CHAPTER 47

VELOCITY AND TIME

So we see that we cannot attach any *absolute* signification to the concept of simultaneity, but that two events which, viewed from a system of coordinates, are simultaneous, can no longer be looked upon as simultaneous events when envisaged from a system which is in motion relatively to that system.

Albert Einstein, "On the Electrodynamics of Moving Bodies" (1905)

47.1 PROPER LENGTH AND PROPER TIME

The special theory of relativity describes consequences of two assumptions: For all inertial observers the laws of nature are the same, and the speed of light is constant. These properties are inherent in the Lorentz transformation, which connects measurements of space and time coordinates in one frame with those in another. This chapter explores further consequences of these equations as they apply to measurements of velocity and time. First we recapitulate some of the ideas concerning length and time in special relativity that were described in Chapter 46.

As usual we consider two reference frames S and S', where S' is regarded as moving with constant velocity v parallel to the positive x axis of S. If the origins of S and S' coincide at time $t = t' = 0$ the Lorentz transformation takes the form

$$x = \gamma(x' + vt'), \tag{46.15x}$$

$$y = y', \tag{46.15y}$$

$$z = z', \tag{46.15z}$$

$$t = \gamma(t' + vx'/c^2), \tag{46.15t}$$

while the inverse transformation is given by

$$x' = \gamma(x - vt), \tag{47.1x'}$$

$$y' = y, \tag{47.1y'}$$

$$z' = z, \tag{47.1z'}$$

$$t' = \gamma(t - vx/c^2), \tag{47.1t'}$$

where γ is the dilation factor, a number greater than 1 given by

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

In the last chapter we learned that if a rod at rest along the x axis in one frame has length L_0 when measured by an observer in that frame, then an observer in the other frame determines the length of the rod to be contracted by the factor $1/\gamma$. The length L_0 is called the *proper length*, whereas L_0/γ is called the *contracted length*.

The Lorentz equations also imply time dilation. If T_0 is the time between two events measured on a clock that is at rest in one frame, then an observer in the other frame determines that the time between the same two events is greater than T_0 by a factor γ. For example, if two events E_1 and E_2 occur at the same place $x_1 = x_2$ in the S frame but at different times $t_1 < t_2$, then by applying Eq. (47.1t′) twice and subtracting we find

$$t_2' - t_1' = \gamma(t_2 - t_1).$$

In other words, if the observer in S thinks that the time between a tick and tock is 1 s, the observer in S' measures this time to be γ s. Because $\gamma > 1$ the observer in S thinks that the S' clock is running slow.

If we call T_0 the *proper time* – the time between two events measured on a clock where the events occur at the same place – and T the time between the events measured by an observer moving with speed v, the foregoing relation becomes $T = \gamma T_0$ or

$$T = \frac{T_0}{\sqrt{1 - v^2/c^2}}.$$

Because the two events occur at the same position in the rest frame, one clock suffices to measure the proper time. In the moving frame, the events occur at different positions, and hence to measure the time between the two events, two clocks would be used, one at each position. A useful mnemonic is, "more clocks, greater time." Whenever we use a familiar phrase such as "the stationary observer thinks the moving clock is running slow," we are really envisaging a series of operations related to distinct events involving as many clocks and signals as necessary.

The time dilation factor γ and the length contraction factor $1/\gamma$ are reciprocals. When v^2 is small compared to c^2, γ is nearly 1 and the relativistic effects become negligible. On the other hand, as $|v|$ gets closer to the speed of light, γ becomes very large and the relativistic effects become more significant. In the limiting case when $v^2 \to c^2$ we find $\gamma \to \infty$ and $1/\gamma \to 0$, which means that a meter stick shrinks to a point and a clock stops completely. We also know that the inequality $v^2 > c^2$ would imply that γ is the square root of a negative number, and this makes no physical sense. In other words, no speed can exceed that of light. That's what Poincaré had in mind when he made his famous speech in St. Louis in 1904 and remarked that in the new mechanics nothing will be able to travel faster than the speed of light.

Example 1

An extremely fast train moving at a speed of $0.6c$ passes two fixed posts on the ground that are separated by 80 m according to observers on the ground. These observers note that at some given time the position of the rear end of the train coincides with that of one of the posts, and at the same instant the position of the front end coincides with that of the other post. Calculate the length of the train according to (a) an observer on the ground and (b) an observer on the train. Calculate the amount of time it takes for the full length of the train to pass one post according to (c) an observer on the ground and (d) an observer on the train. (e) Calculate the distance between posts according to an observer on the train.

(a) Observers on the ground measure the length of the train to be $L = 80$ m, the same as the distance they measured between the posts.

(b) Let L_0 denote the length of the train measured by an observer on the train. The length L measured in (a) is a contraction of L_0 by the factor $1/\gamma$. Now

$$1/\gamma = \sqrt{1 - (0.6)^2} = 0.8,$$

so

$$L_0 = L\gamma = L/0.8 = (80 \text{ m})/0.8 = 100 \text{ m}.$$

(c) According to an observer on the ground, the train must travel its full length L at speed v to pass one post. Therefore, the time it takes is

$$T = L/v = (80 \text{ m})/[(0.6)(3 \times 10^8 \text{ m/s})] = 0.44 \times 10^{-6} \text{ s}.$$

(d) An observer on the train sees the posts move past him. With his clock he measures T_0 for the time between the first and second post passing him. The ground-based observer measures the time interval between these same events to be the dilated time T of part (c). Therefore, $T = \gamma T_0$, so

$$T_0 = T/\gamma = (0.44 \times 10^{-6}\ \text{s})(0.8) = 0.35 \times 10^{-6}\ \text{s}.$$

(e) According to the observer on the train, the posts move past him at speed $0.6c$ in time T_0. Therefore, the distance between the posts is simply $d = vT_0 = 0.6(3 \times 10^8\ \text{m/s})(0.35 \times 10^{-6}\ \text{s})$, or $d = 64$ m. The same result can also be obtained by length contraction, $d = 80\ \text{m}/\gamma = (0.8)80\ \text{m} = 64$ m.

Questions

Questions 1 through 4 refer to the following hypothetical situation: A class of physics students is given a quiz to be completed in 10 min according to the instructor's clock. Relative to the students, the instructor moves at a speed of $0.8c$ and he sends a light signal to the class when his clock reads 10 min. The students stop writing when the light signal reaches them.

1. According to the class, what is the instructor's position when the instructor's clock reads 10 min? (Assume that the two coordinate systems coincide at $t = t' = 0$.)
2. According to the class, what is the time when the instructor's clock reads 10 min?
3. How long does it take the light signal to reach the class?
4. According to the students, how much time did they have to work the quiz?
5. Let T_1 be the lifetime of a meson (a type of radioactive particle) as measured in an inertial frame S_1 in which the meson is at rest. In a different inertial frame, S_2, moving with velocity v_{12} relative to S_1, this lifetime is measured to be T_2. Let T_3 be the lifetime measured in a third frame, S_3, which has a velocity v_{13} relative to S_1 and v_{23} relative to S_2. Among the following equations relating measurements of lifetimes, determine those that are correct, and correct those that are incorrect.

   (a) $T_1 = T_3\sqrt{1 - v_{13}^2/c^2}$.

   (b) $T_2 = T_1\sqrt{1 - v_{12}^2/c^2}$.

   (c) $T_3 = T_2/\sqrt{1 - v_{23}^2/c^2}$.

   (d) $T_3 = T_1/\sqrt{1 - v_{23}^2/c^2}$.
6. A spaceship must travel 8.0×10^{12} m in order to reach its home base. If the life support systems can function for 48 h, what is the minimum speed that the ship must have if the crew is to survive?

47.2 COMBINATIONS OF VELOCITIES IN SPECIAL RELATIVITY

In Galilean relativity, studied in Chapter 45, we learned that if frame S' moves with constant velocity $\mathbf{v}$ with respect to frame S and if the position $\mathbf{r}$ of a body in S is related to its position $\mathbf{r}'$ in S' by the equation

$$\mathbf{r} = \mathbf{r}' + r_0 + \mathbf{v}t, \tag{45.4}$$

then the velocities $\mathbf{u} = d\mathbf{r}/dt$ and $\mathbf{u}' = d\mathbf{r}'/dt$ differ only by a constant amount,

$$\mathbf{u} = \mathbf{u}' + \mathbf{v}. \tag{45.5}$$

In particular, if the relative motion is entirely in the x direction, so that $y = y'$ and $z = z'$, the vector equation (45.5) gives three scalar equations for the velocity components:

$$u_x = u'_x + v, \qquad u_y = u'_y, \qquad u_z = u'_z,$$

where now v is the velocity of frame S' relative to S. A moment's reflection tells us that the first of these equations cannot possibly hold in special relativity. For example, if v is close to the speed of light, say $v = 0.8c$, and if a particle is moving at great speed in S', say $u'_x = 0.7c$, then the formula for u_x gives $1.5c$ for the speed of the particle in frame S, a speed greater than the speed of light, which is impossible. This means that in special relativity velocities do not combine by simple vector addition.

To find exactly how velocities do combine in special relativity, we refer to the Lorentz equations. Differentiating Eq. (46.15x) with respect to t we find

$$\frac{dx}{dt} = \gamma\left(\frac{dx'}{dt} + v\,\frac{dt'}{dt}\right). \tag{47.2}$$

By the chain rule we have

$$\frac{dx'}{dt} = \frac{dx'}{dt'}\,\frac{dt'}{dt},$$

so Eq. (47.2) becomes

$$\frac{dx}{dt} = \gamma\left(\frac{dx'}{dt'} + v\right)\frac{dt'}{dt}. \tag{47.3}$$

With the notation

$$u_x = \frac{dx}{dt} \quad \text{and} \quad u'_x = \frac{dx'}{dt'},$$

Eq. (47.3) becomes

$$u_x = \gamma(u'_x + v)\,\frac{dt'}{dt}. \tag{47.4}$$

By differentiating Eq. (46.15t) with respect to t' we find

$$\frac{dt}{dt'} = \gamma\left(1 + vu'_x/c^2\right). \tag{47.5}$$

In Example 2 we show that $1 + vu'_x/c^2 \neq 0$, so we can take reciprocals in Eq. (47.5) and substitute into (47.4). The factor γ cancels and we obtain

$$u_x = \frac{u'_x + v}{1 + vu'_x/c^2}. \tag{47.6x}$$

This is the law of combination of velocities in special relativity. The sum of velocities in the numerator, $u'_x + v$, is what Galilean relativity gives for u_x, but in special relativity this sum must be divided by the quantity $1 + vu'_x/c^2$. If both v and u'_x have the same sign this quantity is greater than 1, and hence u_x is smaller than what it would be in Galilean relativity.

For example, if $v = 0.8c$ and $u'_x = 0.7c$, as in the previous example, we find

$$u_x = \frac{0.7c + 0.8c}{1 + (0.8)(0.7)} = \frac{1.5c}{1.56} = 0.96c,$$

a speed less than that of light. In fact, even in extreme cases the velocities cannot combine to exceed the speed of light. For example, if frame S' is traveling at velocity v relative to S and if an observer in S' sees a beam of light traveling in the same direction at speed c, then $u'_x = c$ and Eq. (47.6x) tells us that an observer in S sees the same beam traveling at velocity u_x where

$$u_x = \frac{c + v}{1 + cv/c^2} = \frac{c(c + v)}{c + v} = c.$$

In other words, the speed of light is the same for all inertial observers, as it should be according to Einstein's second postulate.

Example 2

Show that we always have $1 + vu'_x/c^2 \neq 0$.

The only way we could have $1 + vu'_x/c^2 = 0$ is if

$$\left(\frac{v}{c}\right)\left(\frac{u'_x}{c}\right) = -1.$$

But in deriving the Lorentz equations in Chapter 46 we found that $v^2 < c^2$, so the first factor on the left has absolute value < 1, and the only way the product could be -1 is for the other factor to have absolute value > 1. This would mean $|u'_x| > c$, which is impossible because nothing travels faster than the speed of light. Therefore, $1 + vu'_x/c^2 \neq 0$.

Equation (47.6x) gives us the x component of velocity in the S frame in terms of the x' component of velocity in the S' frame. What about the velocity components in the y and z directions? You might think that $u_y = u'_y$ and $u_z = u'_z$ because $y = y'$ and $z = z'$. We'll see in a moment that these equations do not hold. The physical reason is that a velocity component is equal to a length divided by a time. The observers will agree on the distance traveled in the y and z directions, but they will not agree on the time it took. So the velocity components should be different. To find the exact velocity relations we proceed as we did in obtaining Eq. (47.6x). Differentiating Eq. (46.15y) with respect to t we get

$$\frac{dy}{dt} = \frac{dy'}{dt} = \frac{dy'}{dt'}\frac{dt'}{dt}.$$

With the notation $u_y = dy/dt$ and $u'_y = dy'/dt'$ this becomes

$$u_y = u'_y \frac{dt'}{dt}.$$

In Galilean relativity we have $t = t'$, hence $dt'/dt = 1$ and $u_y = u'_y$. But in special relativity the times are related by Eq. (46.15t), and dt'/dt is determined by taking reciprocals in Eq. (47.5). This gives us

$$u_y = \frac{u'_y}{\gamma\left(1 + vu'_x/c^2\right)}. \tag{47.6y}$$

In the same way we obtain the corresponding result for $u_z = dz/dt$. In summary, the transformation equations for velocities in special relativity are given by

$$u_x = \frac{u'_x + v}{1 + vu'_x/c^2}, \tag{47.6x}$$

$$u_y = \frac{u'_y}{\gamma\left(1 + vu'_x/c^2\right)}, \tag{47.6y}$$

$$u_z = \frac{u'_z}{\gamma\left(1 + vu'_x/c^2\right)}. \tag{47.6z}$$

These equations are a bit more complicated than their Galilean counterparts because of the extra factors in the denominators. Note that an extra factor γ appears in the denominator of the last two equations. Despite appearances, these equations are symmetric in the primed and unprimed quantities. The corresponding equations for expressing the velocity components u'_x, u'_y, and u'_z in terms of u_x, u_y, and u_z can be obtained by simply replacing v by $-v$ and interchanging primed and unprimed quantities. Verification of this fact is requested in Question 13.

Example 3

Assume frame S' moves with velocity v in a direction parallel to the positive x axis of frame S. Consider a particle moving parallel to the positive y axis of frame S with velocity u_y. Determine the velocity components u'_x, u'_y, and u'_z of this particle in frame S'.

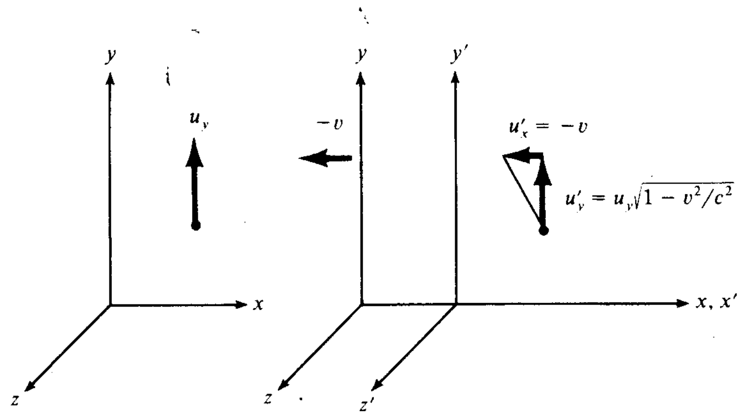
Because the particle is moving parallel to the y axis in frame S we have $u_x = u_z = 0$. Therefore, from Eqs. (47.6x) and (47.6z) we find

$$u'_x = -v \quad \text{and} \quad u'_z = 0.$$

Equation (47.6y) now gives us

$$u'_y = \gamma u_y(1 + vu'_x/c^2) = \gamma u_y(1 - v^2/c^2) = u_y\sqrt{1 - v^2/c^2}\,.$$

The geometric relation between u_y and u'_y is shown in the accompanying figure.



Example 4

A fast train moves at a speed of $0.6c$ relative to the ground. Inside the train a runner moves at a speed $0.6c$ relative to the train. Calculate the velocity components of the runner relative to the ground if he is running (a) in the same direction as the train and (b) in the opposite direction.

Let S denote the ground frame and S' that of the train. We are given that the relative speed of S' to S is $v = 0.6c$, whereas $u'_x = 0.6c$ in part (a) and $u'_x = -0.6c$ in part (b). According to the velocity transformation, we have

$$u_x = \frac{u'_x + v}{1 + vu'_x/c^2}. \qquad (47.6x)$$

Substituting values we find for (a) $u'_x = 0.6c$ implies $u_x = (40/41)c$, and for (b) $u'_x = -0.6c$ implies $u_x = 0$. Similarly, we find $u'_y = u'_z = 0$ so $u_y = u_z = 0$ for both (a) and (b).

Questions

Questions 7 through 9 refer to the following situation: A spacecraft of proper length 100 m moves at a speed $3c/5$ with respect to the earth and contains an alien at the rear of the craft. The alien fires a bullet toward the front of the spacecraft at a relative speed of $3c/5$.

7. Determine the speed of the bullet relative to the earth.

8. Calculate the length of the spacecraft as measured by observers moving along with (a) the alien, (b) the earth, and (c) the bullet.

9. Determine how long the bullet takes to reach the front of the spacecraft as measured by (a) an observer on the earth, (b) the alien, and (c) someone moving along with the bullet.

10. Train A travels north at a speed $4c/5$ relative to a certain station whereas train B travels east at speed $3c/5$ relative to the same station. Find the velocity components of train A relative to an observer on train B.

11. An atom is moving with respect to a lab at a speed of $0.3c$ along the positive x direction, and emits an electron having a speed of $0.8c$ in the positive y direction in the rest frame of the atom.

(a) Find the components of the electron's velocity according to an observer in the lab.
(b) Calculate the magnitude and direction of the electron's velocity in the lab frame.

12. Two spaceships traveling in the same direction pass the earth. According to an Earth-based observer, spaceship A has a speed of $0.7c$ and spaceship B has a speed of $0.9c$.

(a) What is the speed of spaceship B according to an observer on A?
(b) The commander of spaceship B, not wanting to be called an unidentified flying object, flashes a signal to Earth. The observer on Earth measures the duration of the signal to be 10 s. What is the duration of the signal as measured on spaceship B?
(c) What does an observer on spaceship A measure for the duration of the signal?

13. Use the method in the text to express the velocity components u'_x, u'_y, and u'_z in terms of u_x, u_y, and u_z. Your results should agree with those obtained by replacing v by $-v$ and interchanging primed and unprimed quantities in Eqs. (47.6x, y, z).

47.3 THE FIZEAU EXPERIMENT

Einstein's postulate about the speed of light being constant refers to the speed of light in a vacuum. Long before Einstein, many investigators tried to determine whether the speed of light was finite or infinite. The first experimental evidence for the finiteness of the speed of light was obtained in the seventeenth century by a Danish astronomer Ole Roemer as a result of observations on the eclipses of

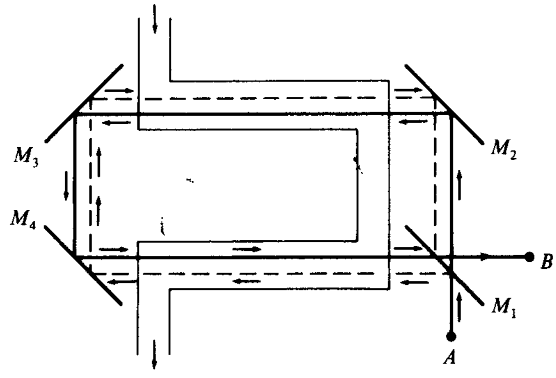


Figure 47.1 Diagram for the Fizeau experiment.

Jupiter's satellites. Later, an English astronomer James Bradley explained an apparent seasonal change in the position of a star called γ Draconis by the finiteness of the speed of light.

In 1849, a French scientist, Armand Fizeau, made the first terrestrial determination of the speed of light, that is, the first determination that did not involve astronomical constants. Using a rotating toothed wheel as a light switch, Fizeau arrived at the value 3.13×10^8 m/s for the speed of light in air. This was not very accurate but it paved the way for more precise measurements. Leon Foucault, who had worked with Fizeau, improved the apparatus and obtained the value 2.98×10^8 m/s in 1862. Much later, Albert A. Michelson modified the Foucault method on a spectacular scale using a rotating mirror apparatus, and in 1927 he announced the highly accurate value $(2.99796 \pm 0.00004) \times 10^8$ m/s. The accepted value today is very close to 2.99776×10^8 m/s.

Fizeau carried out another experiment in 1851 that can be cited as physical evidence for the formulas we obtained on relativistic velocities. Fizeau made light travel through water that, in turn, was flowing with a speed v relative to his laboratory. A diagram of the experimental arrangement is shown in Fig. 47.1. Water flows in a U-shaped tube at known speed v. Light from source A hits a half-silvered mirror M_1 and is split into two beams, indicated by solid and dashed lines in Fig. 47.1, that reflect off mirrors M_2, M_3, and M_4 as shown, then recombine at M_1 and proceed to B. The relative time delay between the two circuits can be measured by interference methods.

When light passes through a transparent medium such as glass, water, or air, its speed is not c but c/n, where n is the index of refraction, introduced in Chapter 44. For example, the index of refraction of crown glass is about 1.52, that of water is about 1.333, while that of air is about 1.0003. Fizeau determined that the speed of the light parallel to the stream of water was given by

$$u = \frac{c}{n} + \alpha v, \tag{47.7}$$

where n is the index of refraction of water and α is a constant called the "drag coefficient." Fizeau's experiment indicated that α is about 0.434. Michelson and Morley repeated Fizeau's experiment in 1886 with improved accuracy and obtained the same value for α.

We can examine these findings in light of Eq. (47.6x). If c/n is the speed of light in water, as measured by an observer at rest relative to the water, and if v is the speed of the water relative to the laboratory observer, then the speed of light u relative to the laboratory observer is given by the composition law in Eq. (47.6x),

$$u = \frac{(c/n) + v}{1 + [v/(nc)]}. \tag{47.8}$$

Now if $x \ll 1$ we have the approximate formula

$$\frac{1}{1+x} \approx 1 - x, \tag{47.9}$$

which is highly accurate if x^2 is negligible compared to x. To see where this comes from we use the algebraic identity

$$1 - x^2 = (1 + x)(1 - x).$$

If x^2 is very small the left member of this identity is nearly 1 so $(1 + x)$ and $(1 - x)$ are nearly reciprocals of each other, and we get (47.9). Taking $x = v/(nc)$ and noting that v is very small compared to c, we conclude that x^2 is negligible compared to x. Therefore, using the approximation (47.9) in (47.8), we obtain

$$u \approx \left(\frac{c}{n} + v\right)\left(1 - \frac{v}{nc}\right) = \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right) - \frac{v^2}{nc} \approx \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right),$$

where in the last step we neglected $v^2/(nc)$. Comparing this with Fizeau's formula (47.7) we find that

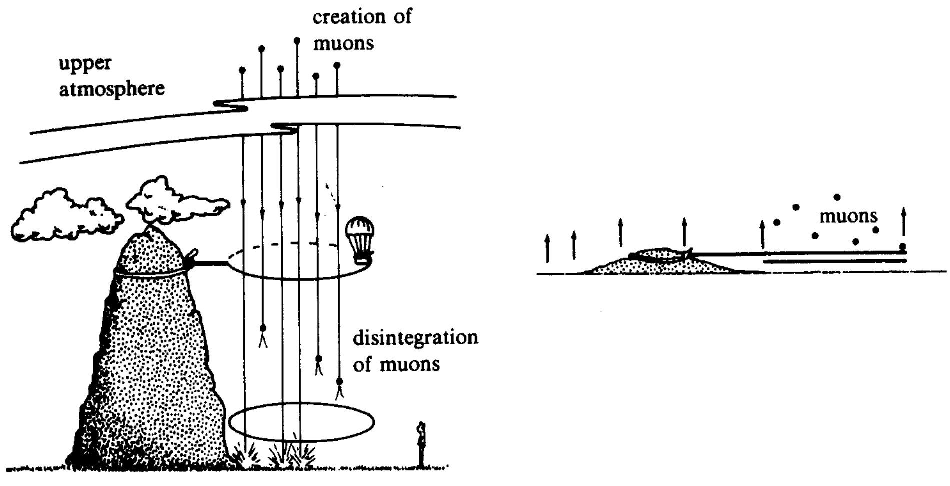
$$\alpha = 1 - \frac{1}{n^2} = 1 - \frac{1}{(1.333)^2} = 0.437,$$

which agrees well with Fizeau's experimentally determined value 0.434.

47.4 THE MUON EXPERIMENT

According to special relativity, moving clocks and meter sticks behave in ways that are contrary to our intuition. A clock in motion appears to run more slowly than one at rest – an effect that has nothing to do with the inner workings of the clock, but rather with the nature of time itself. In ordinary experience this is not noticeable because the time dilation factor γ is very close to 1 for any commonplace observation. In order to demonstrate this effect experimentally one would have to observe a clock moving at a speed close to that of light.

Such clocks actually exist. They are subatomic particles that were first found in the upper atmosphere and are now created routinely by powerful particle accelerators. Observations of these particles led to one of the first experimental verifications of time dilation.



(a) (b)

Figure 47.2 The muon experiment, according to observers (a) on Earth and (b) traveling with the muons.

The earth's atmosphere is constantly bombarded by cosmic rays, particles of very high energy that originate from outer space. When these particles enter the outer layers of the atmosphere about 15 km up, they collide with other particles and produce many kinds of subatomic, subnuclear fragments. One of the products is the muon, or mu meson, an unstable, subnuclear particle that has a mean lifetime of 2.2 μs in its own reference frame. Not every muon will decay in exactly that time, but in a large population of muons the average decay time will be 2.2 μs.

As a result of the collisions that occur at the top of the atmosphere, the muons are propelled downward at high speeds very close to the speed of light. Because we know how long they live, we can calculate how far they travel before they decay. On the average, this distance is the product of their speed c and the decay time, 2.2×10^{-6} s, which is about 700 m. Figure 47.2 illustrates this distance.

Most of the muons are created about 15 km above the earth's surface and, because they are expected to decay after traveling less than 1 km, they should never reach the earth. It's true that some muons live longer than others, but not long enough to travel 20 times the distance they're supposed to. On the other hand, observations reveal that large quantities of muons *do* reach the earth. The explanation for this is time dilation. Because the muon is moving at close to the speed of light, its clock is running slow – that is, it has an internal clock that makes its decay time seem longer than 2.2 μs according to an observer on Earth. We haven't the foggiest notion of the nature of the internal clock, and we don't need to know how it works. But we do know that it will obey the laws of special relativity and exhibit time dilation. In this extra time, the muon travels a much longer distance through the atmosphere than it's supposed to, and that's why it makes it to Earth.

In 1941 B. Rossi and D. B. Hall made measurements comparing the flux of muons at the top of a mountain and at sea level to see how fast the muons decay along the way. The observations indicate that muons survive about 9 times as long as they would if they were at rest with respect to the earth.

From the point of view of the muon itself, time dilation does not occur. The muon thinks it is at rest and that the earth is moving toward the muon at nearly the speed of light. The muon's internal clock tells it to decay right on schedule, 2.2 μs after it is created. But, according to the muon, the earth and its atmosphere are approaching at such great speed that the thickness of the atmosphere undergoes Lorentz contraction. The atmosphere appears so thin that the muon penetrates it and strikes the earth before 2.2 μs have elapsed.

Example 5

An experiment shows that muons travel 2000 m in 6.71×10^{-6} s from the time they are created at the top of a mountain to the time they reach sea level.

(a) Calculate the speed of the muons relative to the earth.

(b) Determine the time dilation factor γ for the muons moving at this speed.

(c) In the reference frame of the muons, what is the altitude of the mountain above sea level?

(a) According to an observer on Earth, the muons travel a distance $L_0 =$ 2000 m in a time $T = 6.71 \times 10^{-6}$ s. Therefore, their speed is given by

$$v = L_0/T = (1000 \text{ m})/(6.71 \times 10^{-6} \text{ s}) = 2.98 \times 10^{-8} \text{ m/s} = 0.994c.$$

(b) Using $v/c = 0.994$ we find $1 - (v/c)^2 = 0.012$ so

$$\gamma = 1/\sqrt{0.012} = 9.13.$$

(c) The moving muons see a contracted length for the height of the mountain. According to length contraction, $L = L_0/\gamma$, so substituting for L_0 and γ, we find $L = 219$ m. Thus, observers on Earth see muons travel a distance of 2000 m in 6.71×10^{-6} s at a speed of $0.994c$, whereas the muon sees the earth move toward it a distance of 219 m in 0.73×10^{-6} s.

Questions

14. Point out the error in the following argument: A clock, at rest with respect to an inertial frame S, records a time T_0 for an egg to hatch in that frame. In frame S' moving with respect to the S frame, a clock records a longer time T for that egg to hatch. Therefore, it is possible to distinguish between the two inertial frames, thus violating Einstein's principle of relativity.

15. A spaceship travels at constant speed relative to Earth and reaches the star system of Procyon, a distance of 10 light years. Suppose the trip takes 18 yr as measured by clocks aboard the spaceship.

(a) What is the speed of the spaceship relative to Earth?
(b) According to observers on Earth, how long does the trip take?
(c) According to the space travelers, how far is Earth from Procyon?

16. In an inertial frame S a child shines a flashlight at an angle of 30° from the x axis. Another child in a frame S' passing by at a speed of $0.8c$ parallel to the x axis observes the beam of light.

(a) What are the components of the velocity of light according to the child in the S' frame?
(b) What is the speed and direction of the light in the S' frame?

Questions 17 through 21 refer to a certain type of radioactive particle, called a charged pion, which has an average lifetime of 2.6×10^{-8} s when observed at rest. Assume that charged pions emerging from an accelerator have a speed of $0.8c$ relative to the lab.

17. According to an observer in the lab, what is the average lifetime of a moving pion?

18. What is the average distance traveled by a pion in the lab before it decays?

19. According to an observer moving along with the pion, what is the average distance from the point where the pion emerges from the accelerator to the point of decay?

20. A fervent superphysicist runs after an emerging pion with a speed of $0.6c$ relative to the lab. According to the superphysicist, how fast is the pion traveling?

21. According to the superphysicist, what is the average lifetime of a pion?

22. Two scientists traveling in separate spaceships observe the formation and decay of a beam of muons. The clock on ship A measures an average lifetime of 2.2 μs, while the clock on ship B measures an average lifetime of 4.4 μs. The scientist on one of the ships notices that the muons are not moving relative to her.

(a) Which scientist sees the muons at rest with respect to her spaceship?
(b) Calculate the relative speed of the two spaceships.

47.5 THE TWIN PARADOX

The idea that time does not run at the same pace for every observer has given rise to a number of paradoxes that puzzled and stimulated every generation of physics students in this century. All these paradoxes arise from incomplete understanding and all have been satisfactorily explained. They appear to be paradoxes only

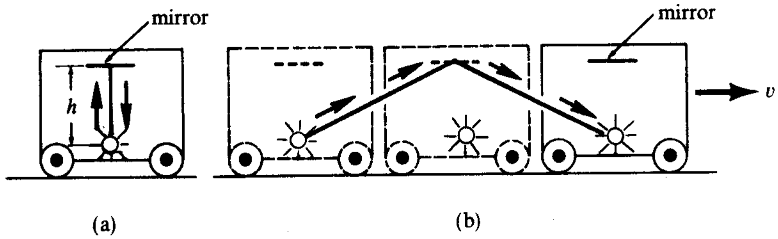


Figure 47.3 The trolley car experiment.

because some of the implications of special relativity seem to conflict with intuitive ideas of distance and time. One of the most celebrated of these, called the twin paradox, will be discussed in this section.

Before introducing the twin paradox, let's recall the trolley car experiment discussed in Section 46.1 and illustrated in Fig. 47.3. If you are in the car, an observer on the ground sees the light take a longer path than you do, as indicated in Fig. 47.3, so he thinks your clock is running slow. But if the ground observer performs an experiment in his laboratory to measure the speed of light, the conditions are reversed. To you in the car, the laboratory seems to be moving and the path of the light in the laboratory seems longer to you, and therefore you conclude that the other person's clock is running slow. The two situations are symmetrical, each of you thinking that the other's clock is slow.

Nevertheless, it seems somewhat bizarre that when 2 s have passed on your clock, you may think only 1 s has passed on his, and when 2 s have passed on his clock, he may think only 1 s has passed on yours. The explanation for this is that you and the other person can meet and set your clocks together once and only once. After that you never meet again because each of you is moving at a constant speed in a straight line relative to the other. So if you want to compare clock readings after you separate, you can do so only by sending signals to each other. One way or another, any such signal is equivalent to a beam of light traveling at speed c. Each of you judges it to have traveled along a path of different length and therefore each thinks it has taken a different time.

The twin paradox involves a pair of twins, a sister and brother, A and B, who separate and later meet again at the same place. They were born at the same time so their biological clocks are synchronized at birth. They are able to compare their clocks at the beginning of the trip and at the end. Brother B takes up a career on Earth as a light-clock repairman, but his more adventurous sister A becomes a space trucker, making the milk run to a nearby solar system, 10 light years away, in a freight rig that manages $\gamma = 10$. This means her speed v is very nearly c, so brother B sees his sister travel 10 light years away, and 10 more back in just slightly more than 20 yr. On the other hand, sister A sees the distance to her destination Lorentz contracted by a factor $1/\gamma = 1/10$, so by traveling at nearly the speed of light she makes the journey in 1 yr each way. Thus, each time she returns to Earth from one of her journeys, she is 2 yr older, but her brother has aged by 20 yr.

This scenario is indeed what the special theory of relativity predicts. The paradox is that we expect each observer to think the other's clock to be running slow (any clock, including the human metabolism). Doesn't sister A think she is standing still while brother B races away on spaceship Earth?

Indeed, as long as the twins are in uniform relative motion, each would judge the other's clock to be running slow. But that has no practical consequence as long as they are far apart (and continue to get farther apart), unable to compare their biological clocks directly. The only way they can get back together again is for one of them to switch inertial frames. That switch, from going away to coming back, by only one of the twins, destroys the symmetry between their points of view. That is why there is no paradox.

Questions

23. An astronaut travels at a speed of $0.5c$ relative to the earth. Does she detect her heartbeat to be slower or the spaceship to be shorter than when she was at rest? Explain your answer.

24. Two observers agree to test time dilation. One in the S' frame moves at a speed $0.6c$ relative to the other in the S frame. When their origins coincide they start their clocks. They agree to send a signal when their clocks read 60 min and send a confirmation signal when each receives the other signal.

- **(a)** When does the observer in the S frame receive the first signal from the observer in S'?
- **(b)** When does the observer in S receive the confirmation signal?
- **(c)** Make a table showing the times in S when the observer sent the first signal, received the signal, and received the confirmation signal. How does this compare with a table that the observer in S' would construct?

25. Astronaut A leaves her twin brother B on Earth and departs on a trip to Alpha Centauri, a distance of 4 light years, and back. She travels at a speed of $0.6c$ relative to Earth both ways, and sends a light signal every 0.01 yr according to her calendar. Her twin B sends a light signal every 0.01 yr according to his own calendar.

- **(a)** How many signals does B receive before A turns around?
- **(b)** How many signals does A receive before she turns around?
- **(c)** What is the total number of signals each twin receives from the other?
- **(d)** By how much younger is twin A than B when she returns? Do both twins agree on this result?

26. For the situation in Question 25, make a sketch on a space–time diagram, showing the trip and a few representative light signals being sent and received on each part of the trip.

47.6 A FINAL WORD

You might wonder why the twins have different ages after one makes a relativistic trip. What caused the astronaut to come back younger than her brother? Was it the acceleration that did it? The change of direction? How did it happen? As far as we know there is no way to answer these questions. It's like asking why bodies in motion remain in motion according to the law of inertia. That's the way the world is. Objects just follow the law of inertia, and we deduce features of motion from that. Similarly, the fundamental nature of time and space produces the result that the astronaut returns younger than her brother. It happens because that's the way time and space are.

Imagine a universe that is completely empty. In such a universe it would be meaningless to talk about time and space. If there is only one object in the universe, it would be meaningless to ask whether that object is moving or at rest. It's just there. If there are two objects in the universe, then it makes sense to ask if they are getting closer together or moving farther apart. In other words, some questions about relative motion make sense. If the universe contains a lot of objects, we can talk about relative motion, and we might be able to devise some way of measuring how big an object is, or how far apart objects are from each other. But there is no reason whatsoever to choose one of those objects and say, "That one is at rest and all others are in motion, and so all others should try to come to rest with respect to the privileged one." There's no reason to do that, and so in the simplest of all universes – which is ours of course – a body in any state of motion will retain that state of motion. That's why the law of inertia makes sense. Moreover, it must continue to make sense in a universe in which light can propagate through the void. However, if there's only one frame of reference in which light has a certain speed, all of the foregoing argument collapses.

Other logically consistent universes may or may not exist. But in the one we live in, the speed of light is the same for all observers. Because of that, absolute rest has no meaning, the law of inertia works, and all the physics we understand works. Knowing that alone, it's possible to deduce that the traveling astronaut will come back younger than her twin brother who stayed behind. The nature of the metabolism of the traveler, why one ages, how clocks work, or what makes crystal oscillators tick – none of that is relevant. The argument we presented is all that's needed to conclude that the traveler will come back younger than the twin who stayed behind. That is an absolutely breathtaking feat of pure logic – one of the remarkable consequences of the special theory of relativity.