CHAPTER 45

THE MICHELSON–MORLEY EXPERIMENT

"Is there any point to which you would wish to draw my attention?"
"To the curious incident of the dog in the night-time."
"The dog did nothing in the night-time."
"That was the curious incident," remarked Sherlock Holmes.

Arthur Conan Doyle in *The Memoirs of Sherlock Holmes* (1893)

45.1 THE ROOTS OF RELATIVITY

The problem of relativity began when Copernicus wrenched the earth from its fixed position in the cosmos and sent it hurtling around the sun. If the earth is actually moving around the sun, why does it seem to be standing still? Scientists such as Galileo, Kepler, Descartes, and Newton tackled this problem. Galileo argued that by watching a stone fall, you cannot tell whether the earth is at rest or moving. You might wonder whether there is any way to tell.

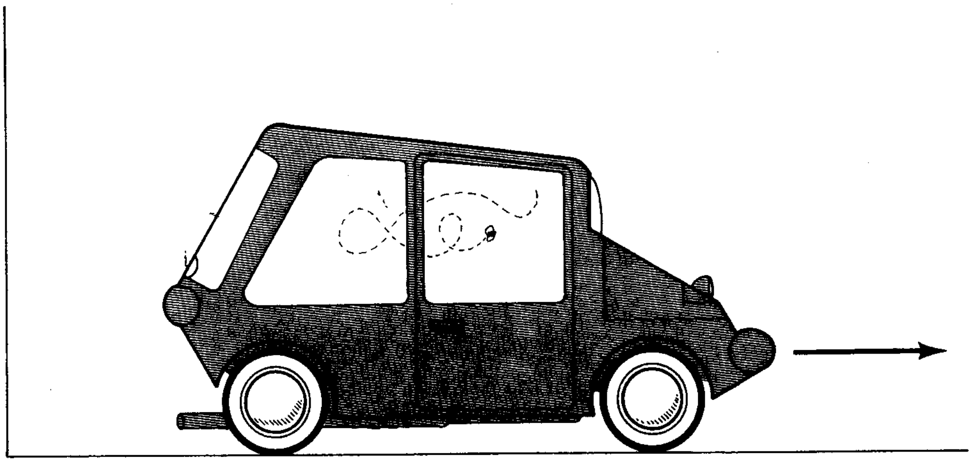


Figure 45.1 Velocity of a fly relative to the ground is equal to its velocity relative to the car, plus the velocity of the car relative to the ground.

In one sense, all motion is relative. Someone on the moon thinks he sees the earth moving, but a person on Earth thinks the moon is moving. To describe motion you need to specify a frame of reference relative to which the motion is measured. We habitually use nearby walls, buildings, trees, and other objects attached to the solid earth – and therefore at rest with respect to one another – as defining a frame of reference, and then we describe the motion of other objects relative to this frame. But on a boat, we tend to use our local surroundings in the boat – once again, a collection of objects at rest with respect to one another – as defining our frame of reference, even though the boat moves relative to the earth. Kinematically, motion can be described relative to any frame of reference. The choice of frame is merely a matter of convenience.

A frame of reference is simply a conceptual set of coordinate axes relative to which we measure displacements, velocities, and accelerations. When one turns from mere kinematic description of motion to dynamics – the laws of motion – one still has great freedom in choosing the frame of reference, but it becomes necessary to distinguish between different categories of frames. We shall define an *inertial frame* as one in which Galileo's law of inertia holds (a body not acted upon by any net force moves with constant speed along a straight line). We will show that if frame S is inertial, then any frame S' with respect to which S moves with constant velocity is also inertial.

To get an idea why this is so, consider a fly buzzing around inside a moving car, as indicated in Fig. 45.1. Common sense tells us that the velocity of the fly relative to the ground is equal to its velocity relative to the car, plus the velocity of the car relative to the ground.

In general, if frame S' is moving with velocity $\mathbf{v}_0$ relative to frame S, then the velocity $\mathbf{v}$ of a body relative to S is equal to its velocity $\mathbf{v}'$ relative to S', plus $\mathbf{v}_0$: $\mathbf{v} = \mathbf{v}' + \mathbf{v}_0$.

If $\mathbf{v}$ and $\mathbf{v}_0$ are constant, then so is $\mathbf{v}'$, and therefore the body obeys the law of inertia in frame S' as well as in S, even though its precise location and velocity are different in the two frames. Thus, if S is an inertial frame, so too is S', and the two frames are equivalent for discussing the law of inertia. The equivalence still holds for objects and reference frames moving in different directions as long as the relative velocity of one to the other is constant, as we shall show in Section 45.2. Any inertial frame is suitable to describe the motion; none is preferred over another. In this sense there is no way to tell whether the earth or any other body is in a state of absolute rest. This is the essence of the *Galilean relativity principle*, which can be stated as follows:

> All laws of mechanics observed in one coordinate system are equally valid in any other coordinate system moving with a constant velocity relative to the first.

This statement contains Galileo's principle of inertia and much more. Galileo was earthbound, but Newton, who sought a grand synthesis of the physics of the heavens with that of the earth, extended the principle to the realm of the universe.

In deep space there are no stationary milestones or other markers to recognize rest. Yet Newton needed a frame of reference from which to apply his laws of mechanics. He conceived the ideas of absolute space and absolute time. According to Newton, absolute space was utterly featureless, unchanging, and immovable, a sort of a cosmic scaffolding. In reference to it, Newton could describe any body as being at rest or in relative motion.

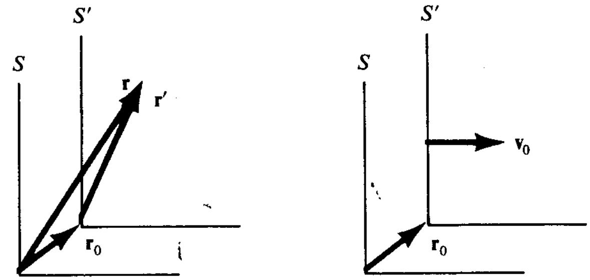
Similarly, thought Newton, absolute time existed as an entity in itself, relentlessly flowing without relation to anything external – a cosmic clock marking time for the universe. With absolute space and absolute time, Newton could apply his laws to everything in the universe. However, by introducing absolute space and absolute time, Newton made a sharp distinction between rest and motion. That difference particularly disturbed him because he was well aware of the principle of relativity: Absolute uniform motion in a straight line cannot be detected.

Questions

1. Devise an experiment to tell whether or not you are in an inertial reference frame.
2. An observer on the ground watching a rock dropped from the mast of a moving ship sees the rock follow a parabolic path, whereas an observer on the mast sees the rock follow a straight-line path. Is this difference in the path of the falling rock as seen by two inertial observers a violation of the principle of relativity?
3. Explain why the earth is or is not an inertial reference frame.

45.2 THE GALILEAN TRANSFORMATION

If two inertial observers are in relative motion, measurements of time, position, velocity, and acceleration made by the two observers can be related by a set of



(a) (b)

Figure 45.2 Two reference frames with (a) shifted origin and (b) constant relative velocity.

equations known as the *Galilean transformation*. To derive these equations we consider two reference frames called S and S', and assume first that they differ only by a simple displacement of the origin, indicated by the vector $\mathbf{r}_0$ in Fig. 45.2a. Let $\mathbf{r}$ denote the position vector of a body in S, and $\mathbf{r}'$ the position vector of the same body in S', so that

$$\mathbf{r} = \mathbf{r}' + \mathbf{r}_0. \tag{45.1}$$

We assume also that the two observers have the same clock, so if an event takes place in S at time t it also takes place in S' at time $t' = t$. If vector $\mathbf{r}_0$, the shift of origin, is time independent, then velocities and accelerations will be the same in both reference frames because, by differentiating Eq. (45.1), we find

$$\frac{d\mathbf{r}}{dt} = \frac{d\mathbf{r}'}{dt} \tag{45.2}$$

and

$$\frac{d^2\mathbf{r}}{dt^2} = \frac{d^2\mathbf{r}'}{dt^2}. \tag{45.3}$$

Therefore, the two frames are equivalent for discussing Newton's laws of motion, even though the precise location of an object will be different in the two frames.

Next, suppose that the origin is shifted as above and also that frame S' moves at a constant velocity $\mathbf{v}_0$ with respect to S, as indicated in Fig. 45.2b. Then the position of a body in S is related to its position in S' by the equation

$$\mathbf{r} = \mathbf{r}' + \mathbf{r}_0 + \mathbf{v}_0 t. \tag{45.4}$$

Differentiating this equation we find that the velocities $\mathbf{v} = d\mathbf{r}/dt$ and $\mathbf{v}' = d\mathbf{r}'/dt$ differ, but only by a constant amount,

$$\mathbf{v} = \mathbf{v}' + \mathbf{v}_0, \tag{45.5}$$

because $d\mathbf{r}_0/dt = \mathbf{0}$. This means that if Newtons' first law holds in one frame, it will also hold in the other. Differentiating once more, we find that the accelerations

$\mathbf{a} = d\mathbf{v}/dt$ and $\mathbf{a}' = d\mathbf{v}'/dt$ are the same:

$$\mathbf{a} = \mathbf{a}'. \qquad (45.6)$$

Thus, once again the two frames are equivalent for discussing Newton's laws of motion, even though the initial location and velocity of an object will be different in the two frames. This is described by saying that Newton's laws of motion are *invariant* under transformation (45.4). (They are also invariant under (45.1), which is merely a special case of (45.4).) This invariance of the laws of motion is called *Galilean relativity*.

Note that Galilean relativity is built into the form of the fundamental forces of gravity and electricity. The inverse-square laws for the gravitational and electrical forces between objects A and B depend not on the absolute positions $\mathbf{r}_A$ and $\mathbf{r}_B$, which would change in a displaced or moving frame, but on the distance of separation $|\mathbf{r}_A - \mathbf{r}_B|$, which is unchanged by the transformation (45.4) because the terms $\mathbf{r}_0$ and $\mathbf{v}_0 t$ cancel in the subtraction.

It can also be shown that Newton's laws are invariant under a change in the orientation of frame S' relative to S; that is, if S' is rotated as well as translated relative to S. This, too, is part of Galilean relativity: The laws of motion in a tilted frame are the same as in a horizontal one.

If the components of the vectors $\mathbf{r}$, $\mathbf{r}'$, $\mathbf{r}_0$, and $\mathbf{v}_0$ are expressed in rectangular coordinates, the Galilean transformation (45.4) can also be expressed by a corresponding set of scalar equations:

$$x = x' + x_0 + v_{0x}t, \qquad (45.4x)$$

$$y = y' + y_0 + v_{0y}t, \qquad (45.4y)$$

$$z = z' + z_0 + v_{0z}t. \qquad (45.4z)$$

Another equation,

$$t = t',$$

is often appended to this list to indicate that the same clock is being used in both frames. The corresponding scalar equations for velocity components are

$$v_x = v'_x + v_{0x}, \qquad (45.5x)$$

$$v_y = v'_y + v_{0y}, \qquad (45.5y)$$

$$v_z = v'_z + v_{0z}, \qquad (45.5z)$$

while those for acceleration components are

$$a_x = a'_x, \qquad (45.6x)$$

$$a_y = a'_y, \qquad (45.6y)$$

$$a_z = a'_z. \qquad (45.6z)$$

Example 1

A train moves along straight tracks at a constant speed of 80 km/h. Inside the train, a fly sits on a sleeping passenger's head. The train passes an observer at a railroad crossing, and the observer starts a stopwatch when the passenger is directly in front of the observer.

(a) According to the observer, what is the position of the fly 30 s later?

(b) If the fly speeds toward the front of the passenger car at 5 km/h, what is its speed relative to the observer?

(a) Let S denote the reference frame of the ground, with the x axis parallel to the tracks, and attach the S' frame to the sleeping passenger with the origin on the fly and the x' axis parallel to the x axis. At time $t = 0$, we have $x = x' = 0$, and from Eq. (45.4x) we know that at time t, we have $x = x' + v_{0x}t$, where x' is the position of the fly in frame S'. Substituting $x' = 0$, $v_{0x} = 80$ km/h, and $t = 30$ s, we find $x = 0.67$ km.

(b) Our intuition tells us that the speed of the fly relative to the ground is equal to the speed of the train relative to the ground plus the speed of the fly relative to the train, and this is confirmed by Eq. (45.5). From Eq. (45.5x) we find $v_x = v'_x + v_{0x}$, so that

$$v_x = 5\ \text{km/h} + 80\ \text{km/h} = 85\ \text{km/h}.$$

Example 2

A rectangular coordinate frame S is placed on the ground. An observer on the ground finds that the acceleration components of a moving truck are a_x, a_y, a_z. What are the acceleration components according to an observer in a car that is moving parallel to the positive x axis of frame S with a constant speed v_0 relative to the truck?

The reference frame S' of the truck moves with constant velocity $v_0\hat{\mathbf{i}}$ relative to S, so we have an example of Galilean relativity. Therefore,

$$a'_x = a_x, \qquad a'_y = a_y, \qquad a'_z = a_z.$$

The truck has the same acceleration as measured in both frames S and S'.

The laws of conservation of energy and momentum discussed in Chapters 13 and 19 were derived from Newton's second law. According to the principle of relativity, these laws should be the same in all inertial reference frames. The next two examples verify this for a simple case of elastic collision of two bodies. We apply a conservation law in one frame S, then use the Galilean transformation to find what that law becomes in a frame S' moving with constant velocity $\mathbf{v}_0$ relative to S.

Example 3

Two masses m and M collide elastically, their initial velocities being $\mathbf{u}_0$ and $\mathbf{U}_0$, respectively, in frame S. After the collision, the masses move off with velocities $\mathbf{u}$

and **U** in frame S. Show that if the total momentum of the system in conserved in frame S, then it is also conserved in any frame S' moving with constant velocity $\mathbf{v}_0$ relative to S.

Conservation of momentum in S means that the total momentum of the system before collision equals that after the collision:

$$m\mathbf{u}_0 + M\mathbf{U}_0 = m\mathbf{u} + M\mathbf{U}. \tag{45.7}$$

In frame S' the initial velocities of the masses are $\mathbf{u}'_0$ and $\mathbf{U}'_0$, and the velocities after impact are $\mathbf{u}'$ and $\mathbf{U}'$. These are related to the velocities in S by the equations

$$\mathbf{u}_0 = \mathbf{u}'_0 + \mathbf{v}_0, \tag{45.8}$$

$$\mathbf{U}_0 = \mathbf{U}'_0 + \mathbf{v}_0, \tag{45.9}$$

$$\mathbf{u} = \mathbf{u}' + \mathbf{v}_0, \tag{45.10}$$

$$\mathbf{U} = \mathbf{U}' + \mathbf{v}_0. \tag{45.11}$$

When these quantities are substituted into Eq. (45.7), all the terms involving $\mathbf{v}_0$ cancel and we are left with

$$m\mathbf{u}'_0 + M\mathbf{U}'_0 = m\mathbf{u}' + M\mathbf{U}', \tag{45.12}$$

which shows the total momentum of the system is also conserved in frame S'.

Of course, the explicit value of total momentum is *not* the same in both frames. The actual value of the total momentum in frame S always differs from that in frame S' by the amount $(m + M)\mathbf{v}_0$. But in either frame, the total momentum of the system *before* collision is equal to that *after* collision.

Example 4

Refer to the collision in Example 3 and show that if the total kinetic energy of the system is conserved in frame S, then it is also conserved in frame S'.

This calculation is a bit more complicated because the kinetic energy $\frac{1}{2}mv^2$ involves the square of the speed and not merely the velocity. To simplify the calculation, we express the square of the speed as the dot product of the velocity vector with itself. Conservation of total kinetic energy in frame S states that the kinetic energy before collision equals that after collision. In the notation of Example 3 this gives us

$$\frac{m}{2}\mathbf{u}_0 \cdot \mathbf{u}_0 + \frac{M}{2}\mathbf{U}_0 \cdot \mathbf{U}_0 = \frac{m}{2}\mathbf{u} \cdot \mathbf{u} + \frac{M}{2}\mathbf{U} \cdot \mathbf{U}. \tag{45.13}$$

In this equation, replace all the velocities in frame S by the corresponding expressions in Eqs. (45.8) through (45.11). After the dot multiplication is carried out and the common terms on both sides are canceled, we obtain the equation

$$\frac{m}{2}\mathbf{u}'_0 \cdot \mathbf{u}'_0 + \frac{M}{2}\mathbf{U}'_0 \cdot \mathbf{U}'_0 + \mathbf{v}_0 \cdot (m\mathbf{u}'_0 + M\mathbf{U}'_0)$$

$$= \frac{m}{2}\mathbf{u}' \cdot \mathbf{u}' + \frac{M}{2}\mathbf{U}' \cdot \mathbf{U}' + \mathbf{v}_0 \cdot (m\mathbf{u}' + M\mathbf{U}').$$

Because of conservation of momentum, verified in Example 3, the terms in parentheses multiplying $\mathbf{v}_0$ are equal, and therefore all terms involving $\mathbf{v}_0$ cancel. The equation that remains states that the kinetic energy of the system is also conserved in frame S'.

Questions

In the following problems, assume that the reference frames are inertial frames with time measured so that $t = t'$ in the two frames.

4. A rectangular coordinate system S is attached to the ground with the origin at the base of a lamppost. Another rectangular coordinate system S' with its axes parallel to those of S is attached to a moving car with its origin on the rear bumper. The car moves with a constant speed of 15 m/s in the direction of the positive x axis. In frame S', a radio in the car has coordinates (2 m, 0.5 m, 0). At time $t = 0$, the origin of S' has coordinates (0, 0.3 m, 0) in S.

(a) Find the coordinates of the radio in frame S when $t = 20$ s.
(b) A jogger runs along the same street in the same direction as the car with a constant speed of 2 m/s in frame S. Determine the speed of the jogger in frame S'.
(c) Inside the car, a fly moves from the front of the car directly toward the rear with a speed of 1 m/s in frame S'. Determine the velocity of the fly relative to the ground.

5. A train moves at 10 m/s relative to the earth. Inside the train a runner moves in the same direction as the train with an acceleration of 1.0 m/s^2 relative to the train. Determine the position of the runner with respect to the ground at time t, if at time $t = 0$ the origins of the two frames coincide and the runner is at the origin with zero speed.

6. A truck moving at 15 m/s along a straight level road carries an aquarium in which a fish swims vertically upward at 3 m/s relative to the truck. What is the speed of the fish relative to the ground?

7. Car A travels east at 35 m/s relative to a station. Car B travels north at 35 m/s relative to the same station. Find the velocity of car A relative to car B.

8. A swimmer can swim 1.5 m/s in an olympic pool. Determine how fast the swimmer can swim relative to the shore in a river that is moving at 1.0 m/s, if he swims

(a) in the direction of the current;
(b) in a direction opposite to that of the current;
(c) perpendicular to the current.

9. In a certain reference frame two objects of masses M and $4M$ move toward each other at velocities $\mathbf{v}$ and $-\mathbf{v}/4$, respectively, then collide and stick together.

(a) Determine the velocity of the composite body after collision.
(b) Calculate the total kinetic energy before and after the collision, and the increase in thermal energy resulting from the collision.

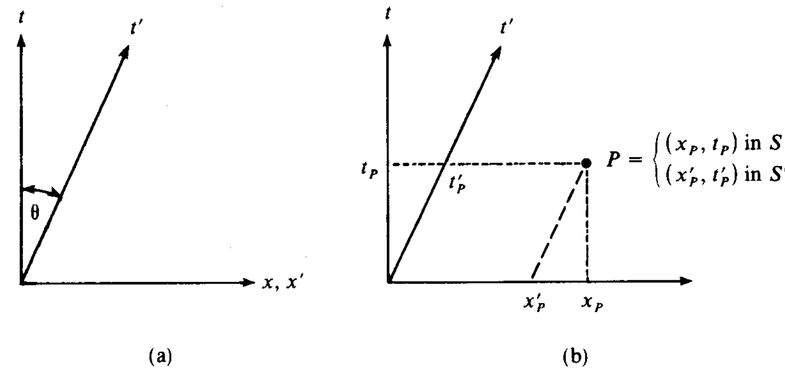


Figure 45.3 Space–time diagram for two inertial frames.

(c) Repeat the calculations of part (b) in a frame moving at velocity $\mathbf{v}_0$ with respect to the original frame.

45.3 SPACE-TIME DIAGRAMS FOR GALILEAN TRANSFORMATIONS

In Fig. 45.2 we represented two inertial reference frames S and S' by drawing two sets of coordinate axes. There is another way to represent Galilean transformations with a single diagram, called a space–time diagram. For simplicity, we consider only one coordinate axis of S, the x axis, and ignore the y and z axes. To study the behavior of the x coordinate of a moving body as a function of time t we would ordinarily plot x as a function of t, using a horizontal axis for t and a vertical axis for x. However, it has become common practice among most physicists to reverse the axes, using a horizontal x axis and a vertical t axis, a practice introduced by Hermann Minkowski. If a body has coordinate x at time t, we plot the point (x, t) on the space–time diagram. Some authors refer to the point (x, t) as an *event*; Minkowski called it a *world point*. The set of all points (x, t) associated with a given body determine a curve on the space–time diagram giving a visual description of how the x coordinate of the body changes with time. Minkowski called this curve a *world line*. For example, if at time $t = 0$ a body has $x = 0$, then that information is represented by the origin $(0, 0)$. If this body remains at rest in the S frame, its x coordinate will not change as t increases, and the corresponding world line for this body will be a vertical line – the t axis – as shown in Fig. 45.3a.

Now suppose the origin O' of another frame S' moves with constant speed v_0 relative to S and coincides with the origin of S at time $t = 0$. On this same space–time diagram, the world line of O' would be depicted by a straight line. If the axes were oriented in a conventional manner with the same scale along each axis, this line would have slope v_0. But because the axes are reversed, this line is tilted away from the vertical t axis by an angle θ whose tangent is v_0, and because this line represents the path of O' it can be regarded as the t' axis for frame S'. Now we draw the x' axis for frame S' on the same diagram, using the same horizontal axis for x' that we used for x in frame S. The coordinate axes in the S' frame will then be oblique rather than perpendicular, as shown in Fig. 45.3a.

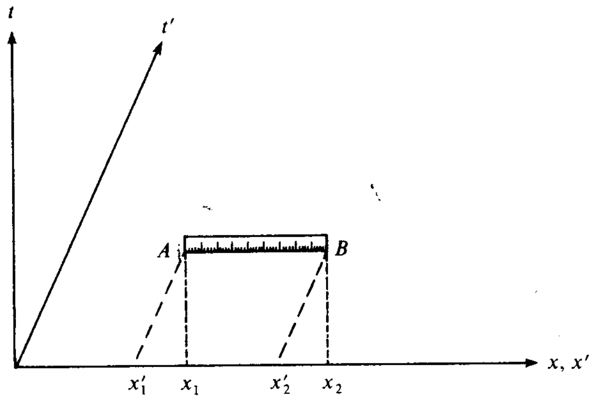


Figure 45.4 Length measurements compared on a space–time diagram.

Next, suppose an event P is described by two observers, one in frame S and the other in frame S'. This event can be described by two pairs of coordinates, (x_P, t_P) in frame S and (x'_P, t'_P) in frame S', as shown in Fig. 45.3b. The geometric relation between these two pairs of coordinates and the respective axes can be described as follows: In frame S, the coordinates (x_P, t_P) are determined geometrically in the usual manner by dropping a perpendicular from P to each of the x and t axes, as shown by the short dashes in Fig. 45.3b. This means that a choice of scale has been made along each of the x and t axes. Now we choose the scale along the x' and t' axes to conform with the Galilean transformation:

$$t' = t,$$
$$x' = x - v_0 t.$$

Geometrically, this means that the horizontal dashed line through P cuts the t' axis at a point representing the time $t'_P = t_P$ in the S' frame, and a line through P parallel to the t' axis (indicated by the longer dashes in Fig. 45.3b) crosses the horizontal axis at a point representing the x' coordinate $x'_P = x_P - v_0 t_P$ in frame S'. In this way, the pair (x'_P, t'_P) represents the coordinates of P in the S' frame, determined by the oblique axes.

Because the scales for measuring distances are different on each of the x and x' axes, the length of an object might be different in the two frames. We will now show that the length of a measuring stick is the same in both frames, provided that the object is observed simultaneously in both frames. Figure 45.4 shows a measuring stick with endpoints A and B. The stick is shown horizontal to indicate that the two endpoints A and B are observed simultaneously in both frames. In frame S, the length of the stick is $x_2 - x_1$. In frame S' its length is $x'_2 - x'_1$. Because the stick is observed at the same time in both frames, we have

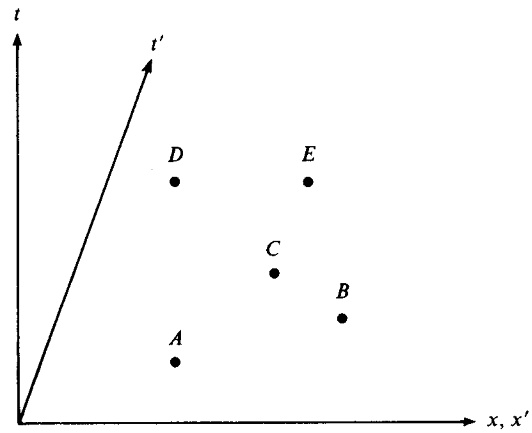
$$x'_2 - x'_1 = (x_2 - v_0 t) - (x_1 - v_0 t) = x_2 - x_1,$$

which shows that the measuring stick has the same length in frame S' that it does in frame S.

Space–time diagrams will be encountered again in the next chapter in the discussion of special relativity.

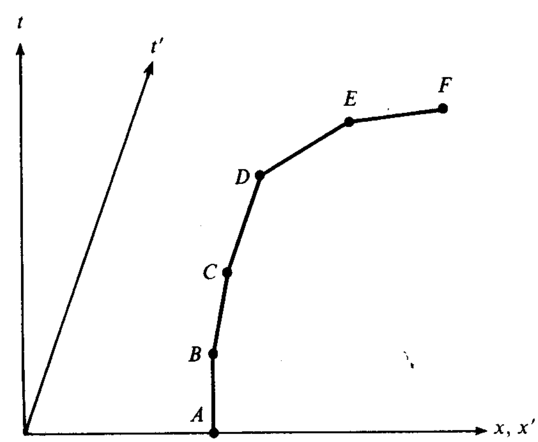
Questions

10. Five events A, B, C, D, E are shown on the following space–time diagram:



(a) Which events are simultaneous?
(b) Which events occur at the same position in the S frame?
(c) Which events occur at the same position in the S' frame?

11. In the space–time diagram shown, the points A, B, C, D, E, F lie on the world line of a moving particle. Line segment AB is vertical and line segment CD is parallel to the t' axis.



(a) According to an observer in frame S, when is the particle at rest?
(b) According to an observer in frame S', when is the particle at rest?

(c) According to an observer in frame S, when does the particle have its greatest speed?

45.4 RELATIVITY AND THE NATURE OF LIGHT

The Galilean principle of relativity served mechanics well, but fell short in the study of light. The nature of light had been debated for centuries. In the sixth century B.C., Pythagoras believed that light consisted of streams of particles. Newton returned to this idea 22 centuries later. He called the light particles "corpuscles" and used them to explain phenomena such as sharp shadows, straight-line propagation of light, and the ability of light to travel across the vacuum of space. An opposing model developed by Newton's rival, Robert Hooke, and by the Dutch physicist Christian Huygens, proposed that light was a wave. The two theories made different predictions concerning the speed of light.

If light consists of particles, then its speed should depend on the speed of the source, just as the speed of a bullet fired from a moving airplane depends on the speed of the plane. Galilean relativity tells us that the velocity of a bullet is equal to its muzzle velocity relative to the plane plus the velocity of the plane relative to the ground. Consequently, the speed of a bullet depends on the speed of the source, and by analogy, the speed of a light particle should depend on the speed of the light source.

On the other hand, if light is a wavelike phenomenon, its speed does not depend on the source but, instead, depends only on properties of the medium through which it propagates. For example, if you hear a whistle from a fast-moving train, the speed with which the signal reaches your ears has nothing to do with the speed of the train; the signal travels at the speed of sound in air. The pitch or frequency of the sound does depend on the speed of the train (for an approaching train the pitch is higher than for a receding train), but the speed of the sound does not.

The debate raged throughout the seventeenth and eighteenth centuries because no one knew how to measure the speed of light. To make matters worse, other phenomena concerning light could be explained by either of the two theories. As already noted in Chapter 44, Thomas Young performed some remarkable experiments that supported the wave theory. We recapitulate his double-slit experiment, illustrated in Fig. 45.5. Young passed a beam of light through two narrow slits toward a screen. Some of the light passing through one slit strikes the screen at some point P, and some light passing through the other slit strikes the same point P, as indicated by the lines AP and BP in Fig. 45.5a. If the two light waves arriving at P travel the same distance and are in phase, as illustrated by the two sinusoidal curves in Fig. 45.6a, their wave crests reinforce each other and a bright spot, called a fringe, appears on the screen. In this case the waves are said to interfere constructively.

A short distance from P light arrives from two waves that start in phase but don't travel the same distance, as illustrated by lines AQ and BQ in Fig. 45.5b. If distance BQ exceeds AQ by exactly half of a wavelength, then the light arriving at Q from A will be ahead by half of a wavelength relative to that arriving from B. In this case, a wave crest will arrive simultaneously with a wave trough and the waves will cancel each other, leaving a dark fringe on the screen. In this case the waves interfere destructively, as suggested by Fig. 45.6b.

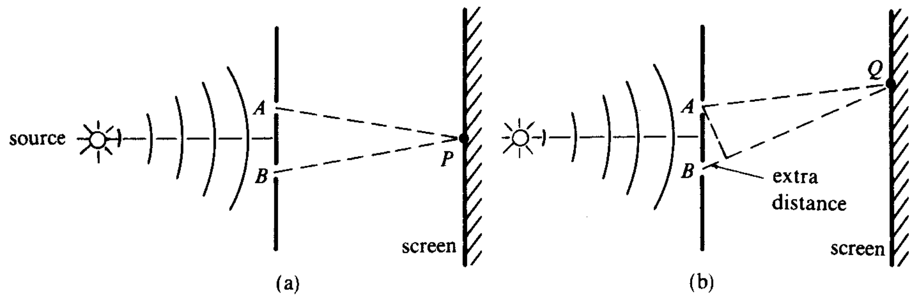


Figure 45.5 Young's double-slit experiment.

Light waves will interfere constructively at any point on the screen where the difference of the travel distances of the two waves is an integer multiple of one wavelength, and they will interfere destructively at any point where this difference is an odd multiple of half a wavelength. The two types of interference should produce alternate bright and dark fringes on the screen. This effect was actually observed in Young's experiments, and because the particle theory could not account for the observed interference patterns, this experiment was taken as conclusive evidence that light is, indeed, a wave.

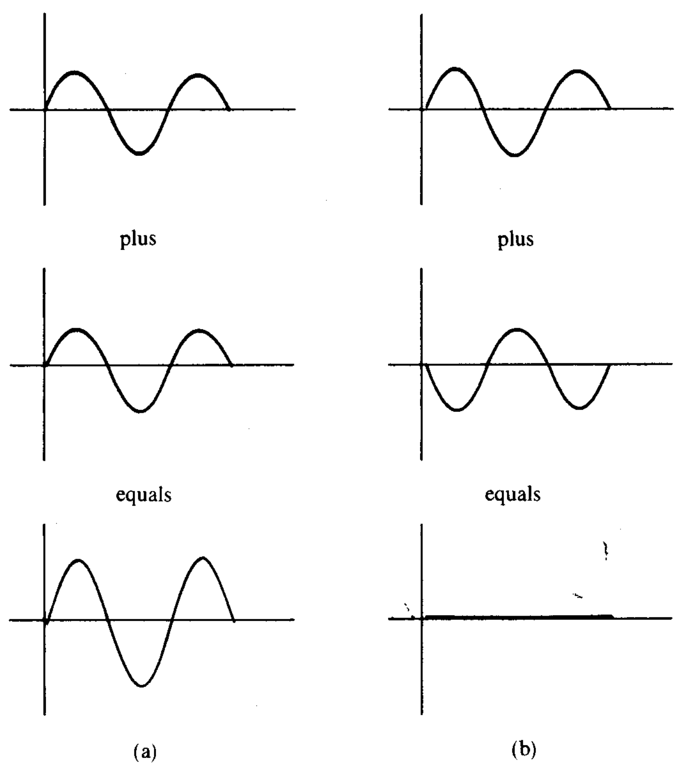


Figure 45.6 (a) Constructive interference. (b) Destructive interference.

Once the wave nature of light was accepted, the next question was to determine what was actually waving. Because sound waves or water waves can only propagate in a medium, it seemed reasonable to assume that light would also require some kind of transmitting medium. To nineteenth century physicists familiar with the mechanical workings of nature, that medium was the *luminiferous aether*. The notion of an aether can be traced back to ancient Greek philosophers, including Aristotle, who believed it to be the medium through which the planets and other heavenly bodies moved. But for nineteenth century physicists, the aether was a colorless, tasteless, odorless medium, rigid enough to propagate light with enormous speed, but tenuous enough to allow the planets to move freely through it. It was soon realized that the aether theory conflicted with the principle of relativity.

According to the principle of relativity, there is no such thing as absolute rest; there is no frame of reference that tells us when things are at rest and when they're not. But if all of space is filled with luminiferous aether that transmits light at a definite speed, then the aether itself could be regarded as a frame at rest, with everything else being in motion relative to it. In particular, one could detect absolute rest or absolute motion simply by measuring the speed of light. It would have one value if you were at rest with respect to the aether, and another value if you were in motion with respect to the aether.

Of course, the stationary aether influenced Maxwell's magnificent theory of electromagnetism, which predicted light to be an electromagnetic wave. Implicit in Maxwell's theory was the prediction that the aether could be detected by the earth's motion through it, but, as we'll see later, most of the effects depend on the square of the ratio of the speed of the earth through the aether to the speed of light, $(v/c)^2$. The ratio v/c was known to be on the order of 10^{-4} so $(v/c)^2$ is on the order of one part in 100 million, and no experimental techniques known in Maxwell's time could attain such sensitivity.

In a sense the success of such an experiment would have been the culmination of the Copernican system, placing the earth in its motion around a fixed sun. Critics might say that although the Copernican theory makes the motions of heavenly bodies mathematically simpler, it's only a clever mathematical device and doesn't correspond to reality because there is no such thing as absolute motion. If the earth could be shown moving through the aether, the Copernican system would have physical reality as well as mathematical convenience. Clearly, the burden of proof was on the experimentalists.

Question

12. Suppose that sodium light of wavelength 5.89×10^{-7} m is used in Young's double-slit experiment. At a particular point on a distant screen, the path length difference between waves arriving from the slits is 4.47×10^{-6} m. Will there be a bright or a dark fringe at that point?

45.5 THE MICHELSON - MORLEY EXPERIMENT

During the nineteenth century various attempts were made to detect the motion of the earth through the aether, and among them was the most significant failure in the history of physics: the Michelson–Morley experiment. Albert A. Michelson became interested in experiments probing the nature of light when he was a physics instructor at the U.S. Naval Academy. Michelson's forte was careful, precise optical measurements. In 1873 he devised an ingenious experiment that yielded the most accurate value at that time for the speed of light. He undertook such a measurement after reading a letter from the great James Clerk Maxwell to David Peck Todd, a colleague of Michelson's at the Nautical Almanac Office. Michelson was intrigued by Maxwell's statement that no terrestrial method for measuring the speed of light could detect the earth's motion through the aether because the effect would depend on the square of the ratio of the earth's velocity to that of light – an effect too small to be observed.

Maxwell presented a very simple argument. Imagine that the earth is moving at some speed v through the aether. If a beam of light travels at speed c in the rest frame of the aether, then an earthbound observer, moving through the aether at speed v, should measure $c + v$ for the speed of light when the light is traveling against the motion of the earth. Similarly, the observer should measure $c - v$ for the speed of a beam of light moving in the same direction as the earth. By comparing the speed of light in the forward and backward directions with respect to the speed of the earth, one should be able to measure the speed v of the earth through the aether.

The speed of the earth through the aether was taken to be its speed about the sun: 3×10^4 m/s, or 10^{-4} times the speed of light. The difference in the two speeds $c + v$ and $c - v$ is $2v$, which is extremely small compared to c. As we will

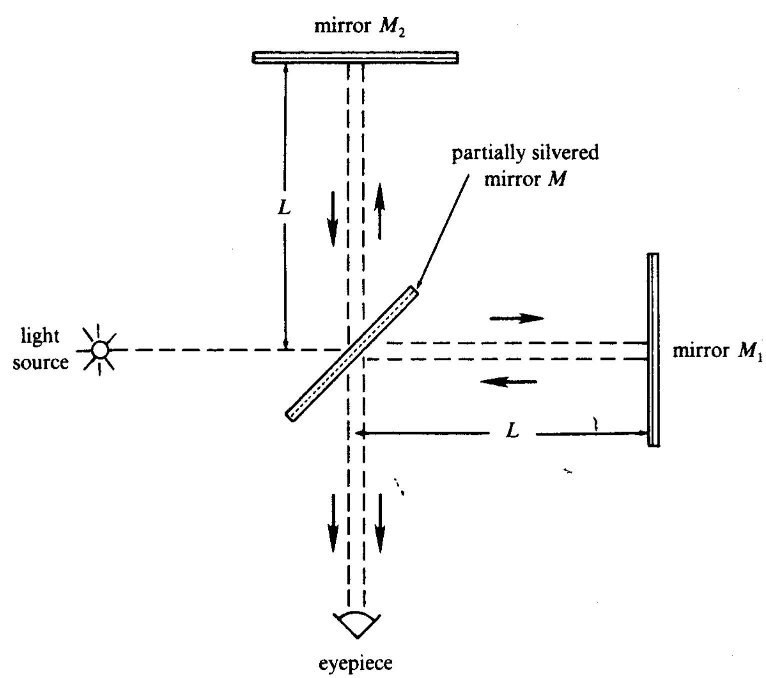


Figure 45.7 Schematic representation of a Michelson interferometer.

show later, the difference in time for light to travel a certain distance down and back depends on $(v/c)^2$, which is even smaller and very difficult to detect by experiment. The challenge of devising an optical instrument sufficiently sensitive to detect the earth's motion led Michelson to study the interference of light.

In 1880 Michelson was granted a leave to study optical techniques in Europe and immediately began working on an apparatus to detect the motion of the earth through the aether. In the laboratory of Hermann von Helmholtz at the University of Berlin, Michelson invented a new instrument of unprecedented sensitivity that has come to be known as the Michelson interferometer.

A diagram representing the principle of a Michelson interferometer is shown in Fig. 45.7. A beam of light is directed at a plate of partially silvered glass that partly transmits and partly reflects light. The silvered surface splits the beam into two components that travel in perpendicular directions, as shown. The transmitted beam travels a distance L to a mirror M_1 and is reflected back along the same path to the silvered glass plate. Similarly, the reflected portion of the beam travels an equal distance L, reflects from mirror M_2, and returns along its original path to the glass plate. At the glass plate both beams recombine as two superposed beams that are directed into a detector, a telescope.

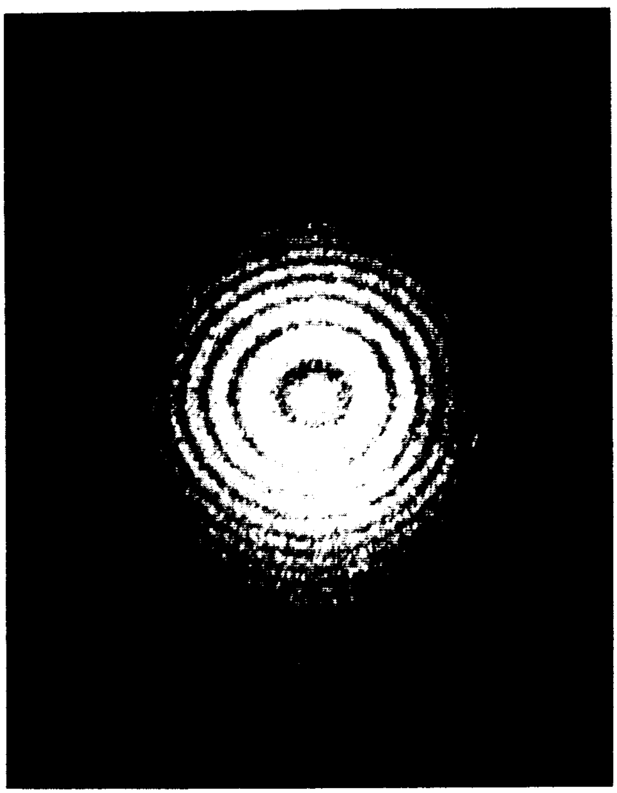


Figure 45.8 View of interference fringes from a Michelson interferometer. (Courtesy of Bob Paz, California Institute of Technology.)

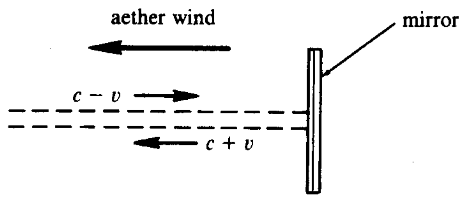


Figure 45.9 Light beam traveling parallel to the aether wind.

If both beams travel the same distance and arrive in phase with one another, they produce a bright fringe from the constructive interference. On the other hand, if the two waves travel different distances and arrive slightly out of phase, destructive interference will result. Consequently a series of bright and dark fringes is seen, similar to those shown in Fig. 45.8. If the length of either arm of the interferometer is changed slightly, the fringe pattern will also change slightly, by an amount that can be detected through the telescope.

Now imagine the whole apparatus moving relative to the aether at speed v in the direction of mirror M_1. From the viewpoint of the laboratory frame, an aether wind is blowing past the apparatus. It is easy to calculate the effect of this aether wind on the travel times for light along the arm of the interferometer parallel to and perpendicular to the wind. If the times are different, the beams will have traveled different distances and that difference will show up as either constructive or destructive interference fringes in the telescope.

When light travels against the wind, as shown in Fig. 45.9, its speed is $c - v$, and when it travels downwind, its speed is $c + v$. Now the time T required for one round trip is the sum of time out plus time back. In each case the time required is simply the distance L divided by the speed for that portion of the trip, so the total time is

$$T = \frac{L}{c - v} + \frac{L}{c + v}.$$

Adding the fractions we obtain

$$T = \frac{2Lc}{c^2 - v^2} = \frac{2L/c}{1 - v^2/c^2}. \tag{45.14}$$

The numerator of this last expression is the time it would have taken if everything were at rest and there were no relative motion to worry about – the total distance $2L$ divided by c, the speed of light. The denominator represents the correction factor due to the aether wind.

To calculate the time it takes light to make the journey up and down perpendicular to the aether wind, it is most useful to describe the process as it would be seen by an observer at rest in the aether. Such an observer would see the interferometer flying by and would be interested in how long light takes to leave the beam splitter and return. As the light travels up and down toward the mirror, as shown in Fig. 45.10, the earth moves a distance vt through the aether, so the light doesn't return to the same place because the mirror is moving.

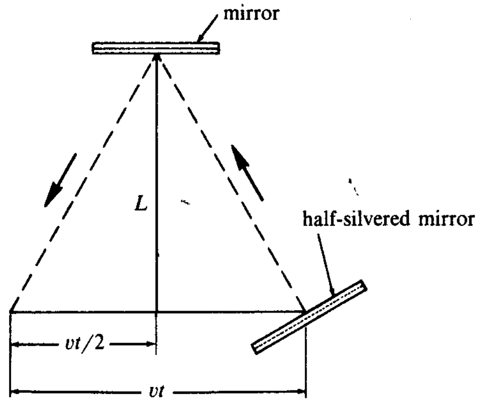


Figure 45.10 Light beam viewed from the aether frame.

The distance the light beam travels can be found from the Pythagorean theorem and is equal to $2\sqrt{(vt/2)^2 + L^2}$. The speed of light in the aether rest frame is simply c, so the time it takes the light beam to travel down and back is this distance divided by c:

$$t = \frac{2}{c}\sqrt{(vt/2)^2 + L^2}.$$

To determine t we square both sides,

$$t^2 = (4/c^2)\left[(vt/2)^2 + L^2\right] = v^2t^2/c^2 + 4L^2/c^2,$$

then solve for t^2, and take the positive square root to get

$$t = \frac{2L/c}{\sqrt{1 - v^2/c^2}}. \tag{45.15}$$

Comparing Eq. (45.15) with (45.14) we see that the travel times differ by a factor $\sqrt{1 - v^2/c^2}$, and therefore the light beams travel different distances. The actual difference is equal to

$$T - t = \frac{2L/c}{1 - v^2/c^2} - \frac{2L/c}{\sqrt{1 - v^2/c^2}} = \frac{2L/c}{1 - v^2/c^2}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$$

Multiplying and dividing the expression in parentheses by $1 + \sqrt{1 - v^2/c^2}$ we can rewrite this as

$$T - t = \frac{2L/c}{(1 - v^2/c^2)} \frac{v^2/c^2}{1 + \sqrt{1 - v^2/c^2}}.$$

Now v/c is a small number (on the order of 10^{-4}), and its square $(v/c)^2$ is even smaller (on the order of 10^{-8}), so we can disregard the terms v^2/c^2 in the denominator and obtain the approximation

$$T - t \approx \frac{2L}{c}\frac{v^2/c^2}{2} = L\frac{v^2}{c^3}. \tag{45.16}$$

This yields an incredibly small time difference $T - t$, and Michelson realized that he couldn't possibly measure the quantity $T - t$ to compare with the predicted value in (45.16). Instead, he realized that by rotating the entire apparatus by 90° he would reverse the roles of the two light paths MM_1M and MM_2M. The time difference between the two waves entering the telescope would also be reversed, thus changing the phase difference between the superposed waves and altering the positions of the interference fringes. He decided to simply rotate the interferometer by 90° and look for a shift of the interference fringes.

As remarked earlier, a bright fringe occurs if the waves arrive in phase and a dark fringe occurs if they arrive half a wavelength $\lambda/2$ out of phase. Therefore, the presence of m bright fringes in a path difference x would mean that m full wavelengths λ fit exactly into the distance x, so $x = m\lambda$ or

$$m = \frac{x}{\lambda}. \tag{45.17}$$

Consequently, a slight change Δx in x would involve a corresponding fringe shift of Δm, where

$$\Delta m = \frac{\Delta x}{\lambda}.$$

The small change in travel distance Δx occurring from rotating the apparatus by 90° would be the product of the corresponding travel time Δt multiplied by the speed of light. The change in travel time Δt is exactly twice the time difference $T - t$ calculated in (45.16); therefore, the corresponding fringe shift caused by rotating the apparatus is

$$\Delta m = \frac{2Lv^2}{\lambda c^2}. \tag{45.18}$$

In Michelson's original apparatus, $L = 120$ cm and $\lambda = 5.7 \times 10^{-5}$ cm, so that a fringe shift of $\Delta m = 0.04$ should have been observed. The sensitivity of his apparatus should have allowed Michelson to measure the fringe shift with an accuracy of about 50%, but he did not observe any shift at all. In 1881 Michelson published an account of his measurements in which he concluded, "The interpretation of these results is that there is no displacement of the interference bands. The result of the hypothesis of a stationary aether is thus shown to be incorrect."

Michelson, together with Edward W. Morley, improved the original apparatus and repeated the experiment at Case School of Science in Cleveland in 1887. By using several mirrors to make the light reflect back and forth through a greater distance, they increased the effective length of the interferometer arms by a factor of 10 over that of the original apparatus, so that a fringe shift $\Delta m = 0.40$ should have been observed. In addition, they mounted the optical parts on a heavy sandstone slab on a wooden float supported by mercury in a trough, as illustrated in

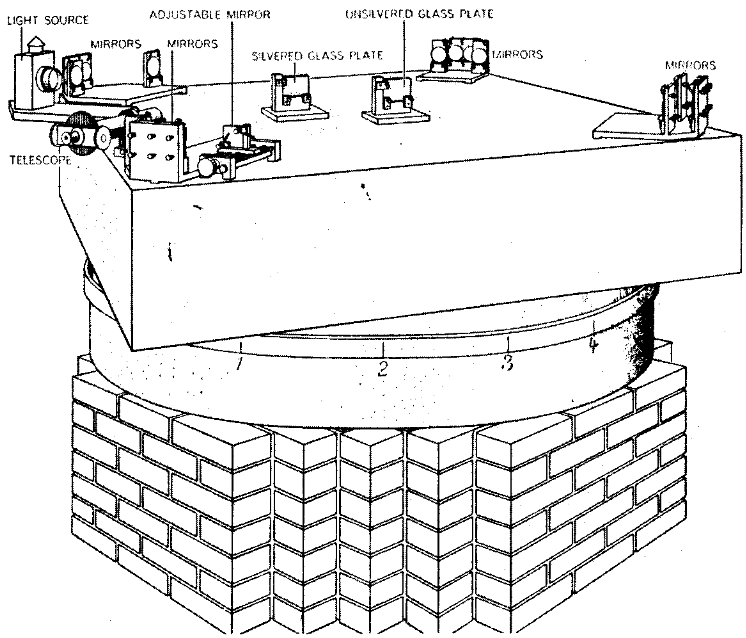


Figure 45.11 Michelson–Morley Experiment by R. S. Shankland. From the Michelson-Morley Experiment by R. S. Shankland.

Fig. 45.11. The arrangement permitted continual observations of the interference fringes as the interferometer was rotated. With this magnificent instrument, Michelson and Morley should have seen a 0.40 fringe displacement with a precision of 0.01. Instead they observed *no displacement whatsoever*. The Michelson–Morley experiment gave a null effect – no effect of the aether was observed.

The experiment was repeated many times under different conditions. Two of the more interesting experiments were done by Miller and by Tomaschek, both in 1924. Miller used sunlight instead of a laboratory light to test whether the effect had anything to do with whether the source of the light was moving with the earth. He used an interferometer arm three times as long as that in the Michelson–Morley interferometer and should have observed a shift of 1.12 fringes with a precision of 0.014. Tomaschek performed the experiment with starlight, wondering if the null result was an effect of the solar system. In this more difficult experiment, Tomaschek used an interferometer with an arm length of 860 cm. He expected a fringe shift of 0.3 and showed it was not more than 0.02. The result of all the trials of the Michelson–Morley experiment, which are summarized in Table 45.1, is that within the precision of each experiment, the effect of the aether is nonexistent.

Questions

13. Michelson and Morley intended to repeat their experiment at intervals of 3 months. Explain why.

Table 45.1 Repetitions of the Michelson – Morley Experiment

Observer (year)	L (cm)	Δm_{calc}	Precision
Michelson (1881)	120	0.04	0.02
Michelson, Morley (1887)	1100	0.40	0.01
Morley, Miller (1902–04)	3220	1.13	0.015
Miller (1921)	3220	1.12	0.08
Miller (1923–24)	3220	1.12	0.03
Miller (1924)	3220	1.12	0.014
Tomaschek (1924)	860	0.3	0.02
Miller (1925–26)	3200	1.12	0.08
Kennedy (1926)	200	0.07	0.002
Illingworth (1927)	200	0.07	0.0004
Piccard, Stahel (1927)	280	0.13	0.006
Michelson et al. (1929)	2590	0.9	0.01
Joos (1930)	2100	0.75	0.002

14. In the Michelson–Morley experiment of 1887, sodium light of wavelength 5.9×10^{-7} m was used. The experiment would have revealed any fringe shift larger than 0.01 fringes. What upper limit does this result place on the speed of the earth through the aether?

15. Show that if the arms of the Michelson interferometer are of different lengths L_1 and L_2, then the expected fringe shift is given by

$$\Delta m = \frac{(L_1 + L_2)}{\lambda} \frac{v^2}{c^2}.$$

16. In their experiment, Michelson and Morley ignored the effect of the earth's rotation. Discuss how this motion of the earth would affect the experiment.

45.6 A FINAL WORD

The stationary aether was every bit as important to nineteenth century scientists as the stationary Earth was to the Aristotelians before Copernicus. Discarding the aether theory was no small step. According to folklore, when Michelson and Morley

Figure 45.12a Portrait of Michelson. (Courtesy of the University of Chicago Archives.)

Figure 45.12b Portrait of Morley. (Courtesy of Case Western Reserve University.)

performed their experiment they triumphantly expelled the aether theory from the pantheon of physics. However, the actual facts are somewhat different.

Michelson never abandoned the aether theory. Instead, he considered his experiment to have been a failure. In 1907 he won the Nobel prize – the first American to receive this honor – not for showing that the aether theory was wrong, but for inventing the interferometer, which enabled him to make his delicate measurements. Equation (45.17) implies

$$\lambda = \frac{x}{m},$$

a formula that expresses the wavelength λ in terms of the travel distance x and the number of bright fringes m. By measuring m, the Michelson interferometer gives a supremely sensitive way to detect small changes in x by using light of known wavelength.

Michelson never recovered from the shock of the experimental result he didn't want, and he always felt that the only useful purpose served by the entire aether controversy was that it led to the invention of his beautiful device. But to science, his feelings are inconsequential. The importance of his experiment was that it showed that if the aether existed it had no detectable effects. And what was the consequence of this revelation? Many physicists proposed ingenious ideas to account for the experimental results. Some imagined that the earth drags along with it a thin layer of aether as it travels through space. They asserted that the Michelson–Morley experiment showed no effect because it was performed on Earth, where the aether was motionless. Others proposed that light travels at a fixed speed not with respect to the aether, but with respect to its source. Another idea, proposed independently by Lorentz and Fitzgerald, was that the length of the interferometer arm parallel to the aether wind contracted by just the right amount to compensate exactly for the slower times to travel up and downwind as compared to traveling crosswind.

Regardless of the details of these imaginative attempts to reconcile theory with experiment, all attempts were specifically designed to explain away the null result of Michelson and Morley. It is firmly entrenched in the folklore of physics that this experiment motivated Einstein to formulate his theory of relativity. However, when Einstein published his theory in 1905 he made no reference to the Michelson–Morley experiment. Afterward he reported that he had no knowledge of it at that time. We'll return to this point in the next chapter.