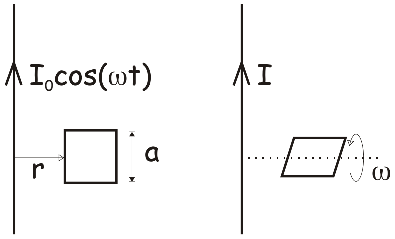


Figure 33: An infinitely long conductor and a loop

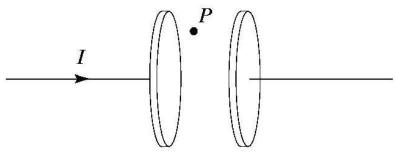


Figure 34: The field inside a charging capacitor

9. Compare the energy density in a magnetic field and an an electrostatic field. Maximum possible (in today's technology) static fields strengths are $20T$ and $10^8 V/m$.

10. Consider an infinitely long straight wire carrying a current $I = I_0 \cos(\omega t)$, and a square loop with sides a at a distance r from the wire (see figure 33, left). Calculate the induced current in the loop.[74]

11. Same question as above, but now the current in the wire is constant at a value I_0, but the loop is rotating around the axis perpendicular to the wire, at angular velocity ω $[sec^{-1}]$ (see figure 33, right).[75].

18 The Maxwell Equations

18.1 Grasps

1. As the capacitor shown below is charged with a constant current I, at point P there is a [76]

   (a) constant electric field
   (b) changing electric field
   (c) constant magnetic field
   (d) changing magnetic field
   (e) changing electric field and a magnetic field
   (f) changing magnetic field and an electric field
   (g) none of the above

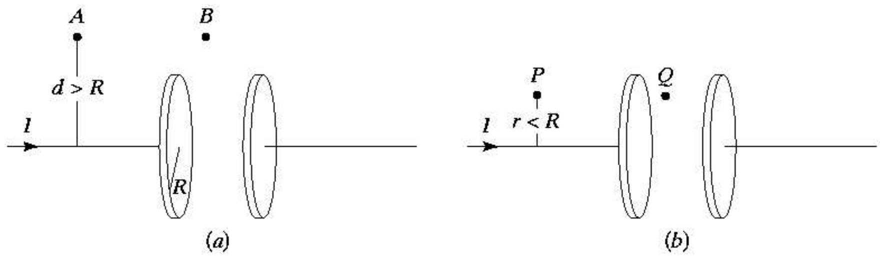


Figure 35: The field inside a charging capacitor

2. Which gives the largest average energy density at the distance specified and thus, at least qualitatively, the best illumination
   (a) a 50-W source at a distance R
   (b) a 100-W source at a distance $2R$
   (c) a 200-W source at a distance $4R$
3. For a charging capacitor, the total displacement current between the plates is equal to the total conduction current I in the wires. The capacitors in the diagram have circular plates of radius R. In (a), points A and B are each a distance $d > R$ away from the line through the centers of the plates; in this case the magnetic field at A due to the conduction current is the same as that at B due to the displacement current. In (b), points P and Q are each a distance $r < R$ away from the center line. Compared with the magnetic field at P, that at Q is [77]
   (a) larger
   (b) smaller
   (c) the same
   (d) need more information
4. Feynman describes in section 18-4 what happens if a sheet of charge is suddenly moving. A similar situation arises when a current is suddenly switched on in a wire. The current causes a magnetic field, and the front of this field travels into space with a velocity v (figure 36).
   (a) Make a drawing of the magnetic field vectors, a short time t after the field has travelled away from the wire.
   (b) Make a graph of the amplitude of the magnetic field as a function of r at time t
   (c) Make a drawing of the vectorfield $\partial \boldsymbol{B}/\partial t$.
   (d) Make a drawing of the electric vector field
   (e) Make a graph of the electric field as a function of r at time t
5. Try to solve the paradox in 17-4. Hint: the disc will rotate.

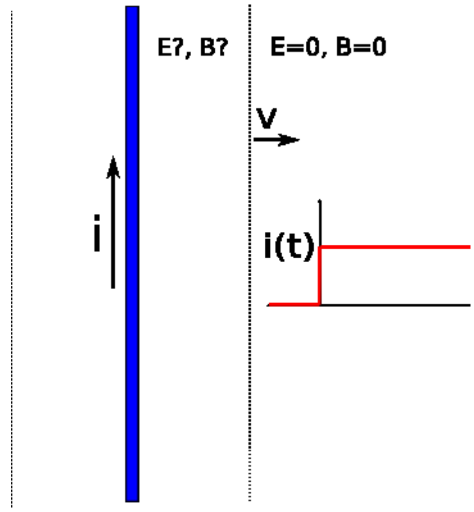


Figure 36: The field surrounding a wire in which the current suddenly is switched on travels at speed v away from the wire

18.2 Problems

1. Let's dive into problem 18.1.4 and figure 36 again. The current increases from 0 to i_0=1 A at t=0.
   (a) Calculate $\boldsymbol{B}$, $\boldsymbol{E}$ everywhere at time t. Hint: follow Feynmans method in 18-4, using Stokes law twice, adapting it for the wire.
   (b) Calculate v in [m/s].
   (c) Make a graph of the amplitude of the E and B field at distance r_0=10 cm from the wire as a function of time (quantitative!).

34 Magnetism in materials

34.1 Review questions

34.2 Grasps

1. What are the boundary conditions for magnetostatic fields at an interface between two different media[78].
2. Explain why magnetic flux lines leave the surface of a ferromagnetic soft medium (with $\mu_r = \infty$) perpendicularly [79]
3. Explain qualitatively the statement that $\boldsymbol{H}$ and $\boldsymbol{B}$ along the axis of a cylindrical bar magnet are in opposite direction.[80]

34.3 Problems

1. A circular rod of magnetic material with permeability μ is inserted coaxially in the long solenoid of figure 37. The radius of the rod, a, is much

CHAPTER 20

20-1. It is sometimes convenient to consider complex solutions of differential equations.

a) If we assume the fields vary sinusoidally in time and with the coordinate x (no y or z dependence), show that each component of, for example,

$$\vec{E} = \vec{E}_o \, e^{+i(\omega t - kx)}$$

satisfies the wave equation. (Remember that the actual field is found by taking the real part of this expression.)

b) Convince yourself that the real part of $\vec{E}$ corresponds to a plane wave travelling along the x-axis. In what direction is it going?

c) Show that the operation $\vec{\nabla}$ when applied to a function like that in part a) becomes:

$$\vec{\nabla} = \vec{e}_x \frac{\partial}{\partial x} = \vec{e}_x \,(-ik)$$

where $\vec{e}_x$ is a unit vector along the x-axis and i is is $\sqrt{-1}$; i.e., show that we can replace the $\vec{\nabla}$ operation by a simple multiplication. What similar statement can you make about the time derivative?

d) Using the results of c, write down (by inspection) how Maxwell's equations appear when applied to fields which vary sinusoidally with x and t. What relationship must exist between k and ω?

e) If the field has the form $\vec{E} = \vec{E}_o \, e^{+i(\omega t + kx)}$ how do your answers change?

20-2. A plane electromagnetic wave of frequency $\underline{\omega}$ is reflected from a mirror travelling with a velocity $\underline{v}$ in the same direction as the wave. Using Maxwell's equations calculate the frequency of the reflected wave as seen by a stationary observer. Compare this result with that obtained in Volume 1 using relativity per se.

CHAPTER 21

21-1. Carry out the details of deriving Eq. (21.25).

21-2. Equation (21.1) gives an equation for calculating the electric field due to moving charges. Consider a dipole made up of a positive and negative charge oscillating about the origin along the z-axis; i.e., the motion of the positive charge is $z_+ = \frac{d}{2}\cos\omega t$ and that of the negative charge is $z_- = -\frac{d}{2}\cos\omega t$. The dipole moment is then defined to be $\vec{p} = d\cos\omega t\,\vec{e}_z$. Show that the equation mentioned above can be used to calculate the entire electric field of the dipole:

$$E_\varphi = 0$$

$$E_\theta = \frac{p}{4\pi\epsilon_o}\sin\theta\left[\left(-\frac{\omega^2}{c^2 r} + \frac{1}{r^3}\right)\cos\omega\left(t - \frac{r}{c}\right) - \frac{\omega}{cr^2}\sin\omega\left(t - \frac{r}{c}\right)\right]$$

$$E_r = \frac{2p}{4\pi\epsilon_o}\cos\theta\left[\frac{1}{r^3}\cos\omega\left(t - \frac{r}{c}\right) - \frac{\omega}{cr^2}\sin\omega\left(t - \frac{r}{c}\right)\right]$$

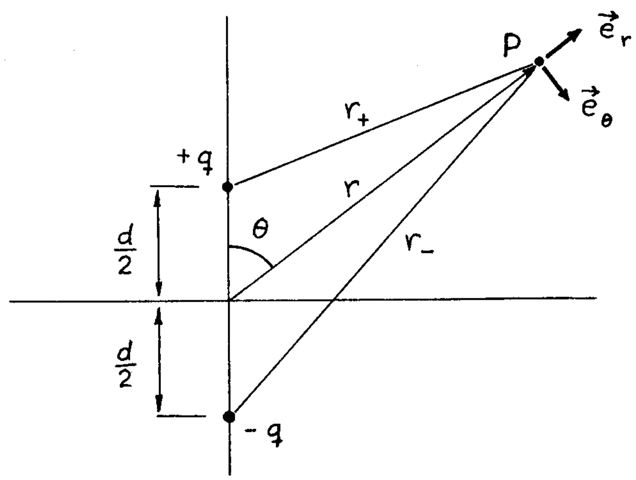
Assume the point P is at a distance $r \gg d$ from the dipole.

Hints:

$$\vec{e}_{r_+} \approx + \vec{e}_r$$

$\frac{d}{dt}(\vec{e}_{r_+})$ and $\frac{d^2}{dt^2}(\vec{e}_{r_+})$ are vectors nearly in the $\vec{e}_\theta$ direction

(See figure on next page)



21-3. From the symmetry of Maxwell's equations and the form of the electric and magnetic field of an oscillating electric dipole,* deduce the field of an oscillating magnetic dipole. The near field must resemble the field of a dipole formed by a small current loop of radius a ($a \ll \frac{c}{\omega}$), and current $i = i_o \cos(\omega t)$

Answer: $B_\varphi = 0$

$$B_\theta = \frac{1}{c^2}\,(E_\theta \text{ electric dipole})$$

$$B_r = \frac{1}{c^2}\,(E_r \text{ electric dipole})$$

$$E_\varphi = -\,(B_\varphi \text{ electric dipole})$$

$$E_\theta = E_r = 0$$

and replace p by μ where $\mu = \pi a^2 i_o$

---

* The electric fields of an electric dipole were given in problem 2; the corresponding magnetic field is found using Eq. (21.1)

$$c\vec{B} = \vec{e}_{r'} \times \vec{E}$$

21-4. In problem 2, an oscillating dipole was made up of two moving charges. Another way of producing a dipole is as follows: the dipole is made up of two conducting balls joined by a wire of length $\underline{d}$. An oscillating current in the wire is set up which establishes a net charge $\pm\, q(t)$ at the ends, but leaves the wire neutral; $q(t)$ can be represented as the real part of $Q_o\, e^{i\omega t}$.

At any field point P, a distance $r \gg d$ from the dipole, the integral for the retarded potential gives an exact expression for φ: (see figure below)

$$\varphi = \frac{Q_o}{4\pi\epsilon_o}\left[\frac{\cos\omega(t - r_1/c)}{r_1} - \frac{\cos\omega(t - r_2/c)}{r_2}\right]$$

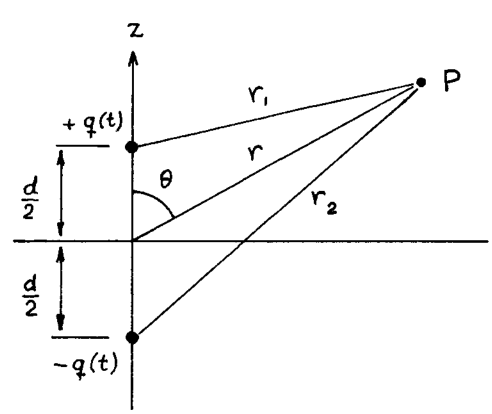
a) Assuming that $\frac{\omega d}{2c} \ll 1$ show that:

$$\varphi \approx \frac{Q_o d \cos\theta}{4\pi\epsilon_o r}\left[\frac{1}{r}\cos\omega(t - r/c) - \frac{\omega}{c}\sin\omega(t - r/c)\right]$$

b) Further show that

$$A_z \approx -\frac{Q_o \omega d \sin\omega(t - r/c)}{4\pi\epsilon_o c^2}$$

c) Convince yourself that these potentials give the same electric and magnetic radiation fields (that part of the fields which is proportional to $\frac{1}{r}$) as were found previously.



21-5. An antenna to be used at a frequency $\omega = \frac{2\pi c}{\lambda}$ is made up of two colinear wires each one-quarter wave length long, and is driven at their junction by a sinusoidal voltage of the appropriate frequency. The resulting current distribution in the wires is to a high degree of approximation sinusoidal

$$i = - i_o \sin (\omega t) \cos \left(\frac{2\pi z}{\lambda}\right)$$

To find the radiation from this antenna it can be regarded as a superposition of many dipoles, each located at a point z, of length Δz with strength varying from dipole to dipole.

a) Show that the proper dipole strength to use is

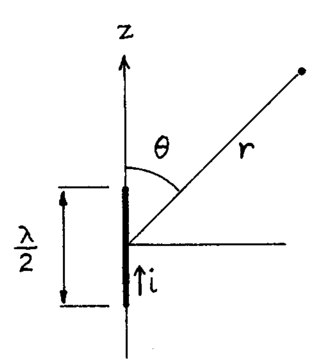
$$\Delta p = \left\{\frac{i_o}{\omega} \cos \frac{2\pi z}{\lambda} \cos \omega t\right\} \Delta z$$

b) Show that at large distances ($r \gg \frac{c}{\omega}$), the field of the entire antenna is

$$E_\theta = \frac{2i_o}{4\pi\epsilon_o cr} \frac{\cos \left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \cos \omega\left(t - \frac{r}{c}\right)$$

$$B_\varphi = \frac{1}{c} E_\theta$$

c) Make a rough polar plot of E_θ vs θ for both this case and a single dipole and compare.



$$i = - i_o (\sin\omega t) \cos \frac{2\pi z}{\lambda}$$

$$\left(- \frac{\lambda}{4} < z < \frac{\lambda}{4}\right)$$

21-6. A particle with charge $\underline{q}$ moves in a circle of radius $\underline{a}$ with a uniform speed $\underline{v}$.

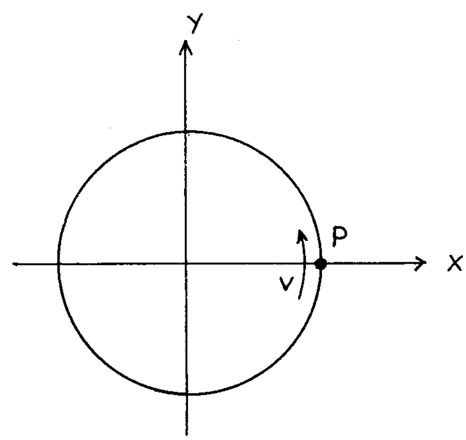
a) Find the scalar potential φ at the center of the circle when the particle is at the point P.

b) Find the vector potential A at the center at the same time.

c) By calculation of the potential in the neighborhood of the center use Eq. (18.19) and (18.16) to determine the electric and magnetic fields at the center. What is the direction of the electric field with respect to the radius vector to point P?

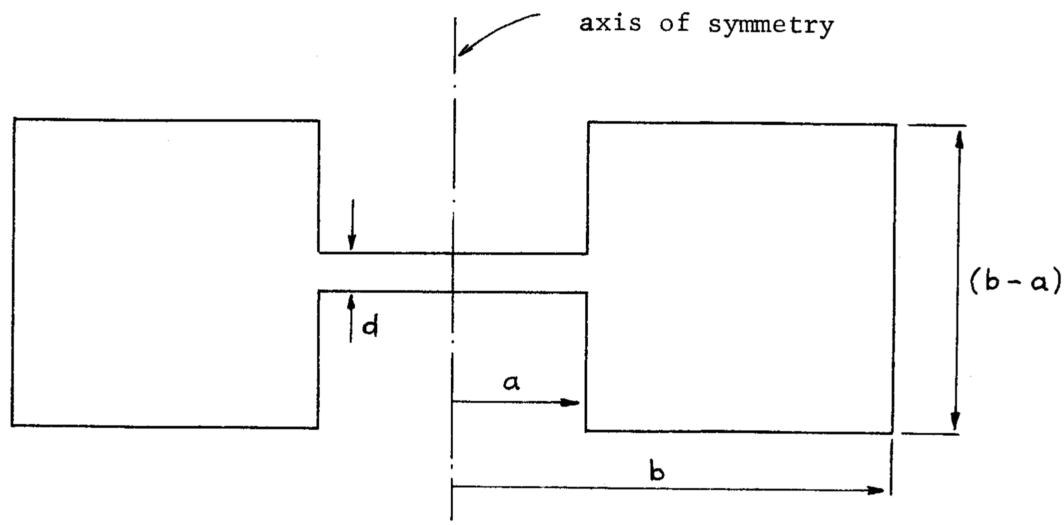
d) Also calculate these fields using Eq. (21.1).

Note this problem is relativistic. The velocity $\underline{v}$ is not necessarily small compared to $\underline{c}$.



CHAPTER 23

23-1. Find the approximate resonant frequency of the cavity shown below. Assume $d \ll a$, $d \ll (b-a)$. What are the main effects which you have neglected?



If the cavity is cooled uniformly (i.e., so the whole cavity is at the same temperature) does thermal contraction lead to an increase, a decrease, or no change in the resonant frequency?

CHAPTER 24

24-1. A transmission line has inductance L_o per unit length and capacitance. C_o per unit length. If the voltage and current are changing slowly (corresponding to transmission of signals with wavelength long compared to the line spacing), show that the governing equations are:

$$\frac{\partial V}{\partial x} = - L_o \frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial x} = - C_o \frac{\partial V}{\partial t}$$

Hence show that I and V both satisfy the wave equation

$$\frac{\partial^2 I}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 I}{\partial t^2} \qquad \frac{\partial^2 V}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 V}{\partial t^2}$$

$$\text{where } v^2 = \frac{1}{L_o C_o}$$

Note the assumption regarding slowly varying signals is not necessary, but the proof of this is beyond the scope of this chapter.

24-2. The characteristic impedance of a transmission line is $z_o = \sqrt{L_o/C_o}$ where L_o is the inductance per unit length and C_o is the capacitance per unit length.

Show that for a transmission line consisting of two thin strips of width $\underline{b}$ and a distance $\underline{a}$ apart $(a \ll b)$

$$z_o \simeq \frac{1}{\epsilon_o c}\,\frac{a}{b}$$

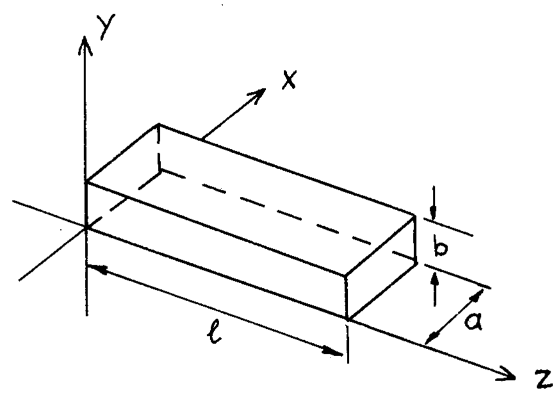
24-3. A cavity is made by putting conducting plates across the ends of a section of a cylindrical coaxial line of length ℓ.

a) Find the frequency of the lowest mode for which the electric field is always radial.

b) Give an expression for $\vec{E}$.

c) Compare the resonant frequency to $\omega_o = 1/\sqrt{LC}$ where L and C are the inductance and capacitance of a length ℓ of the coaxial line.

24-4. A rectangular waveguide made of perfectly conducting material has sides of length a and b as shown in the figure below.



The ends of a section of length ℓ are covered with plates of conducting material; i.e., the waveguide is effectively a resonant cavity. If the electric field is given by the real part of:

$$\vec{E}(x,y,z,t) = E_o(x,z)\, e^{i\omega t}\, \vec{e}_y$$

what is $E_o(x,z)$ for the cavity mode with the lowest resonant frequency? What is this lowest resonant frequency of the cavity?

24-5. A coaxial cable is composed of two concentric conducting cylinders. One end (x = 0) is connected to a voltage generator which produces a voltage

$$V(t) = V_o \cos \omega t \quad .$$

The other end of the cable ($x = \ell$) is covered with a conducting plate. The inductance per unit length is L_o and the capacitance is C_o.

a) If the length of the cable is $\frac{5\pi c}{2\omega}$, where c is the velocity of light, sketch the voltage between the conductors as a function of the distance x. Specify the values of x for which the voltage is maximum.

b) Write an expression for the forward going and reflected traveling waves which make up the voltage across the conductors.

c) What is the current at $x = 0$, $x = \frac{\ell}{2} = \frac{1}{2}\left(\frac{5\pi c}{2\omega}\right)$, and $x = \ell = \frac{5\pi c}{2\omega}$?

d) If the voltage source is an ideal generator whose shaft turns with an angular velocity ω, what average torque must be applied to the generator?

24-6. If a transmission line is terminated at $x = \ell$ by an impedance Z_T, show that the "sending end" impedance ($x = 0$) is given by

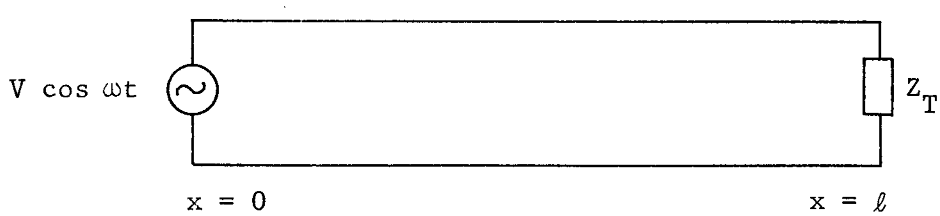
$$Z_s = iZ_o \frac{\tan \omega\sqrt{LC}\,\ell - iZ_T/Z_o}{1 + iZ_T/Z_o \tan \omega\sqrt{LC}\,\ell}$$

where $Z_o = \sqrt{\frac{L}{C}}$ is the characteristic impedance for the line. What is Z_s if

a) $Z_T = 0$

b) $Z_T = \infty$

c) $Z_T = Z_o$?



24-7. A transmission line with a characteristic impedance Z_1 is connected to a transmission line with a characteristic impedance Z_2. If the system is being driven by a generator connected to the first line (Z_1) show that the "reflection coefficient," i.e., $\frac{V_{reflected}}{V_{incident}}$, is given by

$$\frac{V_{reflected}}{V_{incident}} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

while the "transmission coefficient" is given by

$$\frac{V_{transmitted}}{V_{incident}} = \frac{2Z_2}{Z_1 + Z_2}$$

24-8. At JPL's Goldstone Tracking station the electronics cage is separated from the feed of the 85' receiver antenna by about 40 feet of wave guide. The inside dimensions of the wave guide are 5-3/4" by 11-1/2". If a 960 megacycle carrier is used, compare the signal velocity with the velocity in free space.

24-9. The electric fields inside of waveguides which are described in Chapter 24 have the property that the component of the electric field in the direction of propagation is zero; i.e., the electric field is transverse. (Modes of propagation such as these are therefore called TE, or transverse electric, modes.) There are also modes called TM modes in which there is no magnetic field in the direction of propagation. For the rectangular wave guide shown in Fig. 24-3 and 24-4, the vector potential of the TM modes is given by:

$$\vec{A} = \vec{e}_z \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{i(\omega t - k_z z)}$$

where

$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

a) Satisfy yourself that the magnetic fields found from this are really transverse and show that the $\vec{E}$ and $\vec{B}$ fields satisfy the wave equation and the proper boundary conditions. Hint: We require that

$$\vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \vec{\nabla} \times \vec{A}$$

where

$$\vec{\nabla} \cdot \vec{A} = -\frac{1}{c^2}\frac{\partial \varphi}{\partial t}$$

b) Show that the nm mode is not propagated if

$$\omega < c\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

CHAPTER 25

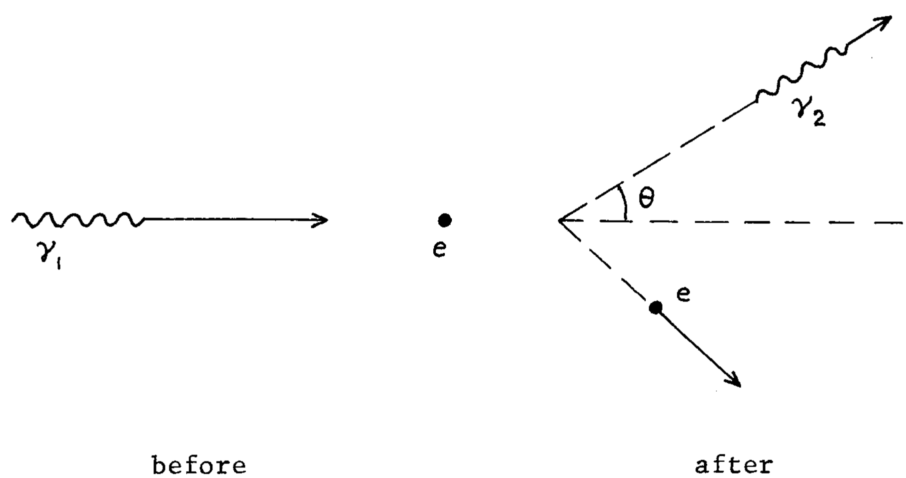
In the following problems the units are such that $c = 1$.

25-1. Write in four-vector form:

$$(\varphi^2 - \vec{A}^2)$$

$$(\vec{A} \cdot \vec{j} - \rho\varphi)$$

25-2. In the Compton effect, a stationary electron is hit by a photon, resulting in a change of momentum of each of them. Find the energy of the emitted photon in terms of its incident energy and the angle of deviation from its initial path.



25-3. A positron can be made by bombarding a stationary electron with a photon:

$$\gamma + e^- \longrightarrow e^- + e^+ + e^-$$

What is the minimum photon energy? Use four-vectors and invariant combinations of them wherever possible.

25-4. An electron-positron pair can be produced by a photon (γ) through the reaction

$$\gamma + e^- \longrightarrow e^- + (e^+ + e^-)$$

It is impossible, however, for the reaction

$$\gamma \longrightarrow e^{+} + e^{-}$$

to occur for a single isolated photon even though the photon energy is larger than twice the electron rest mass and charge is conserved. Using four-vectors show that this is true.

25-5. A particle of mass m at rest is struck by another particle of mass M and momentum P. After a totally inelastic collision they coalesce to form a single new particle. What is its mass and velocity? Compare your results with the values that would be calculated non-relativistically.

CHAPTER 26

In the following problems, the units are such that $c = 1$.

26-1. Write out and evaluate

$$\nabla_\mu F_{\mu\nu}$$

26-2. Find the four-vector whose three-vector part is

$$\rho\vec{E} + \vec{j} \times \vec{B}$$

What is the physical meaning of both time and space components of this four-vector?

26-3. Show that $\vec{E}^2 - \vec{B}^2$ and $(\vec{E} \cdot \vec{B})$ are invariant under Lorentz transformations. Note that if $\vec{E}$ and $\vec{B}$ form an acute angle in one frame, they do so in all frames. For what important physical phenomenon are both of these invariants equal to zero?

26-4. If $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields at a certain point in space in a given frame of reference, determine the velocity of another frame in which the electric and magnetic fields will be parallel. There are many frames which have this property -- if we have found one of them then the same property will be had by any other frame moving relative to the first with a velocity parallel to the common direction of $\vec{E}'$ and $\vec{B}'$. Therefore, we have a choice, and it is sufficient and convenient to find the frame which has a velocity perpendicular to both fields.

$$\text{Ans.} \quad \frac{\vec{v}}{1 + v^2} = \frac{\vec{E} \times \vec{B}}{E^2 + B^2}$$

26-5. In Chapter 26 the fields due to a charged particle moving with uniform velocity were obtained by the transformation of the potentials of a stationary charge to a moving frame. The fields $\vec{E}$ and $\vec{B}$ were obtained from A_μ in the usual way.

Now, find the fields by starting with the fields from a stationary charge and using the transformation laws of the fields.

26-6. Show that the electric and magnetic fields of a charge moving with uniform velocity $\vec{v}$ can be written

$$\vec{E} = \frac{q\vec{r}}{4\pi\epsilon_o r^3} \frac{1 - v^2}{(1 - v^2 \sin^2\theta)^{3/2}}$$

$$\vec{B} = \frac{q}{4\pi\epsilon_o} \frac{\vec{v} \times \vec{r}}{r^3} \frac{1 - v^2}{(1 - v^2 \sin^2\theta)^{3/2}}$$

where $\vec{r}$ is the radius vector from the present position of the particle to the observer and θ is the angle between $\vec{r}$ and $\vec{v}$.

26-7. A very long straight wire carries a current I produced by electrons moving at speed $\underline{v}$. Stationary positive ions in the wire make the total charge density vanish.

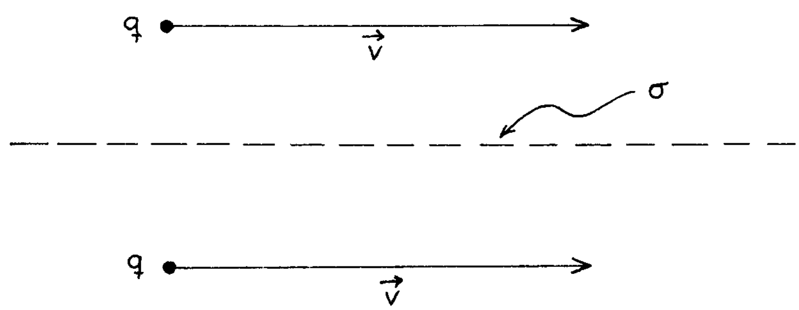
a) Find the fields outside the wire in a frame stationary with respect to the wire.

b) Transform the fields to a frame moving with the electrons. (In Chapter 13 the electric field observed from this moving frame was obtained by another method; Eq. (13.28)).

26-8. Two electrons with equal velocities $\vec{v}$ are moving side by side the distance $\underline{a}$ apart. Midway between them is an infinite sheet of fixed positive charges with a surface charge density σ.

a) How large must σ be in order that the electrons maintain the separation a?

b) Compare the charge density needed if the electrons have an energy of 500 Mev to that needed if they are moving at a very low velocity.



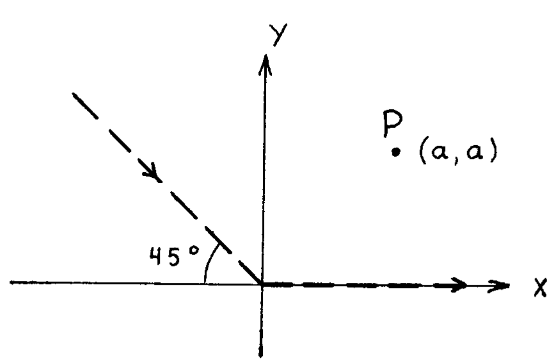
26-9. If f_μ is the four vector force acting on a particle, and u_μ is the four-vector velocity, show that

$$f_\mu u_\mu = 0$$

26-10. A particle of charge q moves in the x-y plane at constant speed v, along the trajectory shown by the dashed line in the figure. (It scatters at the origin). The speed remains constant throughout. At $t = t_1$ it is at $x = a$, $y = 0$.

a) The point P is at $x = y = a$. Find the electric field at P at time t_1 if $v/c = 0.5$ (c is velocity of light).

b) If in part a) the particle trajectory before the scattering were down the y-axis, how would your answer change?



CHAPTER 27

27-1. Using the technique used to derive Eq. (27.11), find equivalent expressions for

$$\vec{\nabla} \times (\vec{A} \times \vec{B})$$

$$\vec{\nabla} (\vec{A} \cdot \vec{B})$$

27-2. How many megatons of energy are contained in the magnetic field of the earth external to the earth? Assume that the earth's field is a dipole with a strength of about 2/3 gauss at the equator. A megaton is the energy released by the explosion of 1-million tons of TNT or 4.2×10^{15} joules. In view of your answer, consider how much you think a one megaton hydrogen bomb exploded high in the atmosphere would disturb the earth's field.

27-3. For a long straight wire of resistance R per unit length, calculate the flux of $\vec{S}$ at the surface of the wire when the wire carries a current I. Compare this with the heating calculated using Ohm's law.

27-4. A long coaxial cable is made of two perfectly conducting concentric cylinders. One end of the cable is connected to a battery whose terminal voltage is V, and the other end is connected to a resistance R so that there is a current I = V/R. Compute, using the Poynting vector, the rate of energy flow.

27-5. The average power radiated by a broadcasting station is 10 kilowatts.

a) What is the magnitude of the Poynting vector at points on the surface of the earth 10 km distant? At this distance, the waves can be considered plane. It is reasonable to assume that the power is radiated by a $1/4\lambda$ antenna above a perfectly conducting plane.

b) Find the maximum electric and magnetic intensities.

27-6. The fields of the lowest TE mode of the rectangular wave guide shown in Fig. 24-6 are given by:

$$\vec{E} = \vec{e}_y E_o \sin \frac{\pi x}{a} \cos (\omega t - k_z z)$$

$$\vec{B} = - \vec{e}_x \cdot E_o \frac{k_z}{\omega} \sin \frac{\pi x}{a} \cos (\omega t - k_z z)$$

$$- \vec{e}_z E_o \frac{\pi}{\omega a} \cos \frac{\pi x}{a} \sin (\omega t - k_z z)$$

a) Show that the solution given above satisfies the boundary conditions for the problem.

b) Calculate the Poynting vector $\vec{S}$ and the energy density U.

c) Calculate the average rate of energy flow across any plane perpendicular to the z-axis.

d) Calculate the average energy density in the wave guide.

e) Use the results of c) and d) to calculate the average velocity with which the energy is propagated. Show that this result is the same as the group velocity (24.27).

27-7. a) Find the rate of energy flow per unit area from an oscillating dipole with a dipole moment $\vec{p} \cos \omega t$.
Hint: Keep only the radiation terms (i.e., those which drop off as 1/r).

b) By integrating over the area of a large sphere centered on the dipole, show that the average power radiated is

$$\frac{1}{3} \frac{p^2}{(4\pi\epsilon_o c^2)} \frac{\omega^4}{c}$$

27-8. A plane light wave is incident upon a free electron. The electron oscillates under the influence of the $\vec{E}$ field. Calculate the ratio of the energy radiated per unit time by the electron to the light energy incident per unit area per unit time. Assume that the light wave is of low frequency and neglect the effect of the $\vec{B}$ field of the wave on the electron.

27-9. A dust particle in the solar system experiences two forces: the gravitational force of the sun and the planets, and the radiation force of light directed away from the sun. Since the gravity force is proportional to the volume of the particle and the radiation force is proportional to its cross section area, there will be a particle size for which these two forces are balanced. Assuming a spherical dust particle which absorbs all the radiation incident upon it, find the radius for which the forces balance.

An explanation for why a comet's tail points away from the sun has been based on the above phenomenon, assuming that the tail consists of small particles, perhaps even gas molecules. Is it a reasonable theory?

The energy radiated by the sun is 4×10^{26} watts, its mass is 2×10^{30} kg.

27-10. An "air-core" toroid of mean radius R and cross-sectional area πr^2 is wound with N turns of wire $[r \ll R]$.

A current with the time dependence $I(t) = Kt$ is turned on at $t = 0$.

a) Directly from the magnetic field, find the energy stored in the magnetic field at time t.

b) Find the direction and magnitude of the Poynting vector at a point just inside the toroid at time t.

c) Using the Poynting vector, find the rate of change of the field energy inside the toroid at time t. Check your answer with that of part a).

CHAPTER 28

28-1. If the rest mass of the electron is identified with the electrostatic energy of its charge and if the charge is uniformly distributed in the volume of a sphere, calculate the radius. Compare with the result given by Eq. (28.2).

28-2. It is well known that an electron in addition to charge and mass has an angular momentum (spin) and a magnetic moment related according to

$$\frac{\text{angular momentum}}{\text{magnetic moment}} = \frac{m}{q}$$

This is correct to about 0.1 per cent. Suppose the mass is given by Eq. (28.4).

a) Take a uniformly charged spherical shell with charge $\underline{q}$ and radius $\underline{a}$ and place a magnetic dipole of strength μ at the center. Show that the angular momentum of the electromagnetic field is

$$L = \frac{2}{3} \frac{q\mu}{4\pi\epsilon_o c^2} \frac{1}{a}$$

b) Find the ratio of angular momentum to magnetic moment and compare with the value $(\frac{m}{q})$ quoted above.

c) Given that μ_z for an electron is $(\hbar\, q/2m)$, calculate the maximum surface velocity of the spinning electron to give this magnetic moment. Make any comment you feel suitable. The quantity $(4\pi\epsilon_o c\hbar/q^2) = 1/\alpha$ has the value 137.

CHAPTER 29

29-1. A charged particle (charge q, rest mass m_0) is initially at rest at the origin. It is acted upon by a constant electric field in the x-direction.

a) Calculate the velocity and position as a function of time (relativistically).

b) If the particle has an initial velocity v_0 in the y-direction, how is your answer modified?

29-2. In a proton cyclotron the protons travel in circular paths in a uniform magnetic field. Find the "cyclotron frequency," the angular velocity as a function of q, B, m at low energies. How will the frequency change as the energy increases? At what energy has the frequency changed by 1 percent?

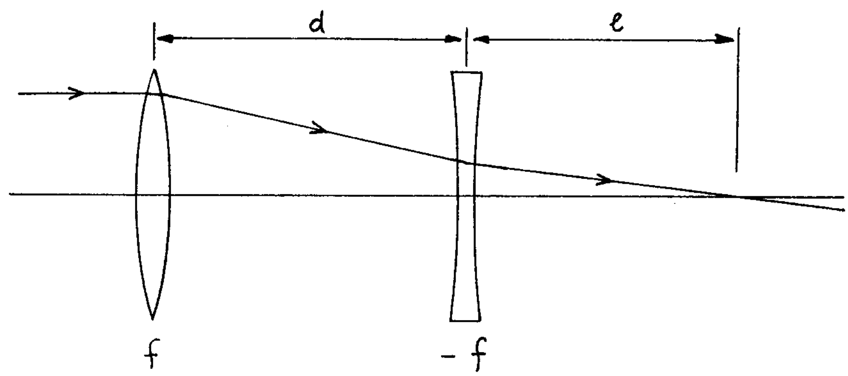
29-3. At $t = 0$ a particle with mass $\underline{m}$, charge $\underline{q}$ is located at the origin at rest. There is a uniform $\vec{E}$ field in the y-direction and a uniform $\vec{B}$ field in the z-direction.

a) Find the subsequent motion, $x(t)$, $y(t)$, $z(t)$ assuming non-relativistic motion. What restriction on E and B does this imply?

b) Can you suggest what the relativistic motion would be like? What happens if $E/B > c$?

c) If we put a plate in the xz plane at $y = 0$ and another parallel one at $y = d$ with potential difference $V_0 = E \cdot d$ and apply a magnetic field parallel to the plates, we have what is called a magnetron. If electrons are emitted from the negative cathode essentially at rest, how strong must the magnetic field be so that the electrons can't reach the positive anode?

29-4. The principle of alternating-gradient focusing can be illustrated by the following optical analog.



Even though the lenses have equal focal lengths, the combination has a converging action under certain circumstances.

a) For parallel incoming light, determine ℓ as a function of $\underline{d}$.

b) Under what conditions is the image real or virtual?

CHAPTER 32

32-1. Show that in a non-polar material, the square of the index of refraction at low frequencies is equal to the dielectric constant.

32-2. At a frequency of about 6 megacycles per second, the ionosphere becomes transparent. Estimate the electron density in the ionosphere using the free electron model.

32-3. An electric field applied to a metal is held constant for a long time, and then is suddenly turned off. Using the free electron model of a metal show that the relaxation time (i.e., the time for the drift velocity to drop to 1/e of its initial value) is equal to 2τ, twice the mean time between collisions.

32-4. In a metal there are plane-wave solutions to Maxwell's equations with the form

$$E_x = E_o\, e^{i(\omega t - kz)}$$

where k is a complex number. For low frequencies

$$k = (1 - i)\sqrt{\left(\frac{\sigma\omega}{2\epsilon_o c^2}\right)}$$

a) Write an expression for the magnetic field associated with such a wave.

b) What is the angle between $\vec{E}$ and $\vec{B}$?

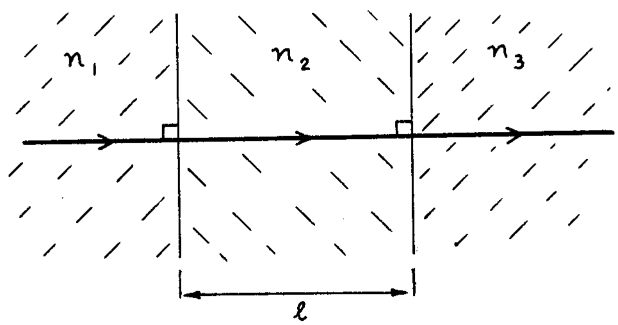
c) What is the ratio of the peak value of $\vec{B}$ to the peak value of $\vec{E}$ at any given value of z?

d) What is the phase difference between $\vec{E}$ and $\vec{B}$? (If the maximum of $\vec{E}$ occurs at t_1 and the maximum of $\vec{B}$ occurs at t_2, the phase difference is defined as $\pm\,\omega(t_1 - t_2)$.)

32-5. Equation (32.50) suggests that the ultraviolet cut-off of a metal (i.e., the value of ω at which $\underline{n}$ changes from real to imaginary) is quite sharp. Experiments show that this cut-off is not sharply defined. Show by means of a better approximation for n^2 that this experimental result is really in agreement with the theory.

CHAPTER 33

33-1. a) Determine the transmission coefficient for a plane electromagnetic wave passing through three dielectric media as shown below.



b) Show that if $n_2 = \sqrt{n_1 n_3}$ and $\ell = \frac{\lambda_2}{4}$ the transmission ratio is unity. (This is the reason for "coating" lenses in good cameras and binoculars.)

c) In binoculars to be used with ordinary white light, what is the thickness ℓ of the coating?

d) If it is only possible to coat one side of a lens, does it matter which side is coated? Why?

33-2. A beam of light with a wavelength 4500 Å (in vacuum) is incident on a prism as shown in the figure below and totally reflected through 90°. The index of refraction of the prism is 1.6. Compute the distance beyond the long side of the prism at which the electric field strength is reduced to $1/e$ of its value just at the surface. Assume the light is polarized so the $\vec{E}$ is perpendicular to the plane of incidence. Is your answer changed if $\vec{E}$ lies in the plane of incidence?

Figure for 33-2.