The True

The denotation of the entire sentence is a function of the denotations of its parts.

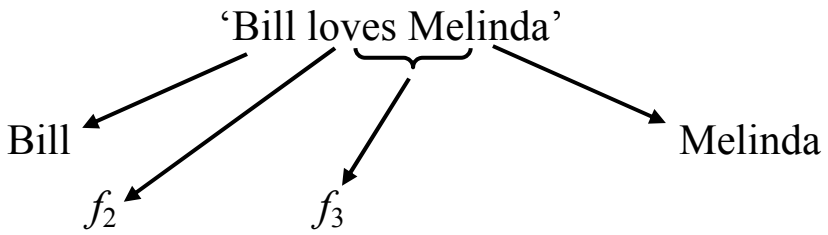
A more complicated example: 'Bill loves Melinda'

'Bill' denotes Bill. Bill = **D**('Bill')

'Melinda' denotes Melinda. Melinda = **D**('Melinda')

'loves' denotes a function, f_2 f_2 = **D**('loves')

'loves Melinda' denotes a function, f_3 f_3 = **D**('loves Melinda')



f_2 maps individuals onto functions: f_2 (Melinda) = f_3

E.g., f_2 maps Melinda onto f_3 (f_3 is the denotation of 'loves Melinda').

f_3 is a function that maps individuals onto truth-values. That is, f_3 is a **concept**.

E.g., f_3 maps Bill onto The True (since Bill loves Melinda), f_3 maps Marc onto The False (since Marc does not love Melinda), etc. In general, f_3 maps x onto The True iff x loves Melinda.

D('Bill loves Melinda') is a function of **D**('Bill'), **D**('Melinda'), and **D**('loves').

D('Bill loves Melinda') = The True

The denotation of the entire sentence is a function of the denotations of its parts.

Sense:

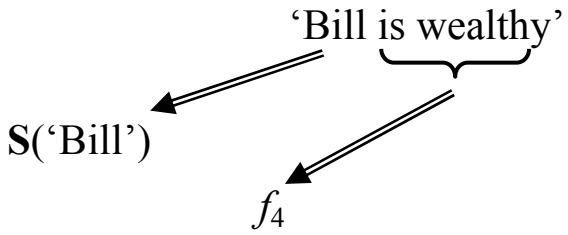
All linguistic expressions (proper names, predicates, sentences) express **senses**.

- The sense of a predicate is a **function** from a sense to a sense.
- The sense of a sentence is a **thought** (i.e., a proposition).

A simple example: 'Bill is wealthy'

'Bill' expresses = **S**('Bill').

'is wealthy' expresses a function, f_4 f_4 = **S**('is wealthy').



f_4 maps senses onto thoughts:

f_4 (**S**('Bill')) = **S**('Bill is wealthy')

That is, f_4 maps **S**('Bill') onto the sense of 'Bill is wealthy'.

S('Bill is wealthy') is the *thought*, or proposition, that Bill is wealthy.

S('Bill is wealthy') is a function of **S**('Bill') and **S**('is wealthy').

The sense of the entire sentence is a function of the senses of its parts.

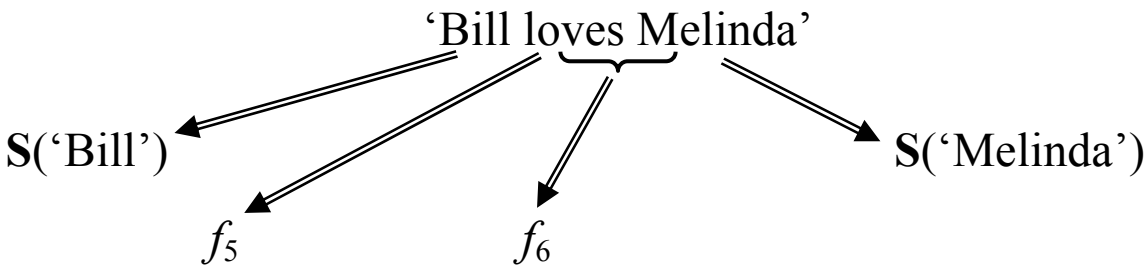
A more complicated example: 'Bill loves Melinda'

'Bill' expresses **S**('Bill').

'Melinda' expresses **S**('Melinda').

'loves' expresses a function, f_5 f_5 = **S**('loves').

'loves Melinda' expresses a function, f_6. f_6 = **S**('loves Melinda')



f_5 maps senses onto functions:

$$f_5\ \mathbf{S}(\text{'Melinda'})) = f_6.$$

That is, f_5 maps **S**('Melinda') onto the function that is the sense of 'loves Melinda'.

f_6 maps senses onto thoughts:

$$f_6\ (\mathbf{S}(\text{'Bill'})) = \mathbf{S}(\text{'Bill loves Melinda'})$$

That is, f_6 maps **S**('Bill') onto the thought that Bill loves Melinda.

S('Bill loves Melinda') is a function of **S**('Bill'), **S**('Melinda'), and **S**('loves').

The sense of the entire sentence is a function of the senses of its parts.

CONSEQUENCES OF COMPOSITIONALITY:

Indirect (oblique) contexts

The principle of compositionality seems to be in immediate difficulty when we consider what Frege called "indirect" or "oblique" (*ungerade*) contexts. These are, typically, what are now called "ascriptions of propositional attitude." That is, what happens when we embed one sentence inside another to talk about what someone says, or believes, etc.

Some examples of indirect contexts:

1) Jane said that Cicero was a Roman orator.

2) Doug believes that the morning star is Venus.

According to the principle of compositionality, the denotation of a complex expression is a function of the denotations of its component parts. This has as a corollary that the denotation of a complex expression remains unchanged if a name occurring in it is replaced by another name having the same denotation. So the **truth-value** of a sentence should remain unchanged after such a substitution.

Since **D**('Cicero') = **D**('Tully'), and **D**('the morning star') = **D**('the evening star'), substituting 'Tully' for 'Cicero' and 'the evening star' for 'the morning star' should always preserve truth-value. That is, (1) and (1a) should have the same truth-value, as should (2) and (2a):

1a) Jane said that Tully was a Roman orator.

2a) Doug believes that the evening star is Venus.

But they don't: (1) and (1a) might differ in truth-value, as might (2) and (2a).

Even more strikingly, the principle of compositionality also seems to give us the result that (3) and (3a) do not differ in truth-value:

3) Tom believes that snow is white.

3a) Tom believes that Clinton was the 42nd president.

That is because (3a) and (3) differ only in the replacement of one expression ('Snow is white') by another ('Clinton was the 42nd president') having the same denotation:

D('Snow is white') = **D**('Clinton was the 42nd president') = The True.

The result is that if Tom believes anything that is true, he believes everything that is true; and if Tom believes anything that is false, he believes everything that is false. Hence, if Tom has at least one true belief and at least one false belief, he believes everything! (Quine, 1960)

But Frege does not give up the principle of compositionality. Rather, he claims that in oblique contexts a name does not have its "customary" denotation. Rather, it has as its denotation its customary **sense**.

Hence, in (1), 'Cicero' does not denote Cicero. Rather:

> In (1), 'Cicero' denotes **S**('Cicero').
> In (1a), 'Tully' denotes **S**('Tully').
> **S**('Cicero') ≠ **S**('Tully')
>
> > This is because **S**'Cicero' and **S**'Tully' are **different modes of presentation** of the same object.
>
> So, **D**('Cicero') in (1) ≠ **D**('Tully') in (1a)

So the fact that (1) and (1a) differ in truth-value does not conflict with the principle of compositionality.

This solves the problem posed by indirect contexts, but raises problems of its own. For one thing, it depends on Frege's thesis that **all** expressions, including proper names, have senses. And as we'll see, there have been major challenges to this idea (esp. Kripke).

There are also two main problems with Frege's theory of propositional attitude ascriptions:

The extreme artificiality of Frege's theory:

> It seems unnatural to suppose that 'Cicero' denotes something different in (1) from what it denotes in 'Cicero was a Roman orator'. For it seems that both sentences, it denotes the same individual, namely, Cicero. It is odd to suppose that when we shift from 'Cicero was a Roman orator' to 'Jane believes that Cicero was a Roman orator', we are using 'Cicero' in a different sense and referring to something different. As Barwise and Perry put it, Frege's theory compromises our "semantic innocence."

Problems of multiple embedding:

1) The morning star is Venus.

2) Doug believes that the morning star is Venus.

3) Sally believes that Doug believes that the morning star is Venus.

What is the denotation of 'Venus' in (3)? On Frege's theory, the denotation of 'Venus' in (2) is **S**('Venus') in (1) — i.e., the ordinary sense of 'Venus'. So it appears as if Frege's theory holds this principle:

> When we embed a sentence, S_1, containing a name, *n*, in an indirect context inside another sentence, S_2, the **denotation** of *n* **in S_2** is the **sense** that *n* has **in S_1**.

Applying that to the case at hand, we get this:

In (1), **D**('Venus') = Venus.

In (2), **D**('Venus') = **S**('Venus') in (1) = the *customary* sense of 'Venus'.

In (3), **D**('Venus') = **S**('Venus') in (2) = what Frege calls its *indirect sense.*

But what is **S**('Venus') in (2)? How is a name's *indirect* sense related to its *customary* sense? We don't really know that, and we don't know how to figure it out. All we know is the **denotation** of 'Venus' in (2)—namely, that it is the "customary" sense of 'Venus'—and there is no way to **recover** the sense of an expression if all we know is its denotation. ("The route from sense to denotation is one-way.")

A puzzle about indirect senses

How can we get around this problem of identifying **indirect** senses? The ordinary sense of a name is just the way the object denoted by the name is **presented** to us. So we need to ask how a **sense** gets presented to us. Normally, an objected denoted by a name isn't the sort of thing that can be directly present to the mind, so a sense is needed by means of which we grasp the object.

But shouldn't senses be **self-presenting**, i.e., totally **transparent** to the mind? If so, we should conclude that we don't need another sense by means of which a sense is presented to us. On this proposal, Frege can distinguish between customary and indirect **denotation**, but needs only one level of sense: the

indirect sense of a name (its sense in an oblique context) is the same as its "customary" sense.

This seems to solve the puzzle about identifying indirect senses. But it fails to solve the problem of multiply embedded sentences—there are cases in which this proposal gives the wrong results. Example:

Al thinks that Bob believes all first-order logical truths, but Al also thinks that Bob does not know the meaning of the expression 'chiropodist'. So:

4) Al believes that Bob believes that all chiropodists are chiropodists.

4a) Al does not believe that Bob believes that all chiropodists are foot doctors.

According to the story we've told, (4) and (4a) are both **true**. Yet, if the indirect sense of 'chiropodist' in (4) and (4a) is the same as its customary sense, then (4) and (4a) could not have the same truth-value, for they would contradict one another.

So Frege's theory seems to require that indirect sense be different from customary sense. But he gives us no way of figuring out how these senses might differ.

Truth-value gaps

If a sentence contains an expression that lacks denotation, the sentence itself lacks denotation. So there are perfectly intelligible sentences that lack truth-value—they are neither true nor false.

> "But what about the denotation [of a sentence]? Can we even ask this question? A sentence as a whole has perhaps only sense and no denotation? It may in any case be expected that there are such sentences, just as there are constituents of sentences which do have sense but no denotation. Certainly, sentences containing proper names without denotations must be of this type (p. 220)."

Although a (meaningful) sentence may contain an expression that lacks denotation, it cannot contain an expression that lacks sense. For the principle of compositionality for senses requires that a sentence cannot express a thought (= have a sense) unless all of its constituents have senses.

This gives sense priority over denotation: sense determines denotation, but denotation does not determine sense.

The "Chosen Object" theory

Interestingly, although Frege introduced truth-value gaps, he did not like them:

> "Now, it is a defect of languages that expressions are possible within them, which, in their grammatical form, seemingly determined to designate an object, do not fulfill this condition in special cases. … This, it is obvious, hinges upon an imperfection of language, of which, by the way, even the symbolic language of analysis is not entirely free. … It is to be demanded that in a logically perfect language (logical symbolism) every expression constructed as a proper name in a grammatically correct manner out of already introduced symbols, in fact designate an object; and that no symbol be introduced as a proper name without assurance that it have a denotation" (p. 224).

And (as a logician) he suggests an alternative to the truth-value gap theory: vacuous singular terms can be avoided by the "stipulation" that an otherwise vacuous singular term shall stand for some *chosen object*. (E.g. the term 'divergent infinite series' shall denote the number 0.)

SUBORDINATE CLAUSES

Frege is interested in two different phenomena: (1) Cases of 'indirect denotation' (e.g., propositional attitude ascriptions), and (2) 'incomplete clauses' (i.e., clauses containing bound variables).

Indirect Denotation

Frege's example (p. 227, left): "Bebel believes that France's desire for vengeance could be assuaged by the restitution of Alsace-Lorraine."

Here he says that the embedded sentence ("France's desire … Alsace-Lorraine") has an **indirect** denotation — it has as its denotation its "ordinary" sense. Indeed, he must hold this, or (as we have seen) he will be faced with the consequence that anyone who believes **any** truth believes **every** truth, and anyone who believes **any** falsehood believes **every** falsehood.

Incomplete Clauses (bound variables)

Frege's example (p. 225, left): "If a number is smaller than 1 and greater than 0, then its square is also smaller than 1 and greater than 0."

Here his concern is different. The problem is that it sounds like a normal 'if … then' sentence, in which case each clause (antecedent and consequent) should express a complete thought (a proposition). But that is not the case here. Rather, we have a **general law** connecting the thought expressed by 'a number is smaller than 1 and

greater than 0' and the thought expressed by 'its square is smaller than 1 and greater than 0'.

Consequently, neither antecedent nor consequent here expresses a complete thought. To use a later terminology, antecedent and consequent here are not propositions but **propositional functions**.

The antecedent is the propositional function: '*x* is a number smaller than 1 and greater than 0'. The consequent is the propositional function: 'the square of *x* is smaller than 1 and greater than 0'. It is only when the two propositional functions are combined that they express a proposition (p. 225, left).

That is, neither '$x < 1 \wedge x > 0$' nor '$x^2 < 1 \wedge x^2 > 0$' expresses a proposition. But the conditional sentence formed from them in combination (universally generalized) does express a proposition:

$$\forall x\,((x < 1 \wedge x > 0) \rightarrow (x^2 < 1 \wedge x^2 > 0))$$

SUMMARY

On Frege's theory of sense and denotation, identity sentences can be both true and non-trivial if they are composed of singular terms that have the **same denotation** (required for truth) but **different senses** (required for non-triviality).

> "If *a* = *b*, then the denotation of '*a*' and of '*b*' is indeed the same and therefore also the truth value of '*a* = *b*' is the same as that of '*a* = *a*'. Nevertheless, the sense of '*b*' may differ from the sense of '*a*'; and therefore the proposition expressed by '*a* = *b*' may differ from that the proposition expressed by '*a* = *a*'; in that case the two sentences do not have the same cognitive significance" (p. 228).

But Frege's theory of sense and denotation leaves us with some problems and/or unanswered questions:

- What are the senses of proper names?
- How can sentences have truth-values as their denotations? Is there a way of avoiding this result? If a sentence does not denote a truth value, what in the world does it denote?
- How can we avoid the hierarchy of senses in multiply embedded propositional attitude ascriptions? If we can't, how can we figure out what such senses are?