

Chapter 7: Conditionals

We next turn to the logic of conditional, or “if ... then,” sentences. We will be treating *if ... then* as a **truth-functional connective** in the sense defined in chapter 3: the truth value of a compound sentence formed with such a connective is a function of (i.e., is completely determined by) the truth value of its components.

Not all sentence-forming connectives are truth-functional. Consider *because*. It is obvious that we could not fill out a truth table for the sentence *P because Q*. How would we fill out the value of *P because Q* in the row where *P* and *Q* are both true? There is no way to do this.

Consider a sentence like *Tom left the party because Lucy sneezed*. Suppose that both component sentences are true. What is the truth value of the entire compound? You can't tell—it could be either. If Tom and Lucy had prearranged that Lucy would sneeze as a signal to Tom that it was time to leave, the sentence would be true. But if Lucy just happened to sneeze and Tom left, but for some other reason, it would be false. So *because* is not a truth-functional connective.

This should be a tip-off that you should not read any kind of **causal connection** into the *if ... then* that we will be introducing into FOL.

§ 7.1 Material conditional symbol: \rightarrow

Truth table definition of \rightarrow

Here is the truth table that appears on p. 178:

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Here *P* is the **antecedent** and *Q* is the **consequent**. (The antecedent is on the **left**, with the arrow pointing **from** it; the consequent is on the **right**, with the arrow pointing **to** it.)

As the truth table shows, a conditional sentence comes out true in every case except the one where the antecedent is true and the consequent false. That is,

$P \rightarrow Q$ is equivalent to both of these Boolean forms:

$$\neg P \vee Q \qquad \neg(P \wedge \neg Q)$$

Hence, \rightarrow adds no new expressive power to FOL (anything we can say using \rightarrow we can also say without it, just using \neg and \vee or \neg and \wedge). But the new symbol makes it easier to produce FOL sentences that correspond naturally to sentences of English.

English forms of the material conditional

It is convenient to read \rightarrow sentences in English using *if ... then*. That is, we read $P \rightarrow Q$ (“*P* arrow *Q*”) as *if P, then Q*. But there are many other ways in English of saying the same thing, and hence many other ways of reading \rightarrow sentences in English:

$$\begin{array}{cccc} Q \text{ if } P & P \text{ only if } Q & Q \text{ provided that } P & Q \text{ in case } P \\ \text{Provided } P, Q & \text{In the event that } P, Q & & \end{array}$$

Note the variation in word order: in English (unlike FOL) the antecedent (in this case P) doesn't always come first.

If you are looking for a way of reading $P \rightarrow Q$ in English that begins with the sentence that replaces P , the only formulation that works is *P only if Q*.

People sometimes read $P \rightarrow Q$ as " p implies q ." This is handy, in that it gives you a way to read the FOL sentence from left to right, symbol-for-symbol, maintaining the word order. But there is something misleading about it, for it suggests a confusion between the *truth* of an *if...then* sentence and a *logical implication*. That is because " p implies q " is even more often used as a shorthand for " p logically implies q ," which expresses the relation of logical consequence: to say that p logically implies q is to say that q is a logical consequence of p .

But the mere fact that $P \rightarrow Q$ is true does not mean that P logically implies Q . It simply means that either Q is true or P is false. Hence it is probably best to avoid reading \rightarrow as *implies*.

If vs. only if

This is a difference that beginners often find baffling. The authors of *LPL* explain (p. 180) the difference in terms of necessary and sufficient conditions. *Only if* introduces a **necessary** condition: *P only if Q* means that the truth of Q is necessary, or required, in order for P to be true. That is, *P only if Q* rules out just one possibility: that P is true and Q is false. But that is exactly what $P \rightarrow Q$ rules out. So it's obviously correct to read $P \rightarrow Q$ as *P only if Q*.

If, on the other hand, introduces a **sufficient** condition: *P if Q* means that the truth of Q is sufficient, or enough, for P to be true as well. That is, *P if Q* rules out just one possibility: that Q is true and P is false. But that is exactly what $Q \rightarrow P$ rules out. So it's obviously correct to read $Q \rightarrow P$ as *P if Q*.

Example

To get really clear on the difference between *if* and *only if*, consider the following sentences:

1. a and b are the same size if $a = b$
 $a = b \rightarrow \text{SameSize}(a, b)$
2. a and b are the same size only if $a = b$
 $\text{SameSize}(a, b) \rightarrow a = b$

(1) is a logical truth: if a and b are one and the same object, then there is no difference between a and b in size, shape, location, or anything else.

But (2) makes a substantive claim that could well be false: it is possible for a and b to be the same size but be two different objects. a and b might be a pair of large cubes, or a might be a large cube and b a large tetrahedron.

Now consider the following pair:

3. $a = b$ only if a and b are the same size
 $a = b \rightarrow \text{SameSize}(a, b)$
4. $a = b$ if a and b are the same size
 $\text{SameSize}(a, b) \rightarrow a = b$

(3), like (1), is a logical truth: a and b can't be identical without being the same size—if $a = b$, then a and b are one and the same object, which of course has the same size as itself! But that's just what (1) says, so (3) and (1) are equivalent. (And that is why they have the same FOL translation.) (4), on the other hand, comes out false if a and b are two different objects of the same size. That is, (4) is equivalent to (2), and so they also have the same FOL translation.

You can confirm this by evaluating these sentences (in file [Ch7Ex1.sen](#)) in some different worlds (start with [Ch7Ex1.wld](#)).

Unless

The best way to think of *unless* is that it means *if not*. So you can read *not P unless Q* as *not P if not Q*, and translate that into FOL as:

$$\neg Q \rightarrow \neg P$$

As we'll see, this FOL sentence is equivalent to

$$P \rightarrow Q$$

And this, in turn, gives us another way to read \rightarrow sentences: “not [antecedent] unless [consequent],” which clearly, and correctly, expresses the fact that the truth of the consequent is a necessary condition for the truth of the antecedent.

We learn something else from this last observation. Since $P \rightarrow Q$ expresses the English *not P unless Q*, and $P \rightarrow Q$ is equivalent to $\neg P \vee Q$, these English and FOL sentences say the same thing:

$$\neg P \vee Q$$

not P unless Q

And what we now see is that, strangely enough, the English *unless* corresponds to the FOL \vee . In effect, we can treat *unless* as meaning *or*.

Summary

- The English forms *Q if P* and *P only if Q* are equivalent, and correspond to the FOL sentence $P \rightarrow Q$.
- But the English forms *P if Q* and *P only if Q* are **not** equivalent. The first goes into FOL as $Q \rightarrow P$; the second as $P \rightarrow Q$.
- *Only if* introduces the consequent; *if* (without the *only*) introduces the antecedent.
- Think of *unless* as meaning *if not*; alternatively, just replace *unless* with *or*, i.e., translate *unless* into FOL by means of \vee .

§ 7.2 Biconditional symbol: \leftrightarrow

\leftrightarrow and *if and only if*

$P \leftrightarrow Q$ corresponds to *P if, and only if, Q*. It is thus really a **conjunction** of a pair of one-way conditionals:

$$(P \rightarrow Q) \wedge (Q \rightarrow P)$$

Truth table for \leftrightarrow

Here is the truth table that appears on p. 182. Note that $P \leftrightarrow Q$ comes out true whenever the two components **agree** in truth value:

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Iff

If and only if is often abbreviated as *iff*. Watch for this.

Just in case

Mathematicians often read $P \leftrightarrow Q$ as *P just in case Q* (or sometimes as *P exactly in case Q*, or as *P exactly if Q*). Watch for this, too.

Biconditionals and equivalence: \leftrightarrow vs. \Leftrightarrow

The FOL sentence $P \leftrightarrow Q$ does not say that P and Q are logically equivalent. It says something weaker, namely, that they (happen to) agree in truth value. The claim that P and Q are logically equivalent is stronger—it amounts to the claim that their biconditional is not just true, but a **logical truth**.

For example, in a world in which b is a large cube, the sentences **Cube**(b) and **Large**(b) are both true, and the sentences **Tet**(b) and **Small**(b) are both false. Hence these two biconditionals:

$$\text{Cube}(b) \leftrightarrow \text{Large}(b) \qquad \text{Tet}(b) \leftrightarrow \text{Small}(b)$$

are both true. But **Cube**(b) is not equivalent to **Large**(b), because there are worlds in which they differ in truth value.

On the other hand, the sentences **Cube**(b) and $\neg\neg$ **Cube**(b) are logically equivalent—there is no world in which they differ in truth value. That is, their biconditional is a logical truth—true in every world.

To say that two sentences are equivalent, we can use the symbol \Leftrightarrow . That is, we can write:

$$\text{Cube}(b) \Leftrightarrow \neg\neg\text{Cube}(b)$$

to mean that **Cube**(b) and $\neg\neg$ **Cube**(b) are logically equivalent. **But the sentence containing \Leftrightarrow is not an FOL sentence.** It is just a way of saying that the FOL sentence $\text{Cube}(b) \leftrightarrow \neg\neg\text{Cube}(b)$ is a logical truth, or, alternatively, of saying that the two sentences **Cube**(b) and $\neg\neg$ **Cube**(b) are logically equivalent.

§ 7.3 Conversational implicature

It is easy to misread what a sentence says because one mistakenly attaches to the meaning of the sentence certain additional information, information that is frequently conveyed by the **assertion** of the sentence, even though it is not strictly speaking part of what is **said**, or part of what the sentence means.

Example: Tom asks whether the picnic will be held, and Betty says “If it rains, the picnic will not be held.” Strictly speaking, what Betty has said is that it is not the case that both rain and the picnic will occur. Tom may well infer, however, that Betty said something additional—that if it does **not** rain, the picnic **will** be held.

But Betty did **not** say this. Tom may well infer that Betty must have **meant** this, for if Betty were aware of any other situation in which the picnic would not be held, she would have mentioned it. Her failure to do so strongly suggests that, in her view, rain is the only thing that would stop the picnic.

We’ll use the terminology of H. P. Grice to describe this situation. Betty **said** that if it rains, the picnic will not be held; but in saying this (in this situation, using these words) she **conversationally implicated** that if it doesn’t rain, the picnic will be held.

The test for conversational implicature is Grice’s “cancellability” test. Suppose a speaker utters a sentence *S*, and the hearer draws the conclusion that *P*. The question now arises whether, in uttering *S*, the speaker has **said** that *P* or only **implicated** that *P*. The test is this: see whether the conclusion the hearer draws (that *P*) can be explicitly “cancelled” by adding *and not P* to the original sentence *S*. If the resulting conjunction *S and not P* is a contradiction, *P* is part of what was said; if the result is not a contradiction, *P* is only implicated, not part of what was said.

Applying the test in this case: can Betty say this, without contradicting herself?:

*If it rains, the picnic will not be held; and even if it doesn’t rain,
the picnic may still not be held.*

Surely there is no contradiction here. Betty may be alluding to the fact that there are many conditions that are sufficient for calling off the picnic: rain, snow, the death of one of the hosts, nuclear annihilation, etc. Only the first has a high enough probability to be worth mentioning, so that is why Betty neglects the other conditions and why she doesn’t attach the second conjunct to her assertion.

Contrast the following case, where the cancellability test gives a different result. The speaker says *Neither Dave nor Sally was in class today*. Did the speaker say that Sally was not in class today? (Notice: the speaker did not utter the words *Sally was not in class today*.) The test is this: is the following self-contradictory?:

Neither Dave nor Sally was in class today, but Sally was in class today.

This is obviously self-contradictory, so the speaker really did **say**, and not just implicate, that Sally was not in class today.