

**Philosophy 120**  
**Introduction to Logic**

**Final Exam Prep**  
**Theory Quiz**

The final exam will include not only problems to solve (translating sentences, evaluating sentences in a world, identifying equivalent pairs of sentences, sketching informal proofs, filling in steps and justifications in a Fitch proof, etc.) but also some questions designed to test your understanding of the basic logical concepts of quantification theory.

Here are some **practice problems** about logical concepts (with a few on equivalence and translation thrown in). There is also an interactive version of this quiz—as you click on your answers, you will get instant feedback. (This page is designed for those whose computers do not display the logic symbols correctly. After reading the questions here, please do take the interactive version.) At the end of the interactive quiz, you will find a link to a separate page containing fuller explanations for each question.

1. Which of the following is **not** a wff?

- a.  $\exists x \text{Tet}(x) \wedge \neg \forall y \text{Tet}(y)$
- b.  $\exists x \text{Tet}(x) \wedge \text{Tet}(y)$
- c.  $\exists x (\text{Tet}(a) \wedge \neg \forall y \text{Tet}(y))$
- d.  $\exists b (\text{Tet}(b) \wedge \neg \forall y \text{Tet}(y))$
- e. None: they are all wffs.

2. Which of the following wffs is **not** a sentence?

- a.  $\exists x (\text{Cube}(x) \rightarrow \text{Large}(x))$
- b.  $\exists x \text{Cube}(x) \rightarrow \text{Large}(x)$
- c.  $\exists x (\text{Cube}(x) \rightarrow \text{Large}(b))$
- d. Both (b) and (c) are non-sentences.
- e. None: they are all sentences.

3. What is the truth-functional form of this sentence:

$$\neg \exists x \text{Tet}(x) \rightarrow \forall y (\text{Small}(y) \leftrightarrow \exists x (\text{Tet}(x) \wedge \text{Adjoins}(x, y)))?$$

- a.  $\neg A$
- b.  $A \rightarrow B$
- c.  $\neg A \rightarrow B$
- d.  $\neg A \rightarrow (B \leftrightarrow C)$
- e.  $\neg A \rightarrow (B \leftrightarrow (A \wedge C))$

4. Which of the following sentences is **not** of the truth-functional form  $A \vee (B \wedge \neg A)$ ?

- a.  $\text{Tet}(d) \vee (\forall y (\text{Small}(y) \leftrightarrow \neg \text{Cube}(y)) \wedge \neg \text{Tet}(d))$
- b.  $\neg \text{Tet}(d) \vee (\forall y \text{Small}(y) \wedge \text{Tet}(d))$
- c.  $\text{Tet}(d) \vee (\forall y \text{Small}(y) \wedge \neg \text{Tet}(d))$
- d.  $\exists x \neg \text{Tet}(x) \vee (\forall y \text{Small}(y) \wedge \neg \exists x \neg \text{Tet}(x))$
- e. None of the above: they are all of that form.

5. Which of the following is a “vacuously” true universal generalization?

- a.  $\forall x ((\text{Tet}(x) \wedge \text{Cube}(x)) \rightarrow \text{Large}(x))$
- b.  $\exists x ((\text{Tet}(x) \wedge \text{Cube}(x)) \rightarrow \text{Large}(x))$
- c.  $\forall x \forall y ((\text{SameCol}(x, y) \wedge \text{SameRow}(x, y)) \rightarrow \text{Larger}(x, y))$
- d. Both (a) and (b).
- e. All of the above.

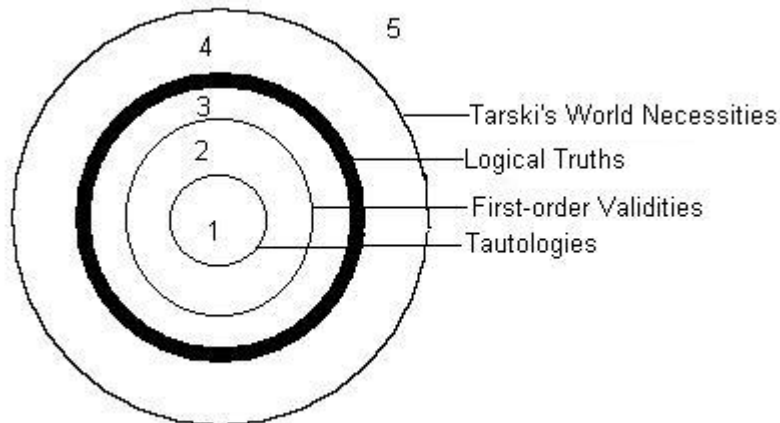
6. Suppose that R is a system of rules of inference for FOL, and suppose that R is **not sound**. Then:

- a. You can use R to deduce a conclusion that is not an FO-consequence of its premises
- b. There are some FO consequences that cannot be deduced from their premises by means of R
- c. Both (a) and (b).
- d. Neither (a) nor (b).

7. Suppose that R is a system of rules of inference for FOL, and suppose that R is **not complete**. Then:

- a. There is some sentence that is not FO-valid but that can be proved using R.
- b. There is some FO-valid sentence that R is not capable of proving.
- c. Both (a) and (b).
- d. Neither (a) nor (b).

The next group of questions (8-15) concerns the following Euler diagram:



8. Which area does the sentence  $\exists x \neg \text{Cube}(x) \rightarrow \neg \forall y \text{Cube}(y)$  fall into?

- a. 1 (it is a tautology)
- b. 2 (it is FO-valid, but not a tautology)
- c. 3 (it is a logical truth, but not FO-valid)
- d. 4 (it is a TW necessity, but not a logical truth)
- e. 5 (it is none of the above).

9. Which area does the sentence  $\exists x \text{Cube}(x) \rightarrow (\exists x \text{Cube}(x) \vee \forall y \text{Tet}(y))$  fall into?

- a. 1 (it is a tautology)
- b. 2 (it is FO-valid, but not a tautology)
- c. 3 (it is a logical truth, but not FO-valid)
- d. 4 (it is a TW necessity, but not a logical truth)
- e. 5 (it is none of the above).

10. Which area does the sentence  $\exists x \exists y \neg \text{Adjoins}(x, y)$  fall into?

- a. 1 (it is a tautology)
- b. 2 (it is FO-valid, but not a tautology)
- c. 3 (it is a logical truth, but not FO-valid)
- d. 4 (it is a TW necessity, but not a logical truth)
- e. 5 (it is none of the above).

11. Which area does the sentence:

$\forall x (\text{Cube}(x) \rightarrow \text{Large}(x)) \rightarrow \neg \exists x (\text{Cube}(x) \wedge \exists y \text{Adjoins}(x, y))$   
fall into?

- a. 1 (it is a tautology)
- b. 2 (it is FO-valid, but not a tautology)
- c. 3 (it is a logical truth, but not FO-valid)
- d. 4 (it is a TW necessity, but not a logical truth)
- e. 5 (it is none of the above).

12. Which area does the sentence  $\exists x (\text{Cube}(x) \rightarrow \forall y \text{Cube}(y))$  fall into?

- a. 1 (it is a tautology)
- b. 2 (it is FO-valid, but not a tautology)
- c. 3 (it is a logical truth, but not FO-valid)
- d. 4 (it is a TW necessity, but not a logical truth)
- e. 5 (it is none of the above).

13. Which of the following sentences falls into area 2 (i.e., is FO-valid but not a tautology)?

- a.  $(\exists x \text{Cube}(x) \wedge \forall y \text{Tet}(y)) \rightarrow \forall y \text{Tet}(y)$
- b.  $(\exists x \text{Cube}(x) \wedge \forall y \text{Tet}(y)) \rightarrow \text{Tet}(b)$
- c.  $\forall x (\text{Tet}(x) \rightarrow \forall x \text{Tet}(x))$
- d.  $\forall x \text{Tet}(x) \rightarrow \forall x \text{Tet}(x)$
- e. Both (b) and (c)
- f. All of the above.

14. Which of the following sentences falls into area 3 (i.e., is a logical truth but not FO-valid)?

- a.  $a = b \rightarrow b = a$
- b.  $\forall x \forall y (\text{Tet}(x) \rightarrow \text{Tet}(y))$
- c.  $\forall x \text{Tet}(x) \rightarrow \forall y \text{Tet}(y)$
- d.  $\forall x \forall y ((\neg \text{Larger}(x, y) \wedge \neg \text{Smaller}(x, y)) \rightarrow \text{SameSize}(x, y))$
- e.  $\forall x \forall y (\text{Adjoins}(x, y) \rightarrow (\neg \text{Large}(x) \wedge \neg \text{Large}(y)))$

15. Which of the following sentences falls into area 4 (i.e., is TW-necessary (true in every Tarski world) but not a logical truth)?

- a.  $\forall x \forall y \forall z (\text{Between}(x, y, z) \rightarrow \neg(x = y \vee x = z))$
- b.  $\forall x (\text{Cube}(x) \rightarrow (\neg \text{Tet}(x) \wedge \neg \text{Dodec}(x)))$
- c.  $\forall x ((\neg \text{Tet}(x) \wedge \neg \text{Dodec}(x)) \rightarrow \text{Cube}(x))$
- d.  $\forall x \forall y \forall z ((\text{Larger}(x, y) \wedge \text{Larger}(x, z) \wedge \neg \text{SameSize}(y, z)) \rightarrow \text{Large}(x))$
- e. Both (c) and (d).

16. Which of the following relations is **not transitive**?

- a. Larger
- b. Adjoins
- c. SameCol
- d. =
- e. All of the above (i.e., all are transitive).

17. Which of the following relations is **symmetric**?

- a. SameRow
- b. Adjoins
- c.  $\text{SameRow} \wedge \text{Adjoins}$  (i.e.: *x is in the same row as y and adjoins y*)
- d. =
- e. All of the above

18. Which of the following relations is **reflexive**?

- a. Larger
- b. SameRow
- c. Adjoins
- d. FrontOf
- e.  $\text{Larger} \vee \text{Smaller}$  (i.e., the relation: *x is either larger or smaller than y*)

19. Which of the following relations is both **irreflexive** and **asymmetric**?

- a. Larger
- b. Adjoins
- c.  $\neq$  (non-identity)
- d.  $\text{Larger} \vee \text{SameSize}$  (i.e., the relation: *x is larger than or the same size as y*)

20. To which of the following FOL sentences is  $\forall x (\text{Cube}(x) \rightarrow \neg \text{Small}(x))$  equivalent?

- a.  $\forall x (\text{Small}(x) \rightarrow \neg \text{Cube}(x))$
- b.  $\neg \exists x (\text{Small}(x) \wedge \text{Cube}(x))$
- c.  $\forall x \text{Cube}(x) \rightarrow \forall x \neg \text{Small}(x)$
- d.  $\forall x \text{Cube}(x) \rightarrow \neg \forall x \text{Small}(x)$
- e. Both (a) and (b).

21. To which of these FOL sentences is  $\neg\forall x \exists y \exists z (\text{Cube}(x) \rightarrow \text{Between}(x, y, z))$  equivalent?

- a.  $\exists x \neg\exists y \exists z (\text{Cube}(x) \rightarrow \text{Between}(x, y, z))$
- b.  $\exists x \forall y \forall z \neg(\text{Cube}(x) \rightarrow \text{Between}(x, y, z))$
- c.  $\exists x \forall y \forall z (\text{Cube}(x) \wedge \neg\text{Between}(x, y, z))$
- d. All of the above.

22. Which of the following is an equivalent pair of sentences?

- a.  $(\forall y \text{Cube}(y) \vee \forall y \text{Large}(y))$  and  $\forall y (\text{Cube}(y) \vee \text{Large}(y))$
- b.  $\exists x (\text{Cube}(x) \wedge \text{Small}(x))$  and  $(\exists x \text{Cube}(x) \wedge \exists x \text{Small}(x))$
- c.  $(\exists x \text{Small}(x) \rightarrow \exists x \text{Dodec}(x))$  and  $\exists x (\text{Small}(x) \rightarrow \text{Dodec}(x))$
- d.  $(\forall x \text{Small}(x) \rightarrow \forall x \text{Dodec}(x))$  and  $\forall x (\text{Small}(x) \rightarrow \text{Dodec}(x))$
- e. None of the above.

23. Which of the following is **not** an equivalent pair of sentences?

- a.  $(\exists x \text{Cube}(x) \wedge \text{Tet}(b))$  and  $\exists x (\text{Cube}(x) \sqcap \text{Tet}(b))$
- b.  $(\exists x \text{Tet}(x) \rightarrow \text{Dodec}(c))$  and  $\exists x (\text{Tet}(x) \rightarrow \text{Dodec}(c))$
- c.  $(\exists x \text{Tet}(x) \rightarrow \text{Dodec}(c))$  and  $\forall x (\text{Tet}(x) \rightarrow \text{Dodec}(c))$
- d.  $(\forall y \text{Cube}(y) \vee \text{Tet}(b))$  and  $\forall y (\text{Cube}(y) \vee \text{Tet}(b))$
- e. None: they are all equivalent pairs.

24. Which of the following sentences is **not** FO-valid?

- a.  $\forall x \forall y \text{SameCol}(x, y) \rightarrow \forall y \forall x \text{SameCol}(x, y)$
- b.  $\exists x \exists y \text{SameCol}(x, y) \rightarrow \exists y \exists x \text{SameCol}(x, y)$
- c.  $\forall x \exists y \text{SameCol}(x, y) \rightarrow \exists y \forall x \text{SameCol}(x, y)$
- d.  $\exists x \forall y \text{SameCol}(x, y) \rightarrow \forall y \exists x \text{SameCol}(x, y)$
- e. None: they are all FO-valid.

25. Which of the following is a correct translation of *Only tetrahedra adjoin cubes*?

- a.  $\forall x (\text{Tet}(x) \rightarrow \exists y (\text{Cube}(y) \wedge \text{Adjoins}(x, y)))$
- b.  $\forall x (\exists y (\text{Cube}(y) \wedge \text{Adjoins}(x, y)) \rightarrow \text{Tet}(x))$
- c.  $\forall x (\text{Tet}(x) \leftrightarrow \exists y (\text{Cube}(y) \wedge \text{Adjoins}(x, y)))$
- d.  $\forall x (\forall y (\text{Cube}(y) \rightarrow \text{Adjoins}(x, y)) \rightarrow \text{Tet}(x))$
- e. Both (a) and (c).

26. Which of the following is **not** a correct translation of *Nothing is a dodecahedron unless it is small?*

- a.  $\forall x (\text{Dodec}(x) \rightarrow \text{Small}(x))$
- b.  $\forall x (\neg \text{Small}(x) \rightarrow \neg \text{Dodec}(x))$
- c.  $\neg \exists x (\text{Dodec}(x) \wedge \neg \text{Small}(x))$
- d.  $\neg \exists x (\text{Dodec}(x) \vee \text{Small}(x))$
- e.  $\forall x (\neg \text{Dodec}(x) \vee \text{Small}(x))$

27. Which of the following is a correct translation of *No one admires anyone?* (You may assume the domain to be restricted to *persons*.)

- a.  $\forall x \exists y \neg \text{Admires}(x, y)$
- b.  $\neg \exists x \exists y \neg \text{Admires}(x, y)$
- c.  $\forall x \neg \exists y \text{Admires}(x, y)$
- d.  $\forall x \forall y \neg \text{Admires}(x, y)$
- e. Both (c) and (d).

28. Which English sentence is a correct translation of this FOL sentence?

$$\forall x \forall y \forall z ((\text{Tet}(x) \wedge \text{Tet}(y) \wedge \text{Tet}(z)) \rightarrow (x = y \vee x = z \vee y = z))$$

- a. There are at least two tetrahedra.
- b. There are at most two tetrahedra.
- c. There are exactly two tetrahedra.
- d. There are at least three tetrahedra.
- e. There are at most three tetrahedra.

29. Which English sentence is a correct translation of this FOL sentence?

$$\exists x (\text{Cube}(x) \wedge \forall y ((\text{Cube}(y) \wedge x \neq y) \rightarrow (\text{Larger}(x, y))))$$

- a. For every cube, there is another cube that is larger.
- b. There is a cube that is larger than every other cube.
- c. There is a largest cube.
- d. Of any two cubes, one is larger than the other.
- e. Both (b) and (c).