Metamaterials: Designer Materials for Light

In vacuum, the wave equation is given by

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E(r,t) = 0 = \left(\nabla^2 - \epsilon_0\mu_0\frac{\partial^2}{\partial t^2}\right)E(r,t) = 0$$

In a material, the wave equation is given by

$$\left(\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)E(r,t) = 0 = \left(\nabla^2 - \epsilon\mu\frac{\partial^2}{\partial t^2}\right)E(r,t) = 0$$

The behavior is much richer because ϵ and μ can be

(1) frequency dependent

(2) spatially dependent

(3) time dependent

(4) complex valued

(5) tensors

What can we make using designer materials?

What can we do with them?

Metamaterials

Designer Materials for Light

http://www.youtube.com/watch?v=z0-kUEiLWPU	Meridth	2:07
http://www.youtube.com/watch?v=Y4zwzlnExVU	Katie	0:58
http://www.youtube.com/watch?v=oLbS3M4V7oI	Science Sensei	3:36
http://www.youtube.com/watch?v=Za72ZFwjkjU	Kaku	1:48
http://www.youtube.com/watch?v=voZXdqGQpgU	Alien Scientist	9:00
http://mitworld.mit.edu/video/455	Pendry at MIT	1:15:00

A few more videos:

http://www.youtube.com/watch?v=cRtwS6SAPy4	Southampton
http://www.youtube.com/watch?v=_JpMJTJXf28	Fractal Antenna Systems
http://www.youtube.com/watch?v=Ja_fuZyHDuk&p=28A426692DB100A9	Duke
http://www.youtube.com/watch?v=Lc290LTsNZ8	Costas Soukoulis
http://www.youtube.com/watch?v=p5dOHQKobxg	Birmingham
http://www.youtube.com/watch?v=cRtwS6SAPy4	Pendry's perfect lens

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Max Born and
Emil Wolf

Electromagnetic Theory of Propagation,
Interference and Diffraction of Light

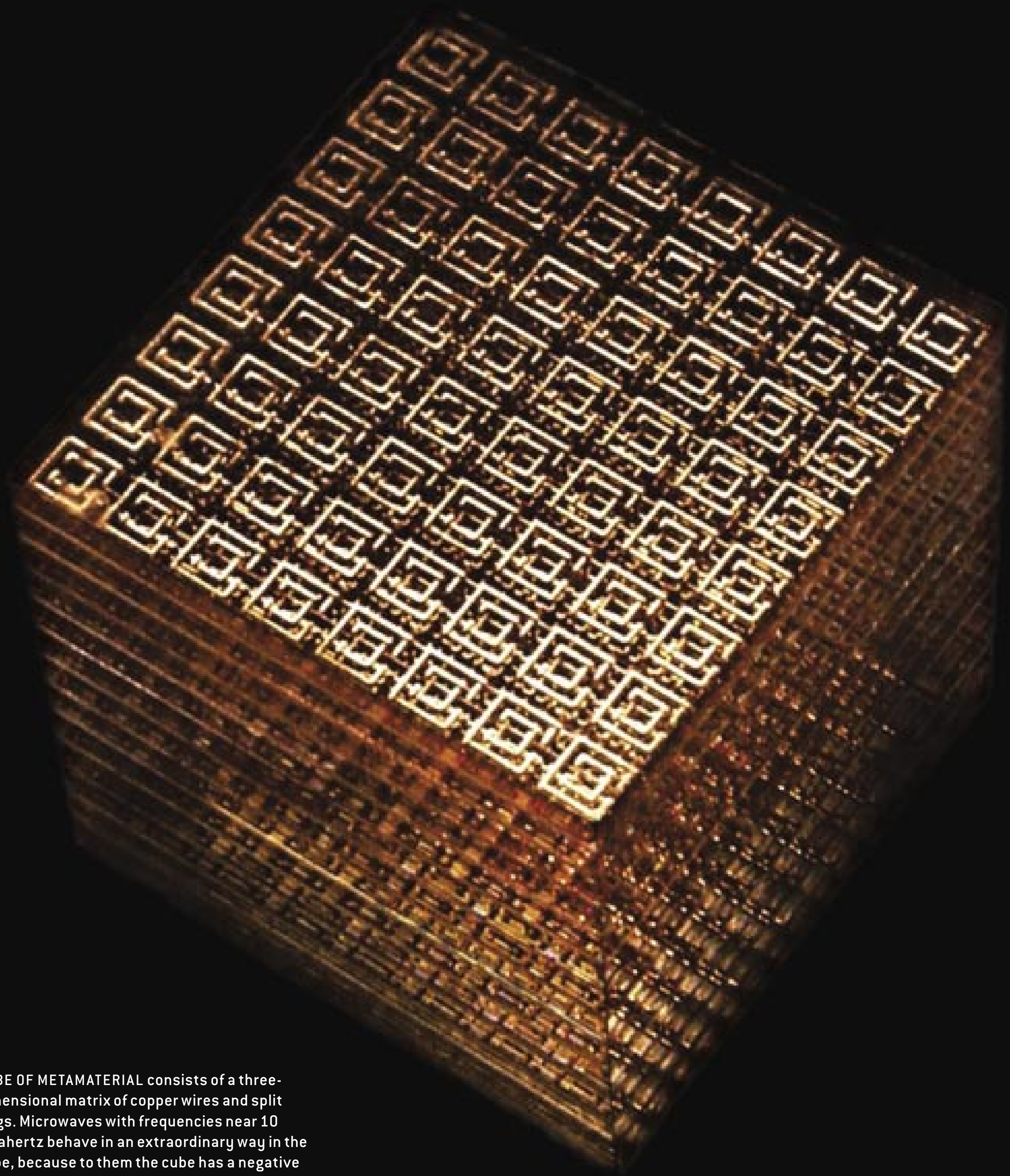
Primary Textbooks

Classical and Relativistic Electrodynamics
Feynman volume 2
Born and Wolfe

Special Relativity
Dray The Geometry of Special Relativity
Taylor and Wheeler Spacetime Physics (first edition)
Taylor and Wheeler Spacetime Physics (second edition)

General Relativity
Geroch General Relativity from A to B
Taylor and Wheeler Exploring Black Holes (first edition)
Taylor and Wheeler Exploring Black Holes (second edition)

THE QUEST FOR THE



CUBE OF METAMATERIAL consists of a three-dimensional matrix of copper wires and split rings. Microwaves with frequencies near 10 gigahertz behave in an extraordinary way in the cube, because to them the cube has a negative refractive index. The lattice spacing is 2.68 millimeters, or about one tenth of an inch.

Superlens

Built from "metamaterials" with bizarre, controversial optical properties, a superlens could produce images that include details finer than the wavelength of light that is used

By John B. Pendry and David R. Smith

Almost 40 years ago Russian scientist Victor Veselago had an idea for a material that could turn the world of optics on its head. It could make light waves appear to flow *backward* and behave in many other counterintuitive ways. A totally new kind of lens made of the material would have almost magical attributes that would let it outperform any previously known. The catch: the material had to have a negative index of refraction ("refraction" describes how much a wave will change direction as it enters or leaves the material). All known materials had a positive value. After years of searching, Veselago failed to find anything having the electromagnetic properties he sought, and his conjecture faded into obscurity.

A startling advance recently resurrected Veselago's notion. In most materials, the electromagnetic properties arise directly from the characteristics of constituent atoms and molecules. Because these constituents have a limited range of characteristics, the millions of materials that we know of display only a limited palette of electromagnetic properties. But in the mid-1990s one of us (Pendry), in collaboration with scientists at Marconi Materials

Technology in England, realized that a "material" does not have to be a slab of one substance. Rather it could gain its electromagnetic properties from tiny structures, which collectively create effects that are otherwise impossible.

The Marconi team began making these so-called metamaterials and demonstrated several that scattered electromagnetic waves unlike any known materials. In 2000 one of us (Smith), along with colleagues at the University of California, San Diego, found a combination of metamaterials that provided the elusive property of negative refraction.

Light in negative-index materials behaves in such strange ways that theorists have essentially rewritten the book on electromagnetics—a process that has included some heated debate questioning the very existence of such materials. Experimenters, meanwhile, are working on developing technologies that use the weird properties of metamaterials: a superlens, for example, that allows imaging of details finer than the wavelength of light used, which might enable optical lithography of microcircuitry well into the nanoscale and the storage of vastly more data on optical disks. Much remains to be done to turn such visions into reality, but now that Veselago's dream has been conclusively realized, progress is rapid.

Negative Refraction

TO UNDERSTAND HOW negative refraction can arise, one must know how materials affect electromagnetic waves. When an electromagnetic wave (such as a ray of light) travels through a material, the electrons within the material's atoms or molecules feel a force and move in response. This motion uses up some of the wave's energy, affecting the properties of the wave and how it travels. By adjusting the chemical composition of a material, scientists can fine-tune its wave-propagation characteristics for a specific application.

But as metamaterials show, chemistry is not the only path to developing materials with an interesting electromagnetic response. We can also engineer electromagnetic response by creating tiny but macroscopic structures. This possibility arises because the wavelength of a typical electromagnetic wave—the characteristic distance over which it varies—is orders of magnitude larger than the atoms or molecules that make up a material. The wave does not "see" an individual molecule but rather the collective response of millions of molecules. In a metamaterial, the patterned elements are considerably smaller than the wavelength and are thus not seen individually by the electromagnetic wave.

As their name suggests, electromagnetic waves contain both an electric field and a magnetic field. Each component induces a characteristic motion of the electrons in a material—back and forth in response to the electric field and around in circles in response to the magnetic field. Two parameters quantify the extent of these responses in a material: electrical permittivity, ε, or how much its electrons respond to an electric field, and magnetic permeability, μ, the electrons' degree of response to a magnetic field. Most materials have positive ε and μ.

Another important indicator of the optical response of a material is its refractive index, n. The refractive index is simply related to ε and μ: $n = \pm\sqrt{\varepsilon\mu}$. In every known material, the positive value must be chosen for the square root; hence, the refractive index is positive. In 1968 Veselago showed, however, that if ε and μ are both negative, then n must also take the negative sign. Thus, a material with both ε and μ negative is a negative-index material.

A negative ε or μ implies that the electrons within the material move in the opposite direction to the force applied by the electric and magnetic fields. Although this behavior might seem paradoxical, it is actually quite a simple matter to make electrons oppose the "push" of the applied electric and magnetic fields.

Think of a swing: apply a slow, steady push, and the swing obediently moves in the direction of the push—although it does not swing very high. Once set in motion, the swing tends to oscillate back and forth at a particular rate, known technically as its resonant frequency. Push the swing periodically, in time with this swinging, and it starts arcing higher. Now try to push at a faster rate, and the push goes out of phase with respect to the motion of the swing—at some point, your arms might be outstretched with the swing rushing back. If you have been pushing for a while, the swing might have enough momentum to knock you over—it is then pushing back on you. In the same way, electrons in a material with a negative index of refraction go out of phase and resist the "push" of the electromagnetic field.

Metamaterials

RESONANCE, the tendency to oscillate at a particular frequency, is the key to achieving this kind of negative response and is introduced artificially in a metamaterial by building small circuits designed to mimic the magnetic or electrical response of a material. In a split-ring resonator (SRR), for example, a magnetic flux penetrating the metal rings induces rotating currents in the rings, analogous to magnetism in materials

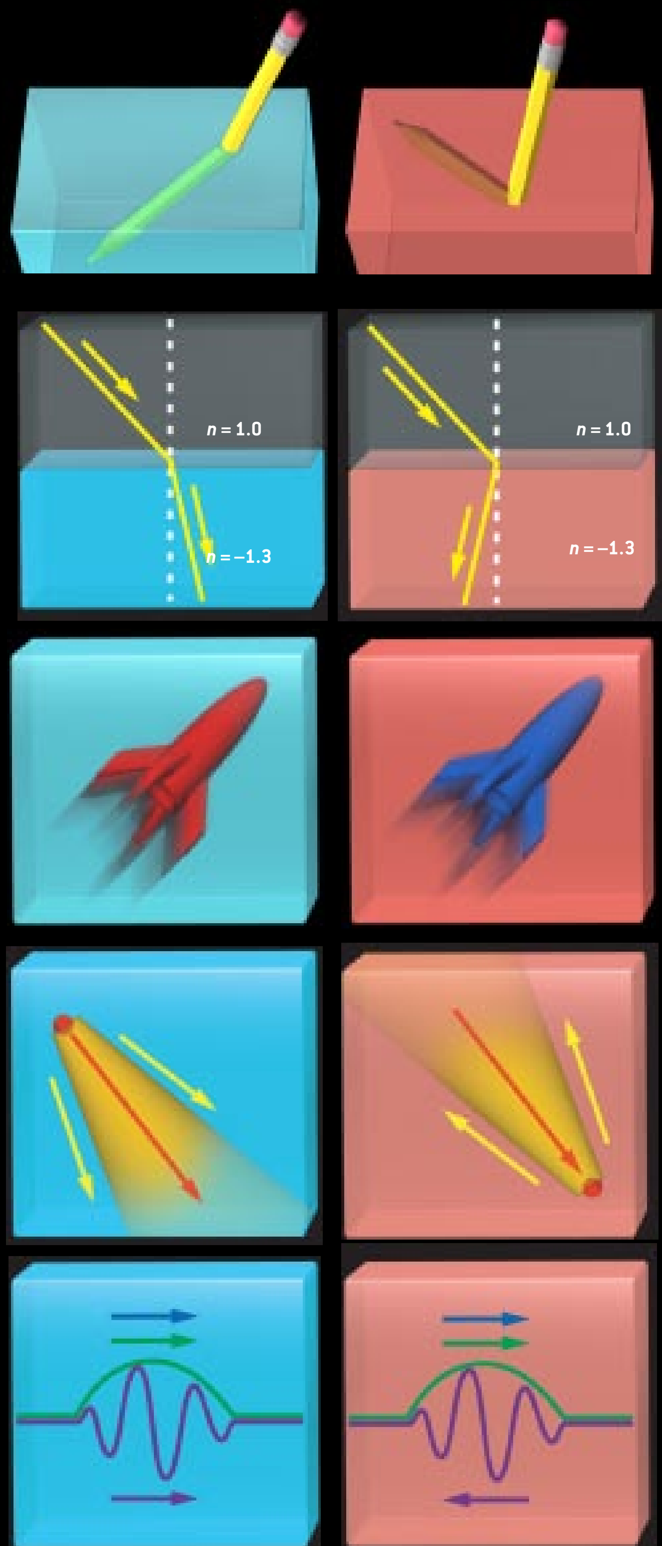
Overview/*Metamaterials*

- Materials made out of carefully fashioned microscopic structures can have electromagnetic properties unlike any naturally occurring substance. In particular, these metamaterials can have a negative index of refraction, which means they refract light in a totally new way.
- A slab of negative-index material could act as a superlens, able to outperform today's lenses, which have a positive index. Such a superlens could create images that include detail finer than that allowed by the diffraction limit, which constrains the performance of all positive-index optical elements.
- Although most experiments with metamaterials are performed with microwaves, they might use shorter infrared and optical wavelengths in the future.

MINAS H. TANIELIAN *Boeing Phantom Works* (page 60)

NEGATIVE-INDEX WEIRDNESS

In a medium with a negative index of refraction, light (and all other electromagnetic radiation) behaves differently than in conventional positive-index material. in a number of counterintuitive ways.



POSITIVE-INDEX MEDIUM

A pencil in a glass of water appears bent because of the water's higher refractive index.

When light travels from a medium with low refractive index (n) to one with higher refractive index, it bends toward the normal (*dashed line at right angles to surface*).

A receding object appears redder because of the Doppler effect.

A charged object (*red*) traveling faster than the speed of light generates a cone of Cherenkov radiation (*yellow*) in the forward direction.

In a positive-index medium, the individual ripples of an electromagnetic pulse (*purple*) travel in the same direction as the overall pulse shape (*green*) and the energy (*blue*).

NEGATIVE -INDEX MEDIUM

A pencil embedded in a negative-index medium would appear to bend all the way out of the medium.

When light travels from a positive-index medium to one with negative index, it bends all the way back to the same side of the normal.

A receding object appears bluer.

The cone points backward.

The individual ripples travel in the opposite direction to the pulse shape and the energy.

MELISSA THOMAS

[*see box on page 64*]. In a lattice of straight metal wires, in contrast, an electric field induces back-and-forth currents.

Left to themselves, the electrons in these circuits naturally swing to and fro at the resonant frequency determined by the circuits' structure and dimensions. Apply a field below this frequency, and a normal positive response results. Just above the resonant frequency, however, the response is negative—just as the swing pushed back when pushed faster than its frequency. Wires can thus provide an electric response with negative ε over some range of frequencies, whereas split rings can provide a magnetic response with negative μ over the same frequency band. These wires and split rings are just the building blocks needed to make a wide assortment of interesting metamaterials, including Veselago's long-sought material.

The first experimental evidence that a negative-index material could be achieved came from the experiments by the U.C.S.D. group in 2000. Because the most stringent requirement for a metamaterial is that the elements be significantly smaller than the wavelength, the group used microwaves. Microwaves have wavelengths of several centimeters, so that the metamaterial elements could be several millimeters in size—a convenient scale.

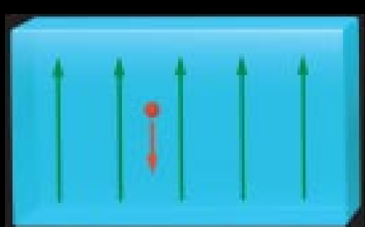
The team designed a metamaterial that had wires and SRRs interlaced together and assembled it into a prism shape. The wires provided negative ε, and SRRs provided negative μ: the two together should, they reasoned, yield a negative refractive index. For comparison, they also fashioned an identically shaped prism out of Teflon, a substance having a positive index with a value of $n = 1.4$. The researchers directed a beam of microwaves onto the face of the prism and detected the amount of microwaves emerging at various angles. As expected, the microwave beam underwent positive refraction from the Teflon prism but was negatively refracted by the metamaterial prism. Veselago's speculation was now reality; a negative-index material had finally been achieved.

Or had it?

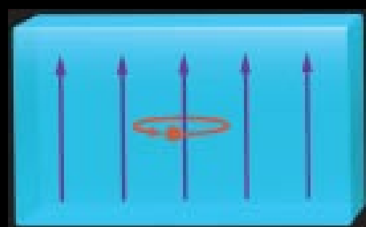
ENGINEERING A RESPONSE

The key to producing a metamaterial is to create an artificial response to electric and magnetic fields.the material.

IN AN ORDINARY MATERIAL

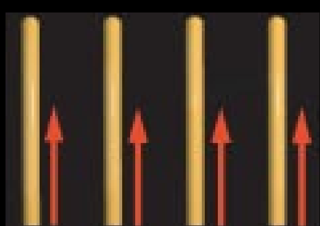


An electric field (*green*) induces linear motion of electrons (*red*).



A magnetic field (*purple*) induces circular motion of electrons.

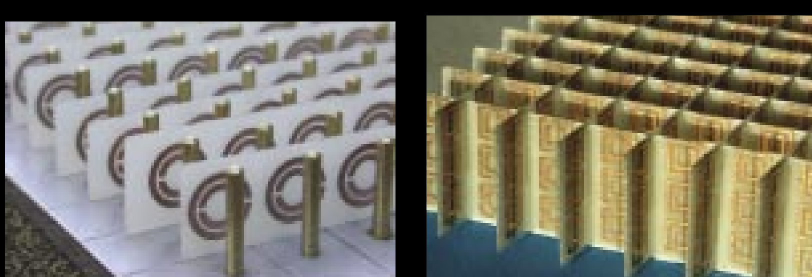
IN A METAMATERIAL



Linear currents (*red arrows*) flow in arrays of wires.

Circular currents flow in split-ring resonators (SRRs).

METAMATERIAL STRUCTURE



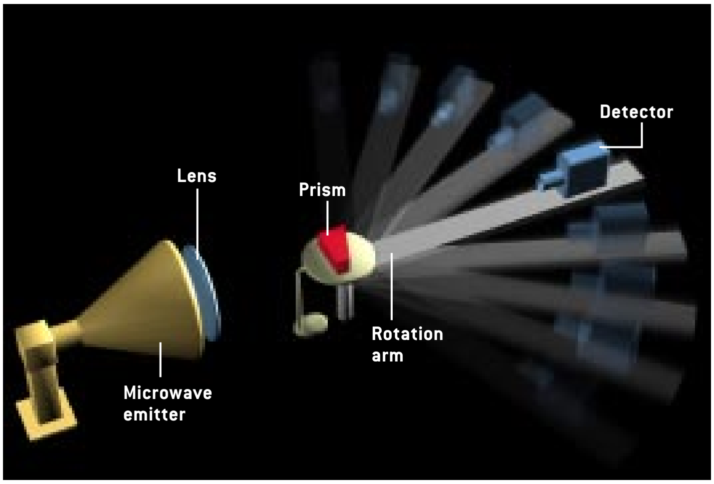
A metamaterial is made by creating an array of wires and SRRs that are smaller than the wavelength of the electromagnetic waves to be used with the material.

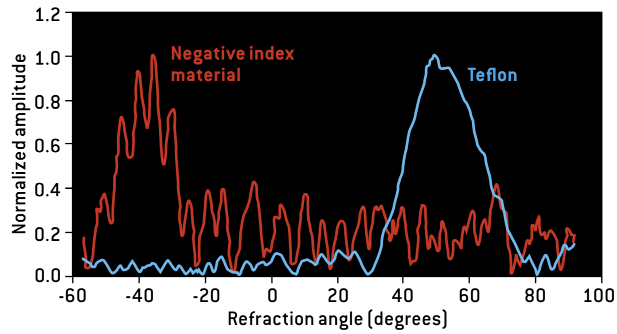
MELISSA THOMAS

Does It Really Work?

THE U.C.S.D. EXPERIMENTS, along with remarkable new predictions that physicists were making about negative-index materials, created a surge of interest from other researchers. In the absence of metamaterials at the time of Veselago's hypothesis, the scientific community had not closely scrutinized the concept of negative refraction. Now with the potential of metamaterials to realize the madcap ideas implied by this theory, people paid more attention. Skeptics began asking whether negative-index materials violated the fundamental laws of physics. If so, the entire program of research could be invalidated.

One of the fiercest discussions centered on our understanding of a wave's velocity in a complicated material. Light travels in a vacuum at its maximum speed of 300,000 kilometers per second. This speed is given the symbol c. The speed of light in a material, however, is





EXPERIMENT CARRIED OUT at Boeing Phantom Works in Seattle using first a metamaterial prism and then a Teflon (positive-index) prism confirmed the phenomenon of negative refraction. The Teflon refracted microwaves by a positive angle (*blue line*); the metamaterial by a negative angle (*red line*).

reduced by a factor of the refractive index—that is, the velocity $v = c/n$. But what if n is negative? The simple interpretation of the formula for the speed of light suggests that the light propagates backward.

A more complete answer takes cognizance that a wave has two velocities, known as the phase velocity and the group velocity. To understand these two velocities, imagine a pulse of light traveling through a medium. The pulse will look something like the one shown in the last illustration in the box on page 63: the ripples of the wave increase to a maximum at the center of the pulse and then die out again. The phase velocity is the speed of the individual ripples. The group velocity is the speed at which the pulse shape travels along. These velocities need not be the same.

In a negative-index material, as Veselago had discovered, the group and phase velocities are in opposite directions. Surprisingly, the individual ripples of the pulse travel backward even as the entire pulse shape travels forward. This fact also has amazing consequences for a continuous beam of light, such as one coming from a flashlight wholly immersed in a negative-index material. If you could watch the individual ripples of the light wave, you would see them emerge from the target of the beam, travel backward along the beam and ultimately disappear into the flashlight, as if you were watching a movie running in reverse. Yet the energy of the light beam travels forward, away from the flashlight, just as one expects. That is the direction the beam is actually traveling, the amazing backward motion of the ripples notwithstanding.

In practice, it is not easy to study the individual ripples of a light wave, and the details of a pulse can be complicated, so physicists often use a trick to illustrate the difference between the phase and group velocities. If we add together two waves of different wavelengths traveling in the same direction, the waves interfere to produce a beat pattern. The beats move at the group velocity.

In applying this concept to the U.C.S.D. refraction experiment in 2002, Prashant M. Valanju and his colleagues at the University of Texas at Austin observed something curious. When two waves of different wavelengths refract at the interface between a negative- and a positive-index material, they refract at slightly different angles. The resulting beat pattern, instead of following the negatively refracting beams, actually appears to exhibit positive refraction. Equating this beat pattern with the group velocity, the Texas researchers concluded that any physically realizable wave would undergo positive refraction. Although a negative-index material could exist, negative refraction was impossible.

Assuming that the Texas physicists' findings were true, how could one explain the results of the U.C.S.D. experiments? Valanju and many other researchers attributed the apparent negative refraction to a variety of other phenomena. Perhaps the sample actually absorbed so much energy that waves could leak out only from the narrow side of the prism, masquerading as negatively refracted waves? After all, the U.C.S.D. sample involved significant absorption, and the measurement had not been taken very far away from the face of the prism, making this absorption theory a possibility.

The conclusions caused great concern, as they might invalidate not only the U.C.S.D. experiments but all the phenomena predicted by Veselago as well. After some thought, however, we realized it was wrong to rely on the beat pattern as an indicator of group velocity.

THE AUTHORS

JOHN B. PENDRY and *DAVID R. SMITH* were members of a team of researchers who shared the 2005 Descartes Research Prize for their contributions to metamaterials. They have collaborated on the development of such materials since 2000, Pendry focusing on the theory and Smith on experimentation. Pendry is professor of physics at Imperial College London, and recently his main interest has been electromagnetic phenomena, along with quantum friction, heat transport between nanostructures, and quantization of thermal conductivity. Smith is professor of electrical and computer engineering at Duke University. He studies electromagnetic-wave propagation in unusual materials and is currently collaborating with several companies to define and develop novel applications for metamaterials and negative-index materials.

We concluded that for two waves traveling in different directions, the resulting interference pattern loses its connection with the group velocity.

As the arguments of the critics began to crumble, further experimental confirmation of negative refraction came. Minas Tanielian's group at Boeing Phantom Works in Seattle repeated the U.C.S.D. experiment with a very low absorption metamaterial prism. The Boeing team also placed the detector much farther from the prism, so that absorption in the metamaterial could be ruled out as the cause of the negatively refracted beam. The exemplary quality of the data from Boeing and other groups finally put an end to any remaining doubts about the existence of negative refraction. We were now free to move forward and exploit the concept, albeit chastened by the subtlety of the new materials.

Beyond Veselago

AFTER THE SMOKE of battle cleared, we began to realize that the remarkable story that Veselago had told was not the final word on how light behaves in negative-index materials. One of his key tools was ray tracing—the process of drawing lines that trace out the path that a ray of light should follow, allowing for reflection and refraction at the interface of different materials.

Ray tracing is a powerful technique and helps us understand, for example, why objects in a swimming pool appear closer to the surface than they actually are and why a half-submerged pencil appears bent. It arises because the refractive index of water (n equals about 1.3) is larger than that of air, and rays of light are bent at the interface between the air and the water. The refractive index is approximately equal to the ratio of the real depth over the apparent depth.

Ray tracing also implies that children swimming in a negative-index pool would appear to float above the surface. (A valuable safety feature!) The entire contents of the pool—and its container—would also appear above the surface.

Veselago used ray tracing to predict that a slab of negatively refracting material, with index $n = -1$, should act as a lens with unprecedented properties. Most of us are familiar with positive-index lenses—in cameras, magnifying glasses, microscopes and telescopes. They have a focal length, and where an image is formed depends on a combination of the focal length and the distance between the object and the lens. Images are typically a different size than the object and the lenses work best for objects along an axis running through the lens. Veselago's lens works in quite a different fashion from those [*see box below*]: it is much simpler, only acting on objects adjacent to it, and it transfers the entire optical field from one side of the lens to the other.

So unusual is the Veselago lens that Pendry was compelled to ask just how perfectly it could be made to perform. Specifically, what would be the ultimate resolution of the Veselago lens? Positive-index optical elements are constrained by the diffraction limit to resolve details that are about the same size or larger than the wavelength of light reflected from an object. Diffraction places the ultimate limit on all imaging systems, such as the smallest object that might be viewed in a microscope or the closest distance that two stars might be resolved by a telescope. Diffraction also determines the smallest feature that can be created by optical lithography processes in the microchip industry. In a similar manner, diffraction limits the amount of information that can be optically stored on or retrieved from a digital video disk (DVD). A way around the diffraction limit could revolutionize optical technologies, allowing optical lithography well into the nanoscale and perhaps permitting hundreds of times more data to be stored on optical disks.

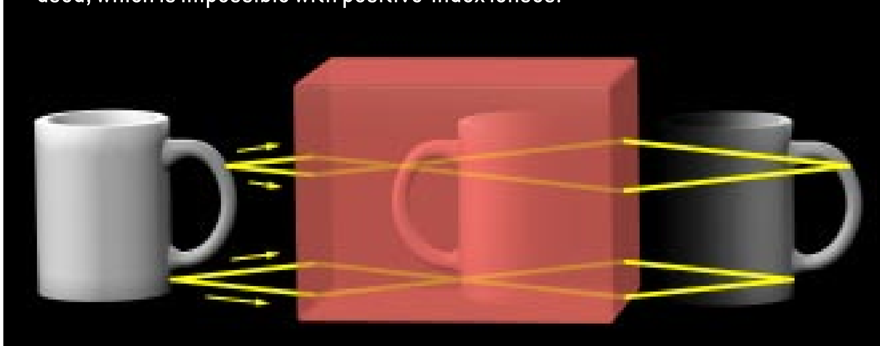
To determine whether or not negative-index optics could surpass the positive version, we needed to move beyond ray tracing. That approach neglects diffraction and thus could not be used to predict the resolution of negative-index lenses. To include diffraction, we had to use a more accurate description of the electromagnetic field.

The Superlens

DESCRIBED MORE accurately, all sources of electromagnetic waves—whether radiating atoms, a radio antenna or a beam of light emerging after passing through a small aperture—produce two distinct types of fields: the far field and the near field. As its name implies, the far field is the part that is radiated far from an object and can be captured by a lens to form an image. Unfortunately, it contains only a broad-brush picture of the object, with diffraction limiting the resolution to the size of the wavelength. The near field, on the other hand, contains all the finest details of an object, but its intensity drops off rapid-

THE SUPERLENS

A rectangular slab of negative-index material forms a superlens. Light (*yellow lines*) from an object (*at left*) is refracted at the surface of the lens and comes together again to form a reversed image inside the slab. The light is refracted again on leaving the slab, producing a second image (*at right*). For some metamaterials, the image even includes details finer than the wavelength of light used, which is impossible with positive-index lenses.



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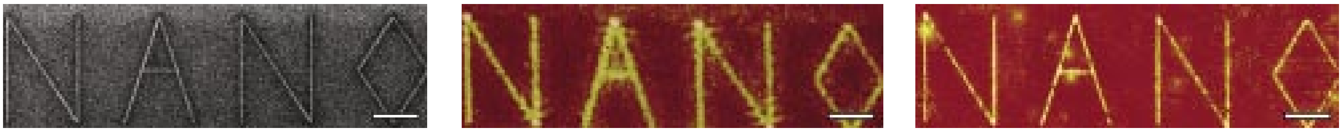


THIN LAYER OF SILVER acts like a superlens over very short distances. Here the word "NANO" is imaged with a focused ion beam (*left*), optically without a superlens (*middle*) and optically with a 35-nanometer layer of silver in place (*right*). Scale bar is 2,000 nanometers long. With the superlens, the resolution is finer than the 365-nanometer wavelength of the light used.

ly with distance. Positive-index lenses stand no chance of capturing the extremely weak near field and conveying it to the image. The same is not true of negative-index lenses.

By closely examining the manner in which the near and far fields of a source interacted with the Veselago lens, Pendry concluded in 2000—much to everyone's surprise—that the lens could, in principle, refocus both the near and far fields. If this stunning prediction were true, it would mean that the Veselago lens was not subject to the diffraction limit of all other known optics. The planar negative-index slab has consequently been called a superlens.

In subsequent analysis, we and other researchers found that the resolution of the superlens is limited by the quality of its negative-index material. The best performance requires not just that the refractive index $n = -1$, but that both $\varepsilon = -1$ and $\mu = -1$. A lens that falls short of this ideal suffers from drastically degraded resolution. Meeting these conditions simultaneously is a severe requirement. But in 2004 Anthony Grbic and George V. Eleftheriades of the University of Toronto showed experimentally that a metamaterial designed to have $\varepsilon = -1$ and $\mu = -1$ at radio frequencies could indeed resolve objects at a scale smaller than the diffraction limit. Their result proved that a superlens could be built—but could one be built at the still smaller optical wavelengths?

The challenge for scaling metamaterials to optical wavelengths is twofold. First, the metallic conducting elements that form the metamaterial microcircuits, such as wires and SRRs, must be reduced to the nanometer scale so that they are smaller than the wavelength of visible light (400 to 700 nanometers). Second, the short wavelengths correspond to higher frequencies, and metals behave less like conductors at these frequencies, thus damping out the resonances on which metamaterials rely. In 2005 Costas Soukoulis of Iowa State University and Martin Wegener of the University of Karlsruhe in Germany demonstrated experimentally that SRRs can be made that work at wavelengths as small as 1.5 microns. Although the magnetic resonance becomes quite weak at these short wavelengths, interesting metamaterials can still be formed.

But we cannot yet fabricate a material that yields $\mu = -1$ at visible wavelengths. Fortunately, a compromise is possible. When the distance between the object and the image is much smaller than the wavelength, we need only fulfill the condition $\varepsilon = -1$, and then we can disregard μ. Just last year Richard Blaikie's group at the University of Canterbury in New Zealand and Xiang Zhang's group at the University of California, Berkeley, independently followed this prescription and demonstrated superresolution in an optical system. At optical wavelengths, the inherent resonances of a metal can lead to negative permittivity (ε). Thus, a very thin layer of metal can act as a superlens at a wavelength where $\varepsilon = -1$. Both Blaikie and Zhang used a layer of silver about 40 nanometers thick to image 365-nanometer-wavelength light emanating from shaped apertures smaller than the light's wavelength. Although a silver slab is far from the ideal lens, the silver superlens substantially improved the image resolution, proving the underlying principle of superlensing.

Toward the Future

THE DEMONSTRATION of superlensing is just the latest of the many predictions for negative-index materials to be realized—an indication of the rapid progress that has occurred in this emerging field. The prospect of negative refraction has caused physicists to reexamine virtually all of electromagnetics. Once thought to be completely understood, basic optical phenomena—such as refraction and the diffraction limit—now have new twists in the context of negative-index materials.

The hurdle of translating the wizardry of metamaterials and negative-index materials into usable technology remains. That step will involve perfecting the design of metamaterials and manufacturing them to a price. The numerous groups now working in this field are vigorously tackling these challenges. SA

MORE TO EXPLORE

Reversing Light with Negative Refraction. John B. Pendry and David R. Smith in *Physics Today,* Vol. 57, No. 6, pages 37–43; June 2004.

Negative-Refraction Metamaterials: Fundamental Principles and Applications. G. V. Eleftheriades and K. Balmain. Wiley-IEEE Press, 2005.

More information on metamaterials and negative refraction is available at:
www.ee.duke.edu/~drsmith/

www.cmth.ph.ic.ac.uk/photonics/references.html

esperia.iesl.forth.gr/~ppm/Research.html

www.nanotechnology.bilkent.edu.tr/

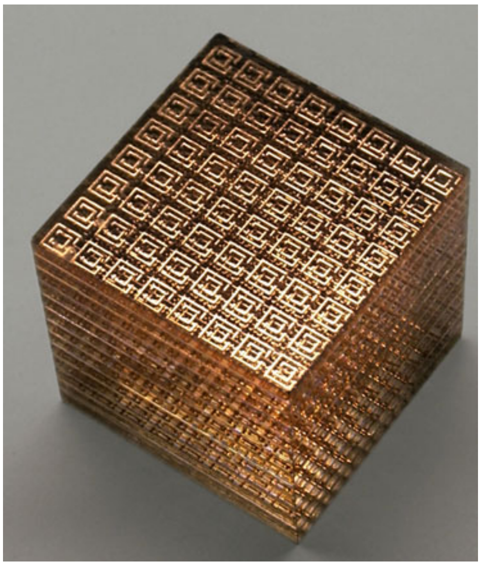
www.rz.uni-karlsruhe.de/~ap/ag/wegener/meta/meta.html

XIANG ZHANG *University of California, Berkeley*

Reversing Light: Negative Refraction

First hypothesized in 1967 by Victor Veselago, materials with negative refractive index have only been appreciated this decade.

John B. Pendry and David R. Smith



'The Boeing cube': a structure designed for negative refractive index in the GHz range

Victor Veselago, in a paper[1] published in 1967, pondered the consequences for electromagnetic waves interacting with a hypothetical material for which both the electric permittivity, ε, and the magnetic permeability, μ, were simultaneously negative. As no naturally occurring material or compound has ever been demonstrated with negative ε and μ, Veselago wondered whether this apparent asymmetry in material properties was just happenstance, or perhaps had a more fundamental origin. Veselago concluded that not only should such materials be possible but, if ever found, would exhibit remarkable properties unlike those of any known materials, giving a twist to virtually all electromagnetic phenomena.

So why are there no materials with negative ε and μ? We first need to understand what it means to have a negative ε or μ, and how it occurs in materials. The Drude-Lorentz model of a material is a good starting point, as it conceptually replaces the atoms and molecules of a real material by a set of harmonically bound electron oscillators, resonant at some frequency ω_0. At frequencies far below ω_0, an applied electric field displaces the electrons from the positive core, inducing a polarization in the same direction as the applied electric field. At frequencies near the resonance, the induced polarization becomes very large, as is typically the case in resonance phenomena; the large response represents accumulation of energy over many cycles, such that a considerable amount of energy is stored in the resonator (medium) relative to the driving field. So large is this stored energy that even changing the sign of the applied electric field has little effect on the polarization near resonance! That is, as the frequency of the driving electric field is swept through the resonance, the polarization flips from in-phase to out-of-phase with the driving field and the material exhibits a negative response. If instead of electrons the material response were due to harmonically bound magnetic moments, then a negative magnetic response would exist.

Though somewhat less common than positive materials, negative materials are nevertheless easy to find. Materials with ε negative include metals (e.g., silver, gold, aluminum) at optical frequencies, while materials with μ negative include resonant ferromagnetic or antiferromagnetic systems.

That negative material parameters occur near a resonance has two important consequences. First, negative material parameters will exhibit frequency dispersion: that is to say they will vary as a function of frequency. Second, the usable bandwidth of negative materials will be relatively narrow compared with positive materials. This can help us answer our initial question as to why materials with both negative ε and μ are not readily found. The resonances in existing materials that give rise to electric polarizations typically occur at very high frequencies, in the optical, for metals, or at least in the THz to infrared region for semiconductors and insulators. On the other hand, resonances in magnetic systems typically occur at much lower frequencies, usually tailing off toward the THz and infrared region. In short, the fundamental electronic and magnetic processes that give rise to resonant phenomena in materials simply do not occur at the same frequencies, although no physical law would preclude this.

Metamaterials extend material response

Because of the seeming separation in frequency between electric and magnetic resonant phenomena, Veselago's analysis of materials with ε and μ both negative might have remained a curious exercise in electromagnetic theory. However, in the mid-1990s, researchers began looking into the possibility of engineering artificial materials to have tailored electromagnetic response. While the field of artificial materials dates back to the 40s, advances in fabrication and computation—coupled with the emerging awareness of the importance of negative

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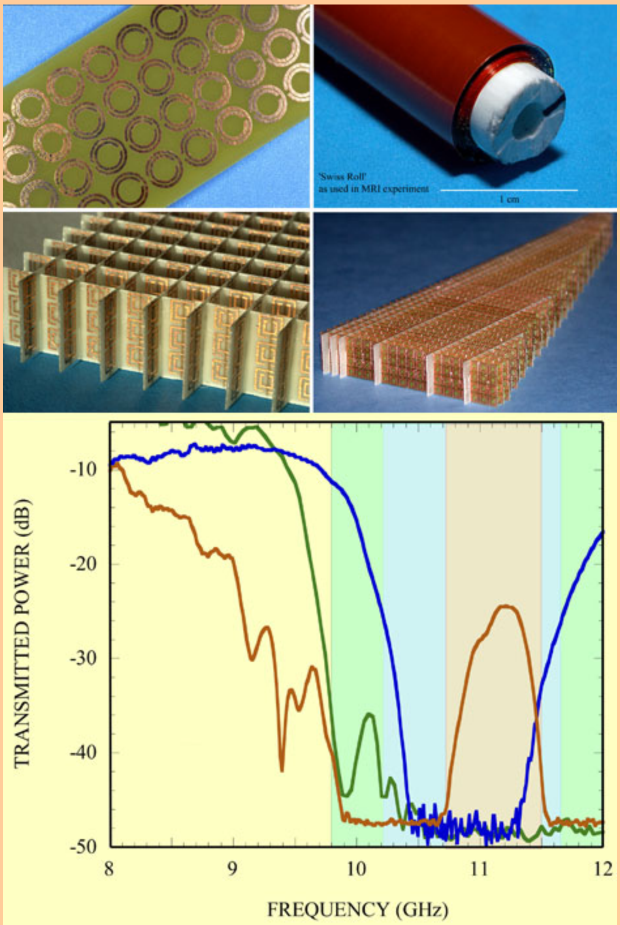

Figure 1. Transmission experiments resolve whether a material is transparent or opaque, providing an indication of the sign of the material parameter. Top panels: examples of metamaterials used in microwave experiments. The unit cells are of the order of 5mm across. Bottom panel: The transmitted power spectra[3] shown here are for (green) a metamaterial of cut wires; (blue) a metamaterial of split ring resonators (SRRs); and (red) a metamaterial of wires and SRRs combined. For the wires, the region of negative ε occurs at frequencies starting just below 10 GHz, while for SRRs the region of negative μ spans a 1 GHz region starting at ~10.3 GHz. The combined medium exhibits a transmission band that would lie in an opaque frequency region for either metamaterial alone.

materials—led to a resurgence of effort in developing new structures with novel material properties.

To form an artificial material, we start with a collection of repeated elements designed to have a strong response to applied electromagnetic fields. So long as the size and spacing of the elements are much smaller than the electromagnetic wavelengths of interest, incident radiation cannot distinguish the collection of elements from a homogeneous material. We can thus conceptually replace the inhomogeneous composite by a continuous material described by material parameters ε and μ. At lower frequencies, conductors are excellent candidates from which to form artificial materials, as their response to electromagnetic fields is large.

A metamaterial mimicking the Drude-Lorentz model can be straightforwardly achieved by an array of wire elements into which cuts are periodically introduced. The effective permittivity for the cut-wire medium, then, has the form,

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2 - \omega_0^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

where the plasma frequency, ω_p, and the resonance frequency, ω_0, are determined only by the geometry of the lattice rather than by the charge, effective mass and density of electrons, as is the case in naturally occurring materials. At frequencies above ω_0 and below ω_p, the permittivity is negative and, because the resonant frequency can be set to virtually any value in a metamaterial, phenomena usually associated with optical frequencies—including negative ε—can be reproduced at low frequencies.

The path to achieving magnetic response from conductors is slightly different. From the basic definition of a magnetic dipole moment,

$$\mathbf{m} = \frac{1}{2}\int_V \mathbf{r} \times \mathbf{j}\, dV$$

we see that a magnetic response can be obtained if local currents can be induced to circulate in closed loops (solenoidal currents). Moreover, introducing a resonance into the element should enable a very strong magnetic response, potentially one that can lead to a negative μ.

In 1999, Pendry *et al*[2]. proposed a variety of structures that, they predicted, would form magnetic metamaterials. These structures consisted of loops or tubes of conductor with a gap inserted. From a circuit point of view, a time varying magnetic field induces an electromotive force in the plane of the element, driving currents within the conductor. A gap in the plane of the structure introduces capacitance into the planar circuit, giving rise to a resonance at a frequency set by the geometry of the element. This *split ring resonator* (SRR), in its various forms, can be viewed as the metamaterial equivalent of a magnetic atom. Pendry *et al.* went on to show that the SRR medium could be described by the resonant form

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

The wire medium and the SRR medium represent the two basic building blocks—one electric the other magnetic—for a large range of metamaterial response, including Veselago's hypothesized material (see figure 1).

Negative refraction

Maxwell's equations determine how electromagnetic waves propagate within a medium and can be solved to arrive at a wave equation of the form,

$$\frac{\partial^2 E(x,t)}{\partial x^2} = \varepsilon\mu \frac{\partial^2 E(x,t)}{\partial t^2}$$

In this equation ε and μ enter as a product and it would not appear to matter whether the signs of ε and μ were both positive or were both negative. Indeed, solutions of the wave equation have the form $\exp\left[i(nkd - \omega t)\right]$ where $n = \sqrt{\varepsilon\mu}$ is the refractive index. Propagating solutions exist in the material whether ε and μ are both positive or are both negative. So what, if anything, is the difference between positive and negative materials?

It turns out that we need to be more careful in taking the square root, as ε and μ are analytic functions that are generally complex valued. There is an ambiguity in the sign of the square root that is resolved by a proper analysis. For example, if instead of writing $\varepsilon = -1$ and $\mu = -1$ we write $\varepsilon = \exp(i\pi)$ and $\mu = \exp(i\pi)$, then:

$$n = \sqrt{\varepsilon\mu} = \exp(i\pi/2)\exp(i\pi/2) = \exp(i\pi) = -1 .$$

The important step is that the square root of either ε or μ alone must have a positive imaginary part—this is necessary for a passive material. This briefly stated argument shows why the material Veselago pondered years ago is so unique: the index of refraction is negative.

A negative refractive index implies that the phase of a wave advancing through the medium will be negative rather than positive. As Veselago pointed out, this fundamental reversal of wave propagation contains important implications for nearly all electromagnetic phenomena. Many of the exotic effects of negative index have been or are currently being pursued by researchers. But perhaps the most immediately accessible phenomenon from an experimental or computational point-of-view is the reversal of wave refraction, illustrated in figure 2.

Snell's law, which describes quantitatively the bending of a wave as it enters a medium, is perhaps one of the oldest and most well known of electromagnetic phenomena. In the form of a wedge experiment, as depicted in figure 2, Snell's law is also the basis for a direct measurement of a material's refractive index. In this type of experiment, a wave is incident on the flat side of a wedge shaped sample. The wave is transmitted through the transparent sample, striking the second interface at an angle. Because of the difference in refractive index between the material and free space, the beam exits the wedge deflected by some angle from the direction of incidence.

One might imagine that an experimental determination of Snell's law should be a simple matter; however, the peculiarities of metamaterials add a layer of complexity that renders the experimental confirmation somewhat more difficult. Present samples, based on SRRs and wires, are frequency dispersive with fairly narrow bandwidths and exhibit considerable loss. The first experiment showing negative refraction was performed in 2001 by Shelby et al. at UCSD[4], who utilized a planar waveguide apparatus to reduce the dimensionality of the measurement to two dimensions, similar to that depicted in figure 2. Shelby et al. measured the power refracted from a two-dimensional wedge metamaterial sample as a function of angle, confirming the expected properties.

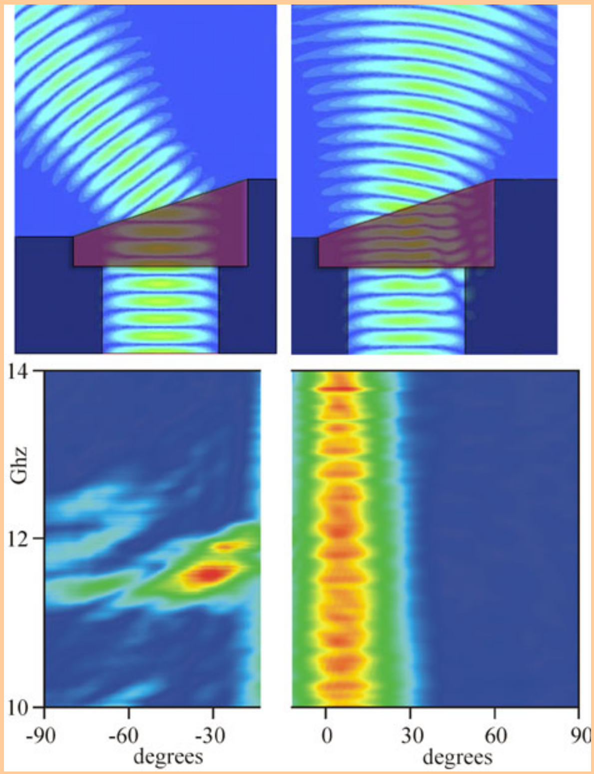

Figure 2. A negative index implies negative refraction. *Top left:* in this simulation[5] of a Snell's law experiment, a negative index wedge $\varepsilon = -1.0, \mu = -1.0$ deflects an electromagnetic beam, incident at an angle to the interface, by a negative angle so that it emerges on the same side of the surface normal as the incident beam, confirming negative refraction. *Top right:* by contrast a wedge with positive index $\varepsilon = 2.0, \mu = 1.0$ will positively refract the same beam relative to the surface normal. Experiments also confirm this behavior. *Bottom left:* deflection observed for a negative wedge. *Bottom right:* deflection (horizontal axis) observed for a Teflon wedge as a function of frequency (vertical axis). Note that in the latter case there is strong dispersion with frequency because the condition $\varepsilon = -1.0, \mu = -1.0$ is realized only over a narrow bandwidth around 12GHz.

While the UCSD data were compelling, the concept of negative index proved counter-intuitive enough that many other researchers needed further convincing. In 2003, Houck et al.[6] at MIT repeated the negative refraction experiment on the same sort of negative index metamaterial, confirming the original findings. The MIT group considered wedges with different angles, showing that the observed angle of refraction was consistent with Snell's law for the metamaterial. In the same year, Parazzoli et al.[7], from Boeing Phantom Works, also presented data on a negative index sample. The Boeing data differed from the previous two experiments in that their sample was designed not for a two-dimensional scattering chamber, but rather for free space. The Boeing experiment removed any doubts that the observed deflection might be due to any artifact related to the planar waveguide; moreover, the distance measured from the sample was significantly larger than for the previous waveguide demonstrations.

While it has proven a valuable concept, a rigorously defined negative index-of-refraction may not necessarily be a prerequisite for negative refraction phenomena. An alternate approach to attaining negative refraction uses the properties of 'photonic crystals'[8,9,10,11,12] - materials that lie on the transition between a metamaterial and an ordinary structured dielectric. Photonic crystals derive their properties from Bragg reflection in a periodic structure engineered in the body of a dielectric, typically by drilling or etching holes. The periodicity in photonic crystals is on the order of the wavelength, so that the distinction between refraction and diffraction is blurred. Nevertheless, many novel dispersion relationships can be realized in photonic crystals, including ranges where the frequency disperses negatively with wave vector as required for a negative refraction.

The concept of negative refraction has also been generalized to transmission line structures, common in electrical engineering applications. By pursuing the analogy between lumped circuit elements and material parameters, Eleftheriades and coworkers[13] have demonstrated negative refraction phenomena in microwave circuits. The transmission line model has proven exceptionally valuable for the development of microwave devices: Tatsuo Itoh and Christophe Caloz at UCLA have applied the transmission line model to develop novel microwave components, including antennas, couplers and resonators.

These experiments and applications have shown that the material Veselago hypothesized more than thirty-five years ago can now be realized using artificially constructed metamaterials, making discussion of negative refractive index more than a theoretical curiosity. The question of whether such a material can exist has been answered, turning the development of negative index structures into a topic of materials—or metamaterials—physics. As metamaterials are being designed and improved, we are now free to consider the ramifications associated with a negative index-of-refraction. This material property, perhaps because it is so simply stated, has enabled the rapid design of new electromagnetic structures—some of them with very unusual and exotic properties.

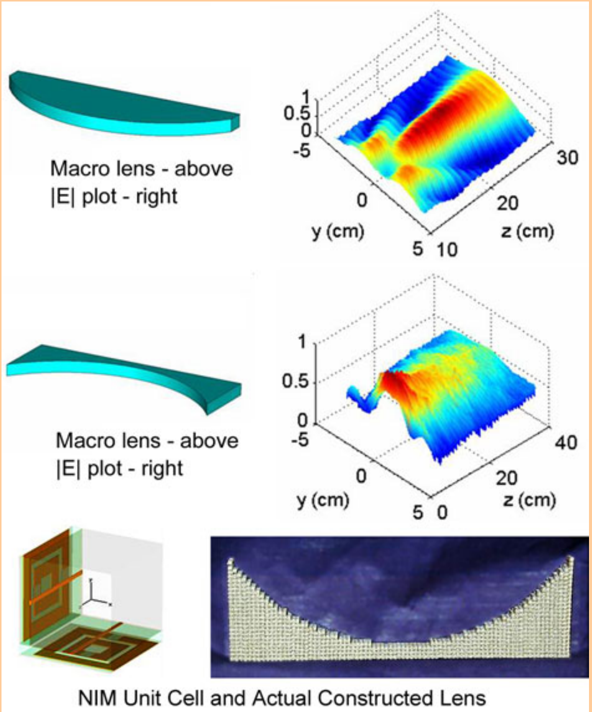

Figure 3. A material with index of $n=+1$ (relative to vacuum) has no refractive power, while a material with index of $n=-1$ has considerable refractive power. Consider the formula for the focal length of a thin lens: $f=R/|n-1|$, where R is the radius-of-curvature of the surface. For a given R, a lens with an index of $n=-1$ will have the same focal length as would an index of $n=+3$; by the same reasoning, if we compare two lenses of the same absolute value of index but opposite sign, the negative index lens will be more compact than the positive index lens. The practicality of negative index metamaterial lenses has been demonstrated by researchers at Boeing, Phantom Works. Applying the same basic elements previously used to construct negative index metamaterial wedges—SRRs to create a magnetic response, wires to create an electric response (see inset above)—Parazzoli, Greegor and their colleagues[14] have designed a concave lens with an index very near to $n=-1$ at microwave frequencies (~15 GHz). The negative index metalens has a much shorter focal length as compared with a positive index lens (in this case $n=2.3$) having the same radius-of-curvature, as shown in the figures above. Moreover, the metalens is comparatively much lighter than the positive index lenses, a significant advantage for aerospace applications.

A better focus with negative index

Refraction is the phenomenon responsible for lenses and similar devices that focus or shape radiation. While usually thought of in the context of visible light, lenses are utilized throughout the electromagnetic spectrum, and represent a good starting point to implement negative index materials.

In his early paper, Veselago noted that a negative index focusing lens would need to be concave rather than convex. This would seem to be a trivial matter, but there is, in fact, more to the story. For thin lenses, geometrical optics—valid for either positive or negative index—gives the result that the focal length is related to the radius of

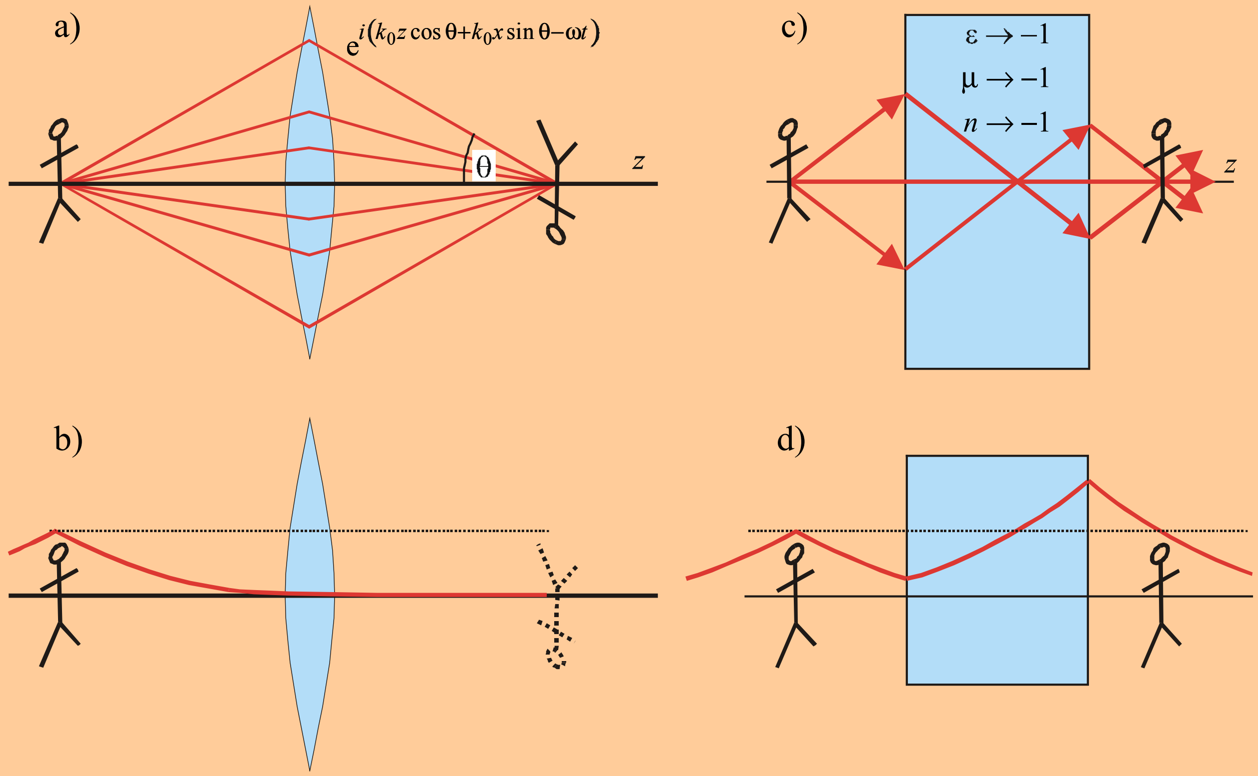

Figure 4. Limitation of resolution in lenses. a) Conventional lenses need a wide aperture for good resolution but even so are limited in resolution by the wavelength employed. b) The missing components of the image are contained in the near field which decays exponentially and makes negligible contribution to the image. c) A new lens made from a slab of negative material not only brings rays to a focus but has the capacity d) to amplify the near field so that it contributes to the image thus removing the wavelength limitation. However the resonant nature of the amplification places sever demands on materials: they must be very low loss.

curvature of the lens by $f = R/(n-1)$. The denominator in the focal length formula implies an inherent distinction between positive and negative index lenses, based on the fact that an $n=+1$ material does not refract electromagnetic fields while an $n=-1$ material does. The result is that negative index lenses can be more compact with a host of other benefits, as shown in figure 3.

To make a conventional lens with the best possible resolution a wide aperture is sought. Each ray emanating from an object, as shown in Figure 4a), has wave vector components along the axis of the lens, $k_z = k_0\cos\theta$, and perpendicular to the axis, $k_x = k_0\sin\theta$. The former component is responsible for transporting the light from object to image and the latter represents a Fourier component of the image for resolution: the larger we can make k_x, the better. Naturally the best that can be achieved is k_0 and hence the limit to resolution of ,

$$\Delta \approx \pi/k_0 = \lambda/2$$

where λ is the wavelength. This restriction is a huge problem in many areas of optics. Wavelength limits the feature size achieved in computer chips, and the storage capacity of DVDs. Even a modest relaxation of the wavelength limitation would be of great value.

In contrast to the image, there is no limit to the electromagnetic details contained in the object but unfortunately not all of them make it across the lens to the image. The problem lies with the z component of the wave vector which we can write,

$$k_z = \sqrt{k_0^2 - k_x^2}$$

Evidently for large values of k_x, corresponding to fine details in the object, k_z is imaginary and the waves acquire an evanescent character. By the time they reach the image they have negligible amplitude, figure 4b), and for this reason are commonly referred to as 'the near field', and the propagating rays as 'the far field'.

If by some magic we could amplify the near fields we could in principle recoup their contribution, but the amplification would have to be of just the right amount and possibly very strong for the most localized components. This is a tall order but by a remarkable chance the new negative slab lens achieves this feat[15].

In figure 4c) we see rays contributing to the image for the negative slab. Just as for the conventional lens, the rays only contribute details greater that about half a

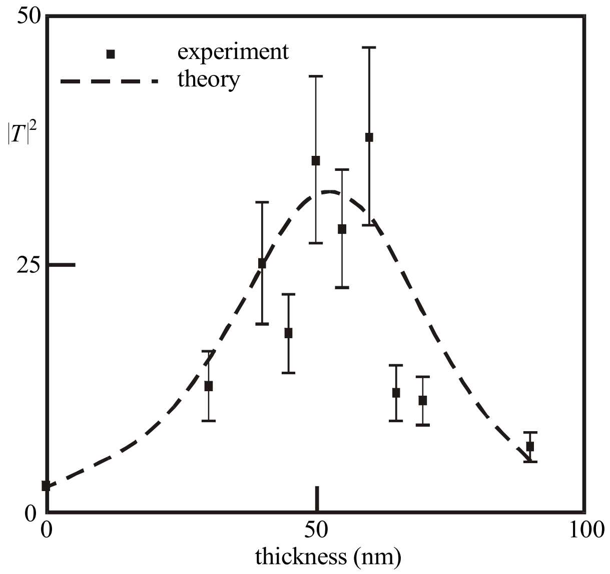

Figure 5. Amplifying the near field: near fields incident on thin films of silver are transmitted to the far side with amplification, at least up to a critical thickness. For thicker films losses dominate and spoil the resonant effect. Data taken form a paper by Fang et al.[11].

wavelength in diameter. In contrast the behavior of the near field is remarkably different as shown in figure 4d). It has the capacity to excite short wavelength resonances of the negative surface which are akin to the surface plasmons familiar on the surfaces of metals such as silver. Interaction with the plasmon like excitation kicks the decaying wave into the corresponding growing wave and the negative medium amplifies the wave, compensating for the decay that occurred in an equal thickness of vacuum. the resonances have a finite width and therefore this super lensing effect is a narrow band phenomenon: the requirement of $\varepsilon \to -1,\ \mu \to -1$ can be met only at one frequency because negative media are necessarily dispersive.

For a conventional lens resolution is limited by the aperture. Our new lens based on negative materials will also in practice have limitations, in this case chiefly due to losses. Any real material will always have small positive imaginary components to ε and μ which represent resistive losses in the system and damp the resonances responsible for amplifying the near fields. The sharpest resonances are the first to be killed by the losses and so with increasing loss the resolution is rapidly degraded. Fang *et. al.*[16] have explored these effects exploiting the fact that for very small systems much smaller than the free space wavelength we can concentrate on either the electric or magnetic fields. Silver has a negative real part to ε and therefore a thin film should behave like our negative slab and amplify the near field. They experimented on several

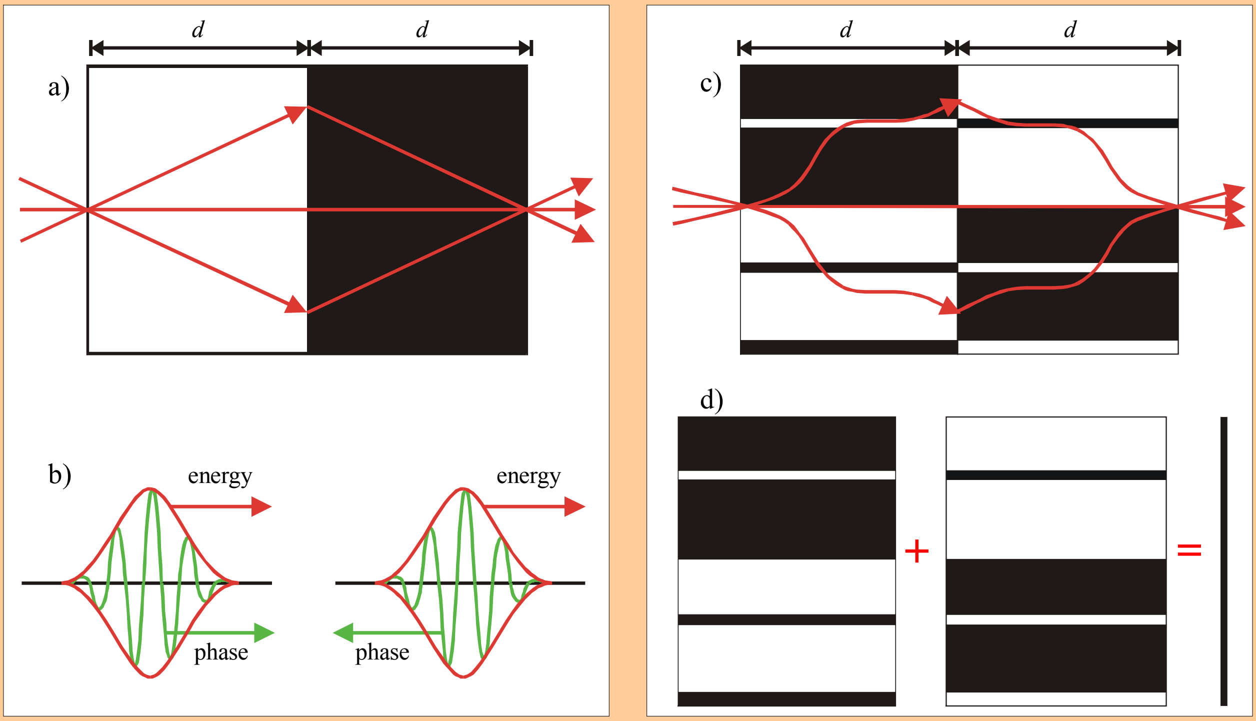

Figure 6. Generalizing the perfect lens: a) an $n = -1$ slab draws light to a perfect focus; b) shows how the focus is achieved by the negative slab ‘unwinding’ or negating the phase acquired in passing through free space; c) focusing can occur through two more complex objects provided that one is the inverse mirror image of the other; d) a graphical statement of the optical cancellation mirror antisymmetric regions of space optically annihilate one another. A negative medium is in effect a piece of optical antimatter.

films of different thickness but each time selecting the same k_x. Their results are seen in figure 5: the film is clearly amplifying waves up too a critical thickness, when losses intervene and the amplification process collapses. nevertheless considerable amplification is possible and this leads us to be optimistic that some limited sub wavelength focusing can be achieved with silver films.

Negative Refraction: Negative Space

We have seen that a slab of negative material with $\varepsilon = \mu = -1$ acts like a lens: objects on one side are brought to a focus on the other side. Figures 6a) and b) show that as the waves enter the negative medium, their phase is wound backwards as they progress. Overall the slab undoes the effect of an equal thickness of vacuum. Similarly decaying waves have their amplitude restored by passing through the slab. This suggests another view of the focusing action, that of the slab annihilating an equal thickness of vacuum. Negative media behave like optical antimatter.

In fact the result is more general than this. It has been shown[17] that two slabs of material optically annihilate if one is the negative mirror image of the other. Suppose that they meet in the plane $z = 0$, then at equal and opposite distances from this plane:

$$\varepsilon(x, y, z) = -\varepsilon(x, y, -z)$$

$$\mu(x, y, z) = -\mu(x, y, -z)$$

In figures 6c) and d) there is an illustration of what this might mean. The two media have varying refractive indices and so in general light does not follow a straight line. Nevertheless in each medium complementary paths are traced such that the overall phase acquired in the first medium is cancelled by the contribution from the second. Likewise if the waves have a decaying nature, decay in one half would be followed by amplification in the second.

This may seem straightforward but some configurations have surprises. Consider figure 7. In the top panel the two halves are inverse mirror images as required by the theorem, and therefore we expect that incident waves are transmitted without attenuation and without reflection. Yet a ray tracing exercise holds a surprise. Ray 2 in the figure hits the negative sphere and is twice refracted to be ejected from the system rather than transmitted. The rays are not supportive of our theorem! Further investigation shows that the sphere is capable of trapping rays in closed orbits, shown by dotted lines in the center of the figure. This is the signature of a resonance and a clue as to how the paradox is resolved. A full solution of Maxwell's equations shows that when the incident light is first switched on the ray predictions are initially obeyed. With time some of the incident energy will feed into the resonant state in the middle of the system which in turn will leak energy into a transmitted wave, and a contribution to the reflected wave which cancels with the original reflection. As always in negative

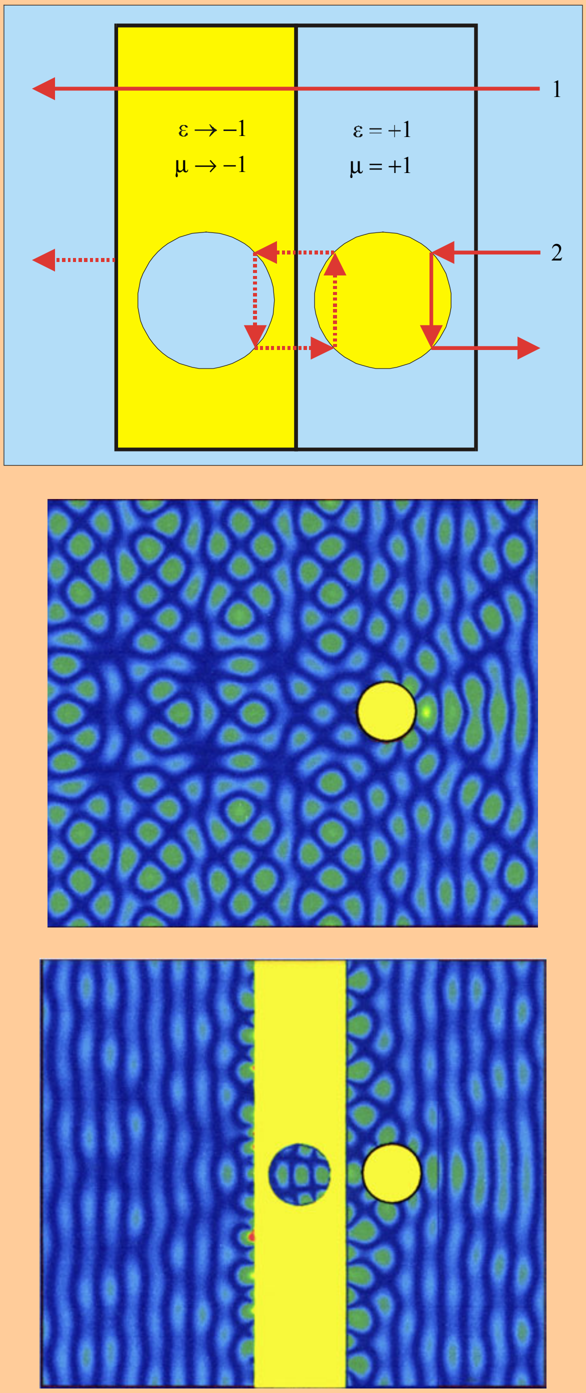

Figure 7. An optical paradox. In the top figure ray tracing predicts that some rays will be rejected from this system even though the mirror theorem predicts that all waves should be transmitted. The middle figure gives the solution of Maxwell's equations for a single negative cylinder $(n = -1)$ and the lower figure the solution when the complementary layer is added. In the latter figure within the accuracy of the calculation all scattering is removed. Note the accumulation of amplitude around the sphere indicating a resonant contribution to transmission.

media, resonant states play a central role in their properties.

The two lower panels in the figure show the resulting equilibrium solution first with only the negative sphere indicating that there is strong scattering, and then with the mirror symmetric layer included which within the accuracy of our calculation removes the reflected contributions and the spurious forward scattering to leave transmission unhindered as predicted.

An interesting question arises if there is absorption in the system represented by positive imaginary parts of either or both of ε and μ . Conditions for the theorem may still be satisfied but require that for every instance of a positive part to ε,μ there is a mirror antisymmetric negative ε,μ somewhere else in the system. This implies that parts of the system must exhibit gain. Loss can only be compensated by active amplification.

Conclusions

Negative refraction is a subject with constant capacity for surprise: innocent assumptions lead to unexpected and sometimes profound consequences. This has generated great enthusiasm but also controversy yet even the controversies have had the positive effect that key concepts have been critically scrutinized in the past 18 months. Finally in the past year experimental data have been produced which validate the concepts. As a result we have a firm foundation on which to build. Many groups are already moving forward with applications. Naturally the microwave area has been most productive as the metamaterials required are easier to fabricate. We have given an illustration of a microwave lens, but novel waveguides and other devices are under consideration.

One of the most exciting possibilities is imaging beyond the wavelength limit. Practical applications will require low loss materials, a great challenge to the designers of new metamaterials. Proposals to employ thin silver films as lenses are under investigation in several laboratories. Nor are the challenges purely experimental: we are not yet done with theory since the assumption of negative refraction has many ramifications still being explored and which are sure to cast more light on this strange but fascinating subject. Not surprisingly many are joining the field and 2003 saw over 200 papers published on negative refraction. We expect even more in 2004!

Further reading can be found in a special edition of Optics Express[18] and in the article by McCall *et al*[19] .

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Metamaterials and Negative Refraction

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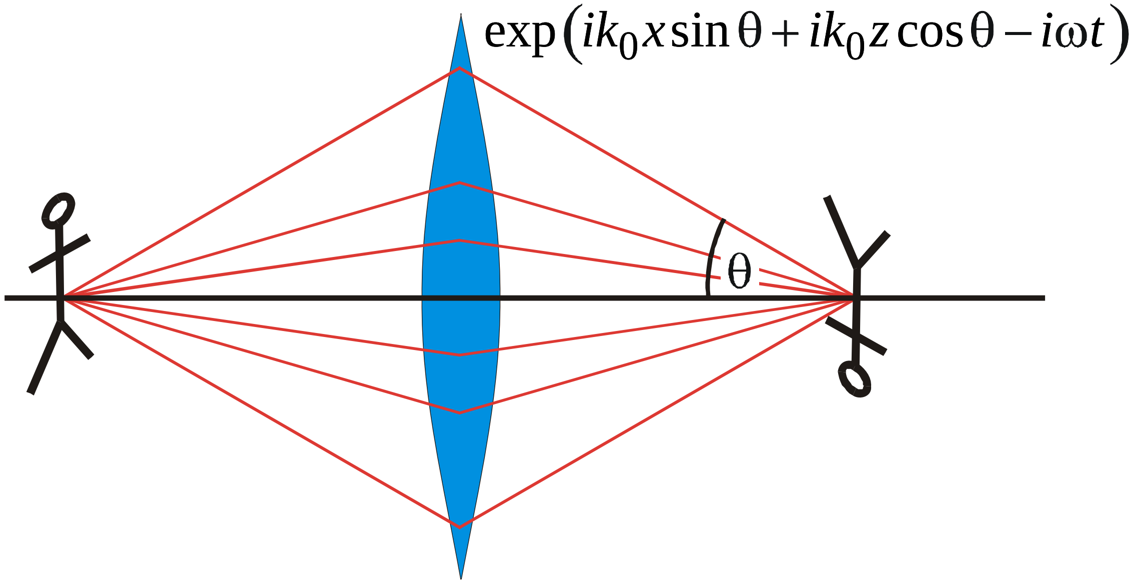
57 [6] 37-43 (June 2004)

Focussing light

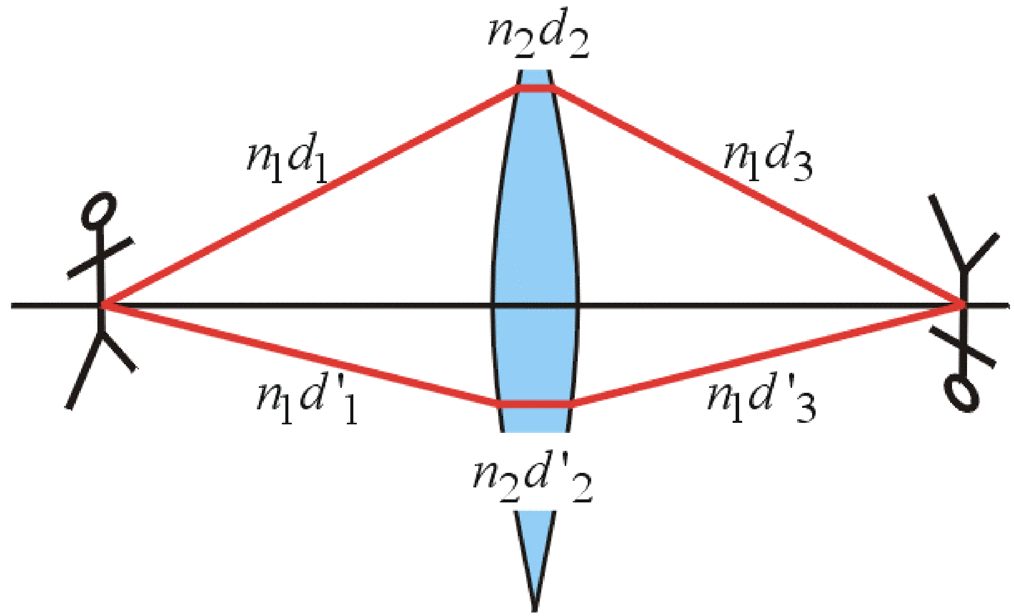
Galileo by Leoni - 1624



lens, *n.* L. *lens* lentil, from the similarity in form. A piece of glass with two curved surfaces

Fermat's Principle:

"Light takes the shortest optical path between two points"



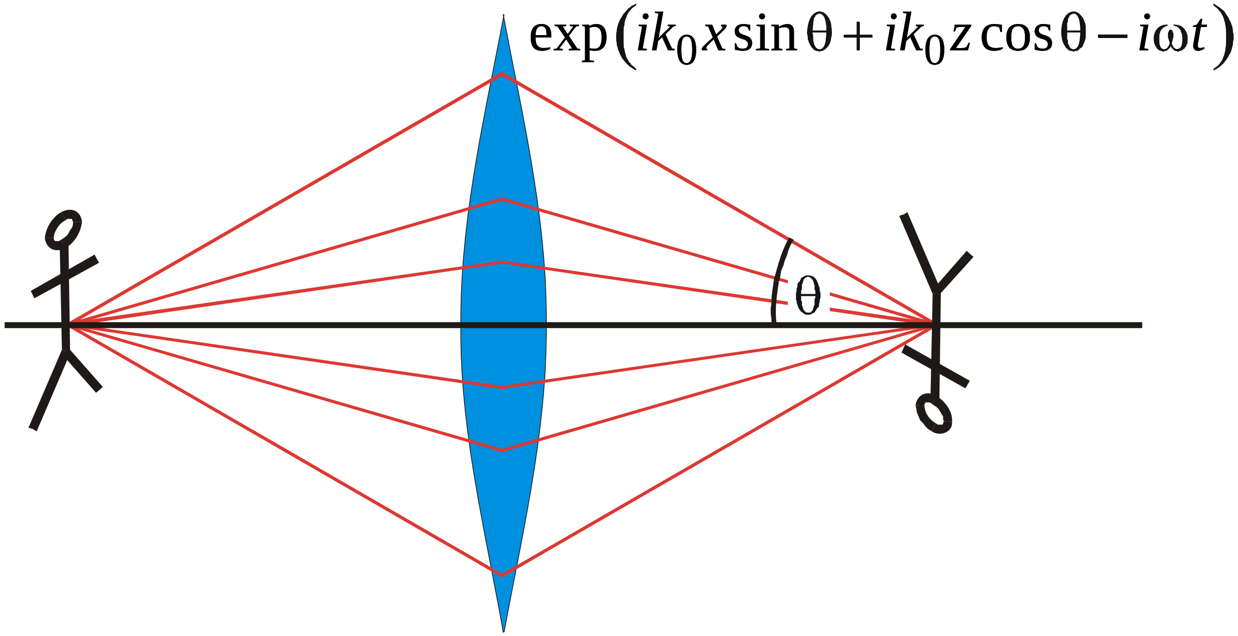
e.g. for a lens the shortest optical distance between object and image is:

$$n_1 d_1 + n_2 d_2 + n_1 d_3 = n_1 d'_1 + n_2 d'_2 + n_1 d'_3$$

both paths converge at the same point because both correspond to a minimum.

Focussing light: wavelength limits the resolution

Contributions of the far field to the image ….



….. are limited by the free space wavelength:

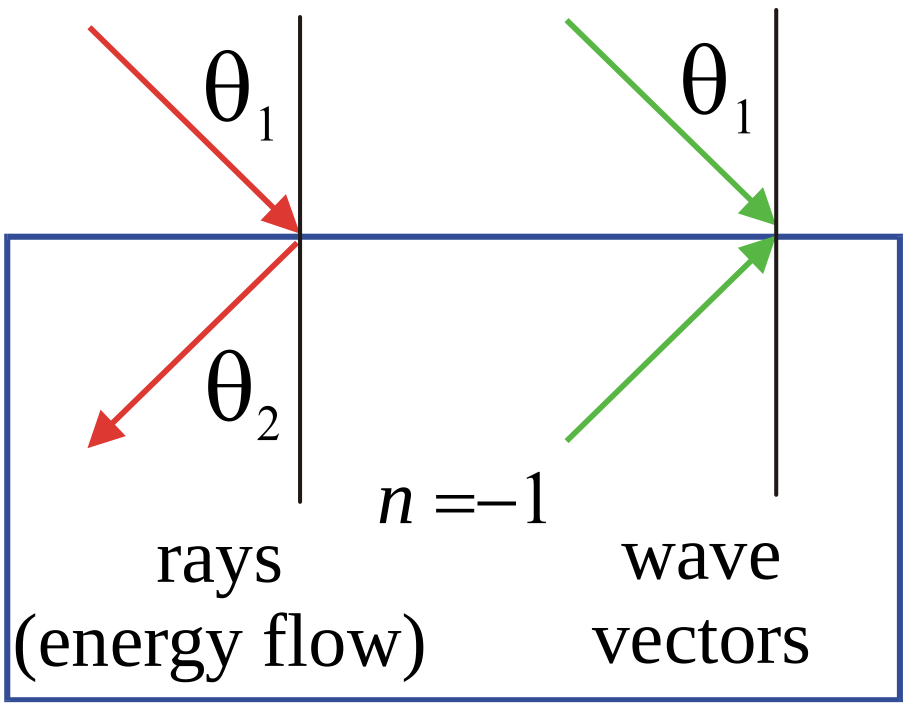
$\theta = 90°$ gives maximum value of $k_x = k_0 = \omega/c_0 = 2\pi/\lambda_0$ – the shortest wavelength component of the 2D image. Hence resolution is no better than,

$$\Delta \approx \frac{2\pi}{k_0} = \frac{2\pi c}{\omega} = \lambda_0$$

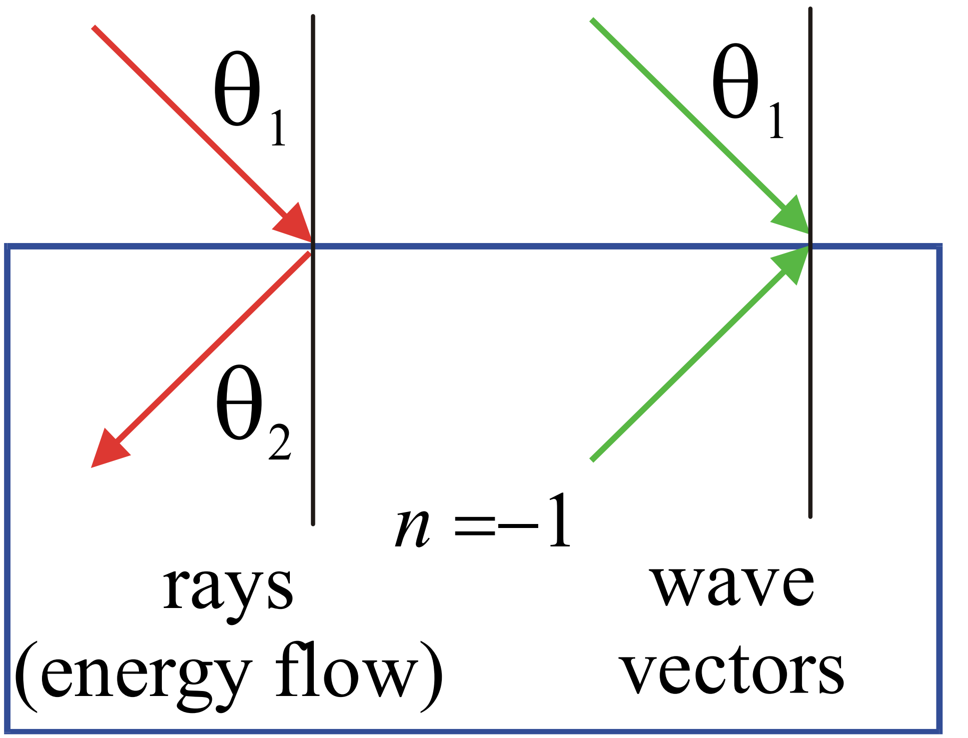
Negative Refractive Index and Snell's Law

$$n = \frac{\sin(\theta_1)}{\sin(\theta_2)}$$

Hence in a negative refractive index material, *light makes a negative angle with the normal*. Note that the parallel component of wave vector is always preserved in transmission, but that energy flow is opposite to the wave vector.



The consequences of negative refraction
1. negative group velocity



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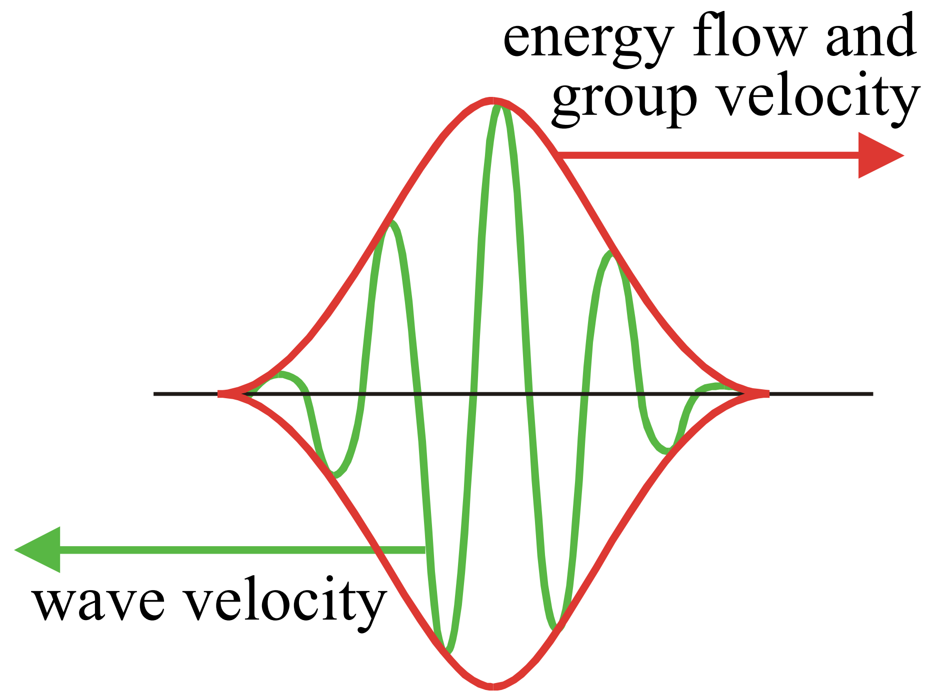
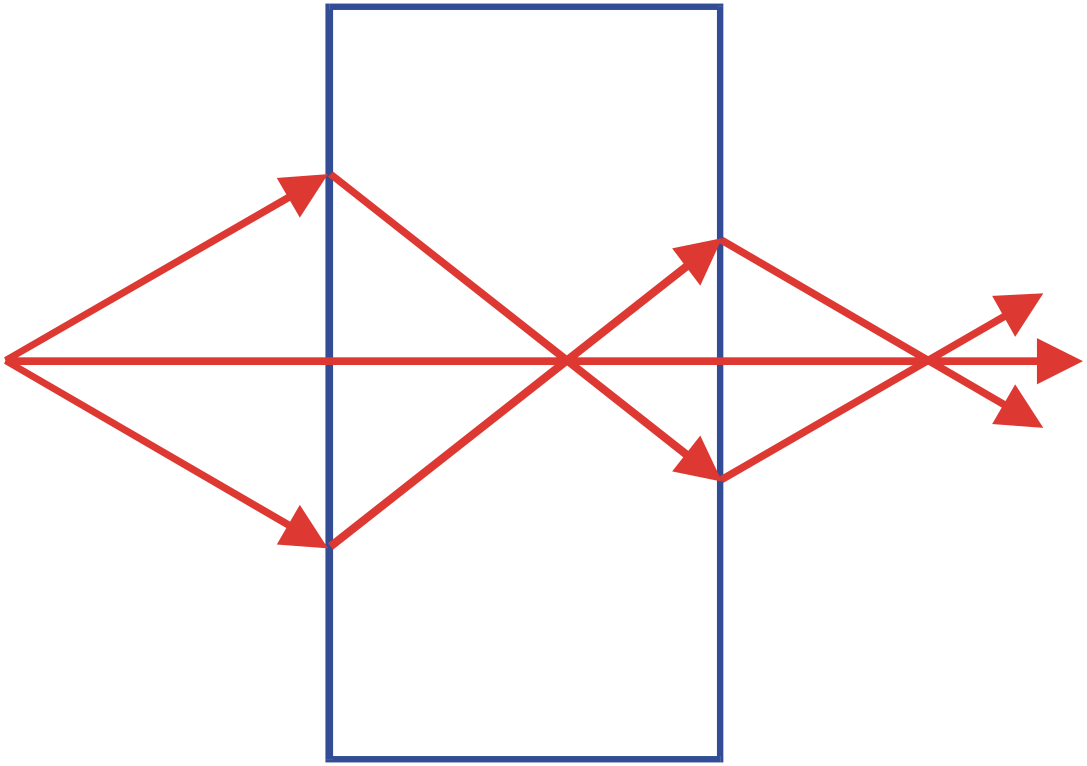


Materials with negative refraction are sometimes called ***left handed materials*** because the Poynting vector has the opposite sign to the wave vector.

Negative Refractive Index and Focussing



A negative refractive index medium bends light to a negative angle relative to the surface normal. Light formerly diverging from a point source is set in reverse and converges back to a point. Released from the medium the light reaches a focus for a second time.

Recipe for Negative Refractive Index

James Clark Maxwell showed that light is an electromagnetic wave and its refraction is determined by both:

the electrical permittivity, ε,
and the magnetic permeability, μ.

The wave vector, k, is related to the frequency by the refractive index,

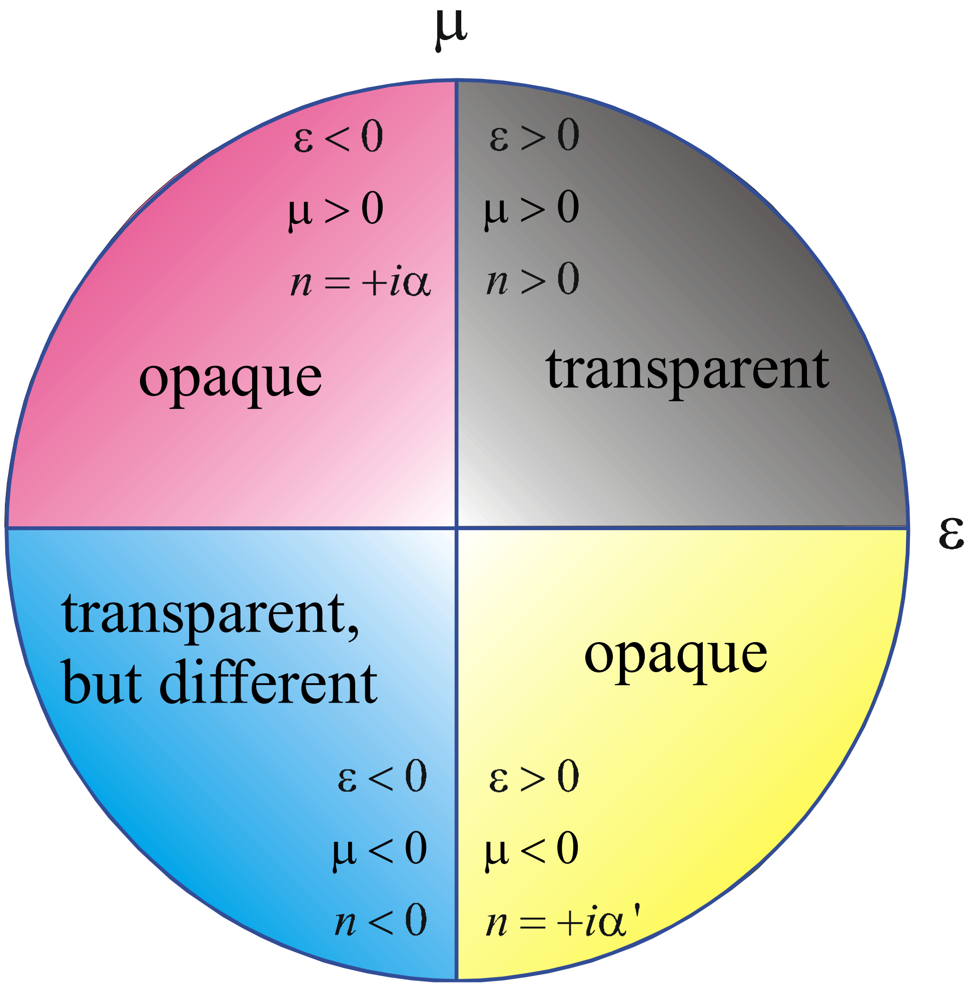
$$k = \sqrt{\varepsilon\mu}\,\omega c_0^{-1} = n\omega c_0^{-1}$$

Normally n, ε, and μ are positive numbers.

In 1968 *Victor Veselago* showed that if ε and μ are negative, we are forced by Maxwell's equations to choose a negative square root for the refractive index,

$$\boxed{n = -\sqrt{\varepsilon\mu}, \quad \varepsilon < 0, \quad \mu < 0}$$

Negative Refraction - $n < 0$



The *wave vector* defines how light propagates:

$$E = E_0 \exp(ikz - i\omega t)$$

where,

$$k = \omega/c \times \sqrt{\varepsilon\mu} = \omega/c \times n$$

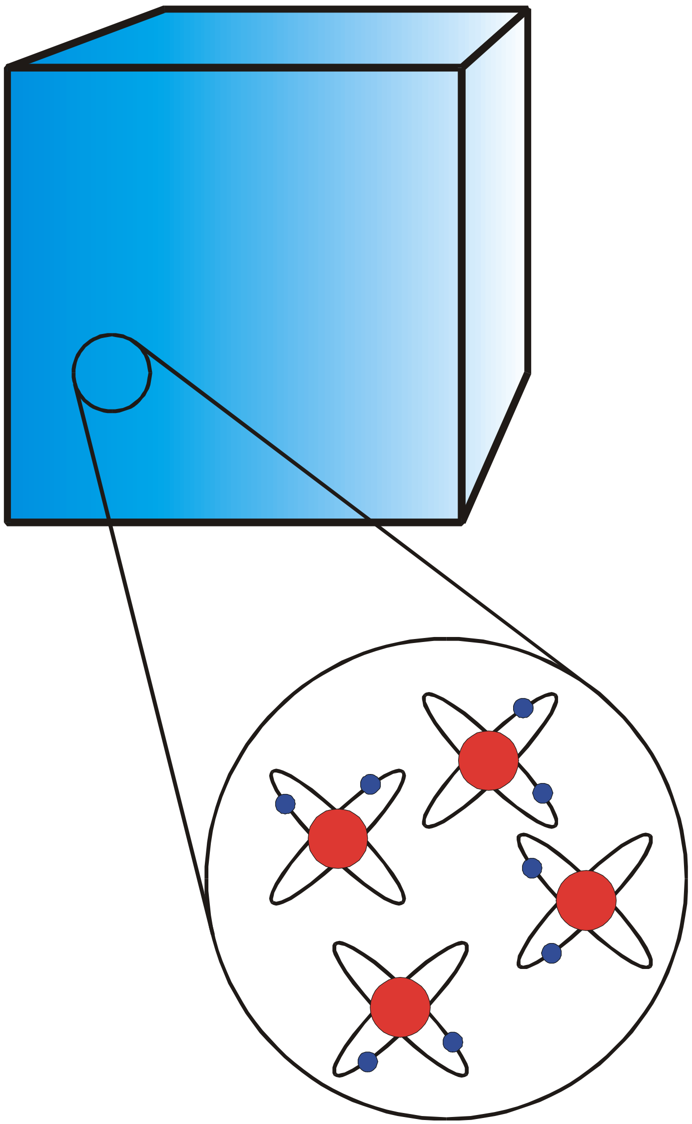
Either $\varepsilon < 0$, or $\mu < 0$, ensures that k is imaginary, and the material opaque.

If $\varepsilon < 0$ *and* $\mu < 0$, then k is real, but we are forced to choose the *negative* square root to be consistent with Maxwell's equations.

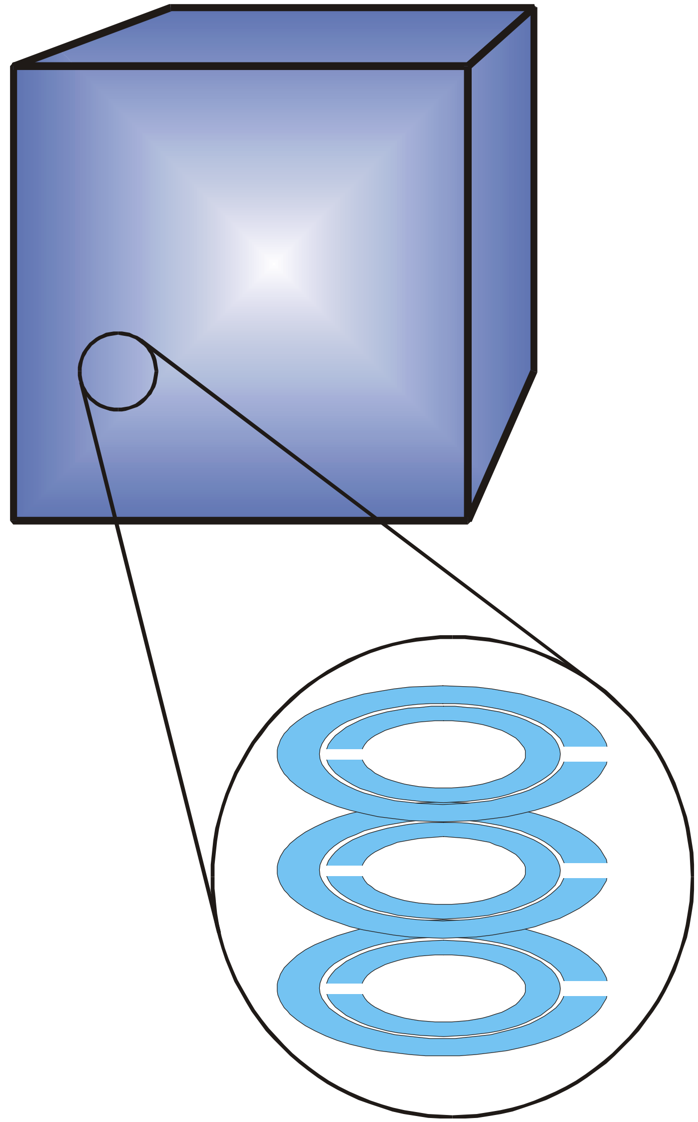
$\boxed{\varepsilon < 0, \mu < 0 \text{ means that } n \text{ is negative}}$

What is a 'metamaterial'

Conventional materials: properties derive from their constituent *atoms*.

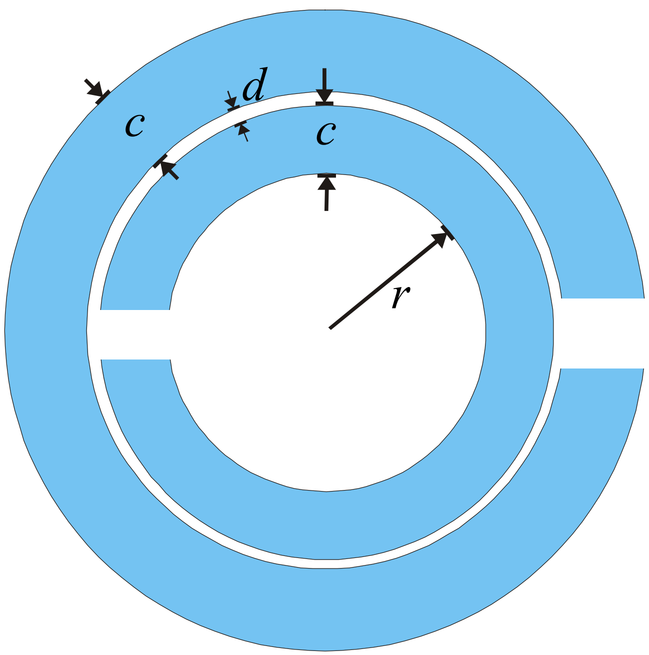


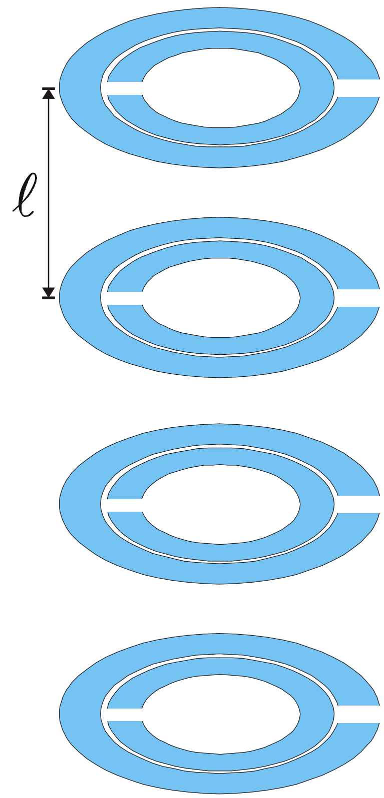
Metamaterials: properties derive from their constituent *units*. These units can be engineered as we please.



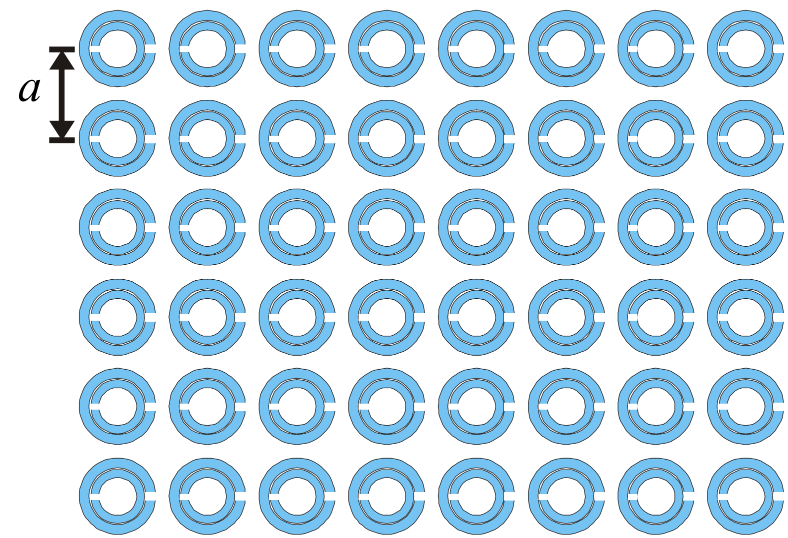
A metamaterial with $\mu < 0$ at 10GHz

The ‘split ring’ structure is designed to resonate around 10GHz. The circulating currents give a magnetic response, even though the rings are made from copper.

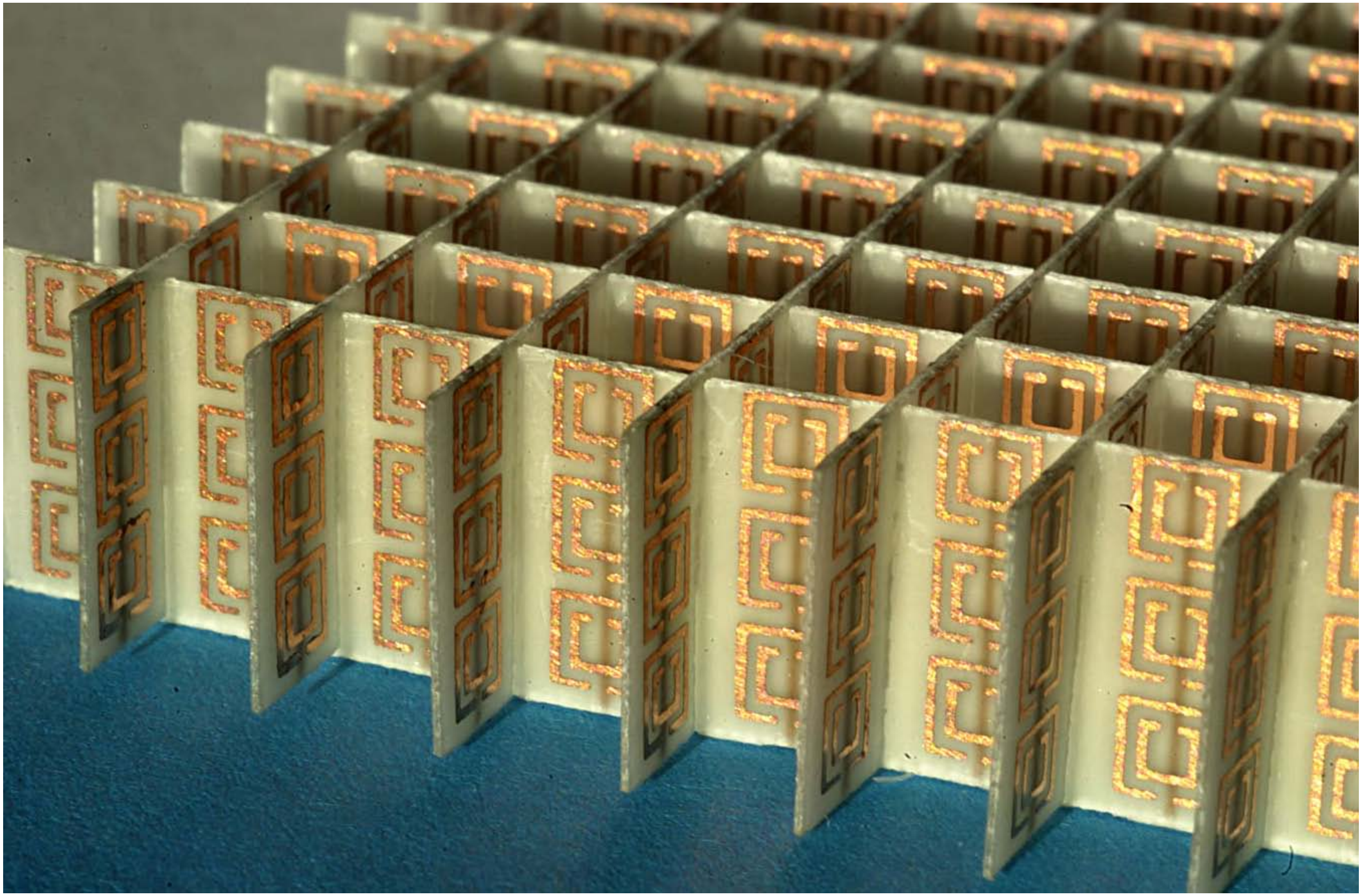




Planar split ring structure
inner radius r = 2.0mm
width of each ring c = 1.0mm
spacing between ring edges d = 0.1mm
lattice constant a = 10.0mm



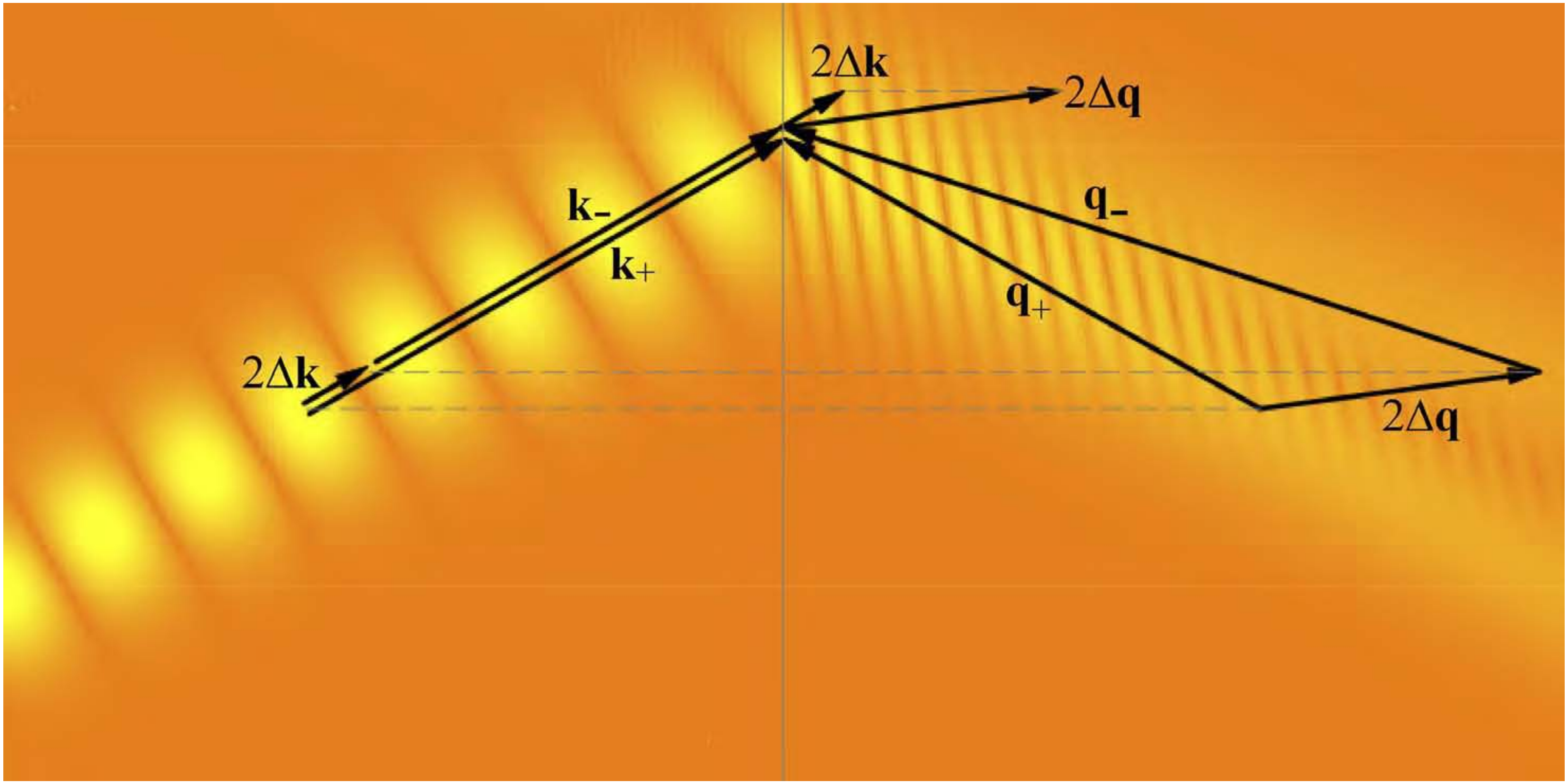
Negative refraction: $\varepsilon < 0, \mu < 0$



Structure made at UCSD by David Smith

Refraction of a Gaussian beam into a negative index medium.

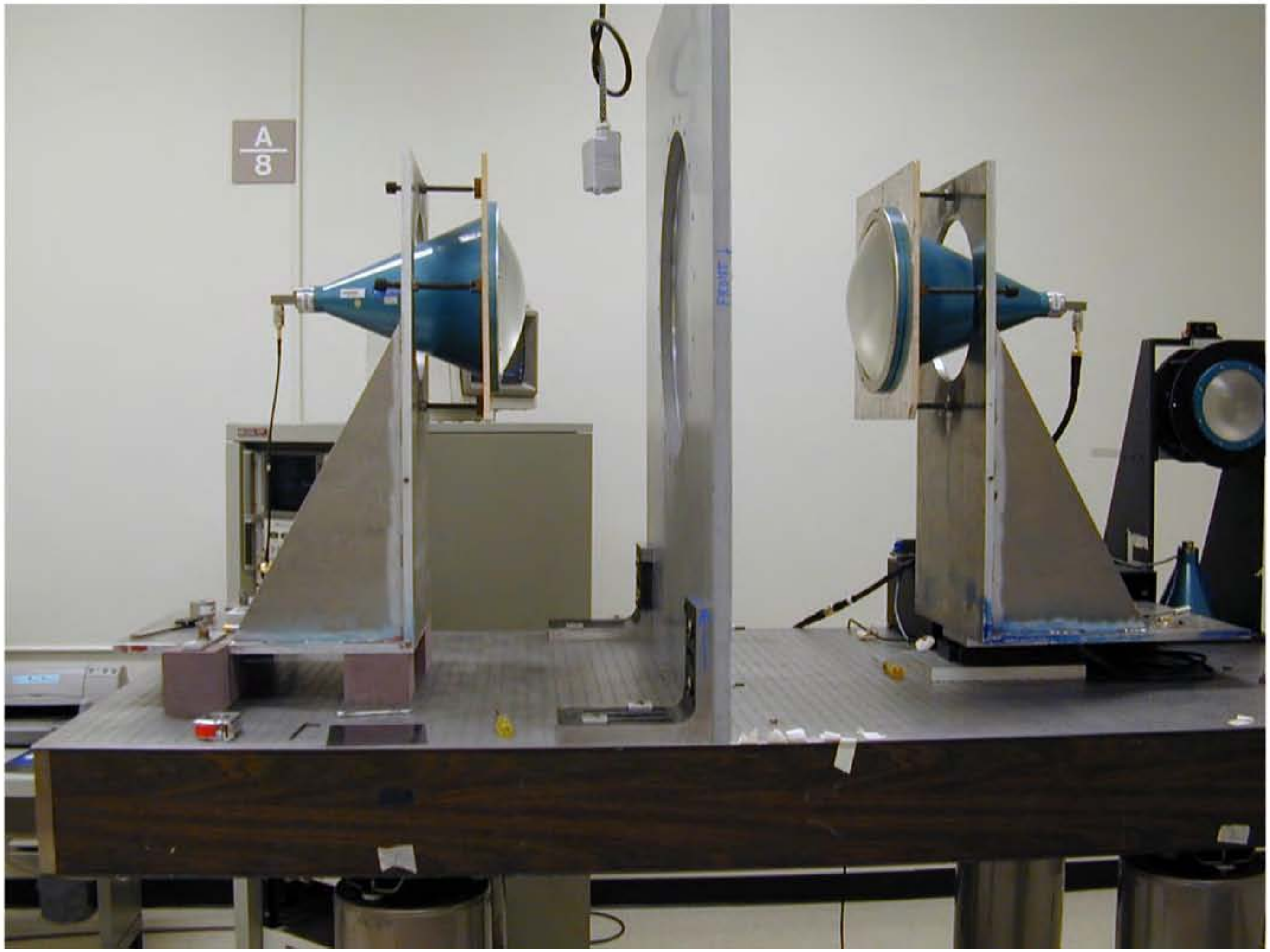
The angle of incidence is 30° (computer simulation by David Smith UCSD)



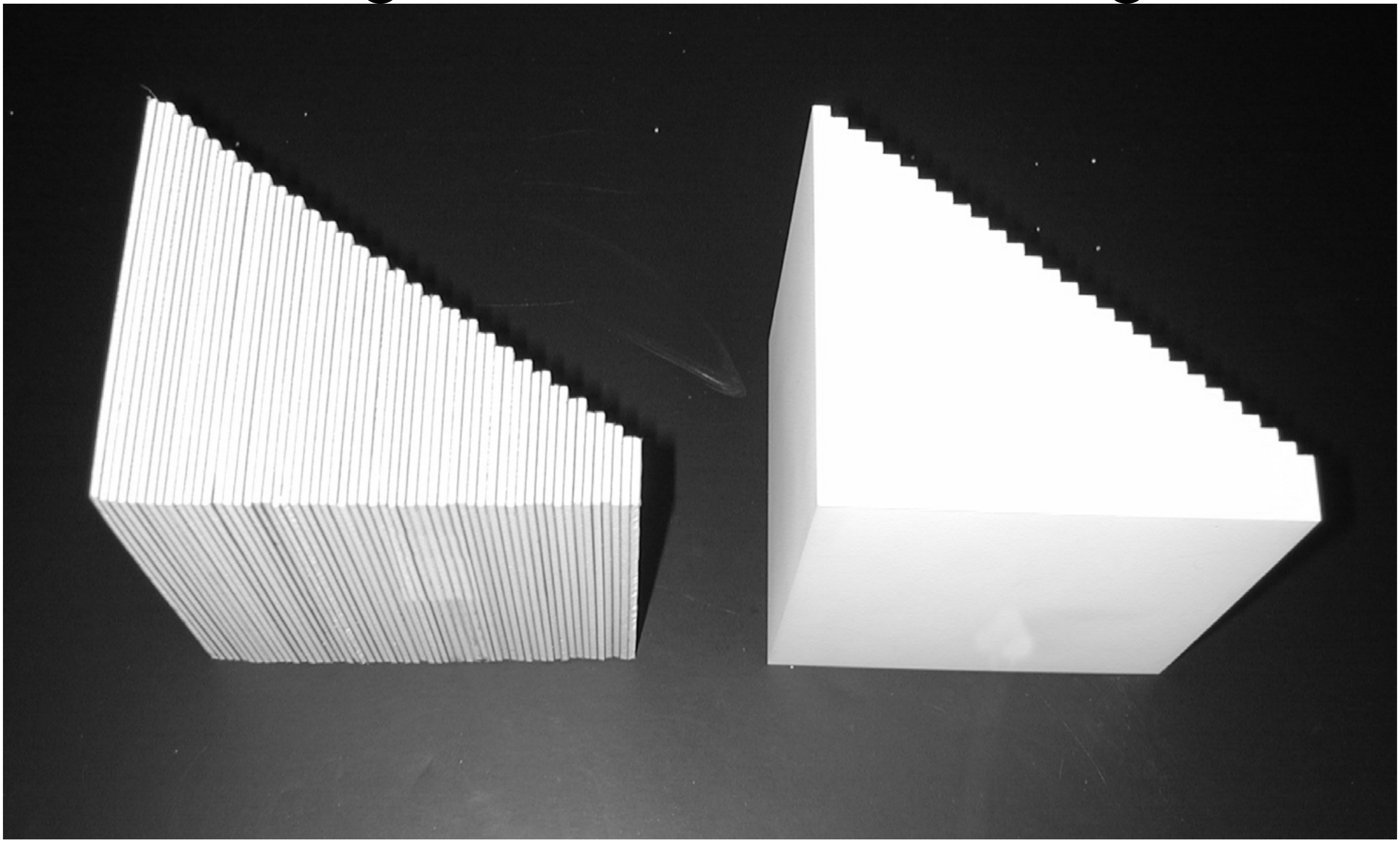
$$n(\omega_-) = -1.66 + 0.003i, \quad n(\omega_+) = -1.00 + 0.002i, \quad \Delta\omega/\omega = 0.07$$

Negative Refraction at the Phantom Works

Free-Space Experimental Set-up



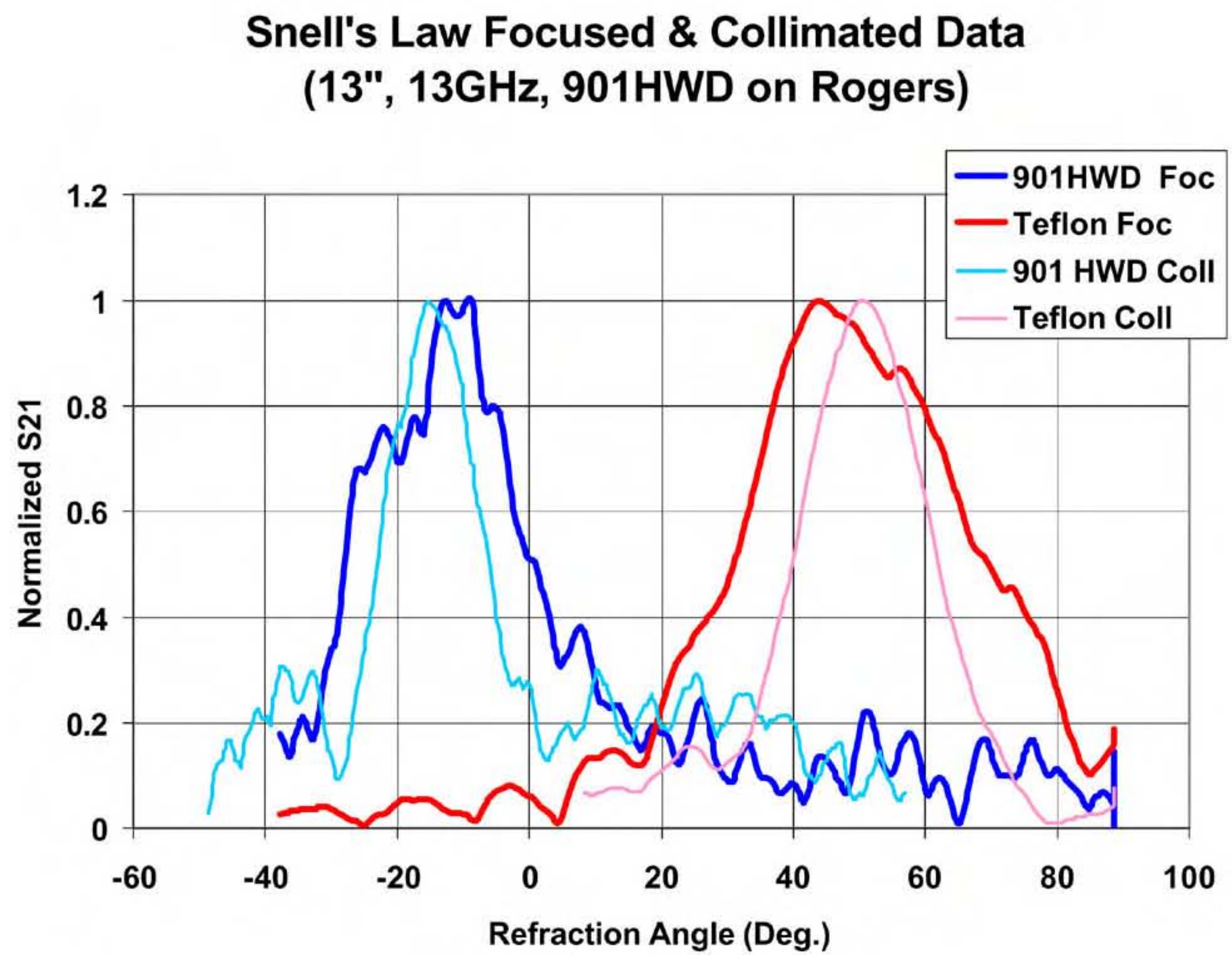
Boeing PhantomWorks 32° wedges



Left: negatively refracting sample
Right: teflon

Snell's Law Verification for n<0 in Free Space (2)

(Line plots, f=13 GHz, detector at 13", Boeing 2002)

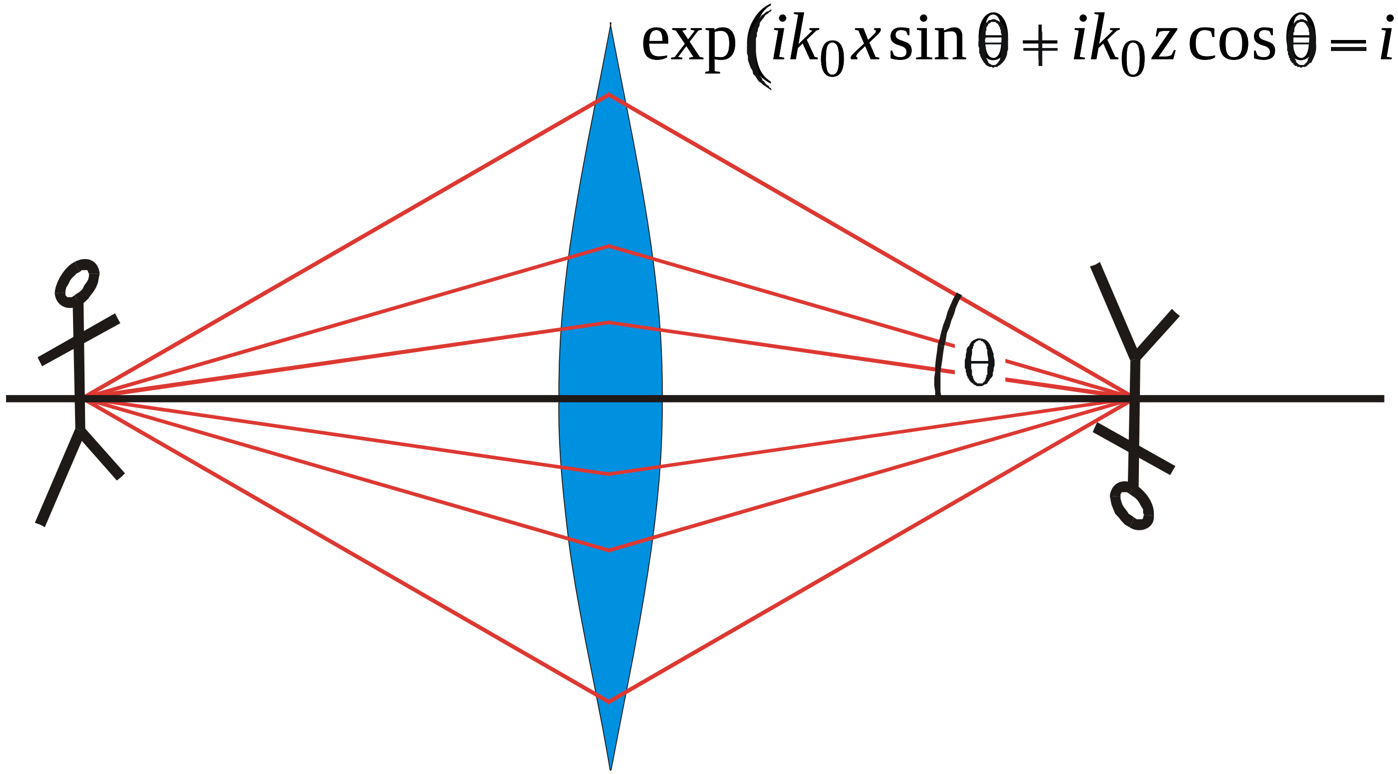


Iowa State University of Science and Technology · University of California San Diego · UCLA · LSU · Technische Universität Darmstadt · Phantom Works

Limitations to the Performance of a Lens

Contributions of the far field to the image …..

$$\exp\left(ik_0 x \sin\theta + ik_0 z \cos\theta - i\omega t\right)$$



….. are limited by the free space wavelength: $\theta = 90^\circ$ gives maximum value of $k_x = k_0 = \omega/c_0 = 2\pi/\lambda_0$ – the shortest wavelength component of the 2D image. Hence resolution is no better than,

$$\Delta \approx \frac{2\pi}{k_0} = \frac{2\pi c}{\omega} = \lambda_0$$

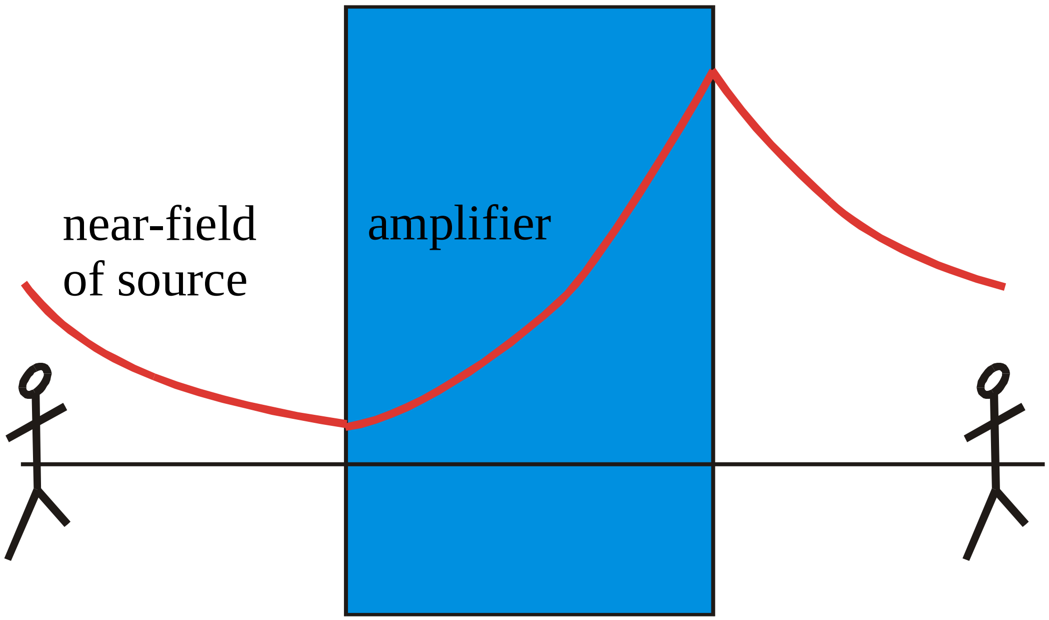
Limitations to a Conventional Lens (2)

Contributions of the near field to the image …..

come from large values of k_x responsible for the finest details in the source. Forget about ray diagrams because,

$$k_z = +i\sqrt{k_x^2 - \omega^2 c^{-2}}, \qquad \omega^2 c_0^{-2} < k_x^2$$

and 'near field' light decays exponentially with distance from the source. i.e. the near field is confined to the immediate vicinity of the source. Unless we can make an *amplifier* it is inevitable that the finest detail is lost from the image.



Attempting the impossible:
a lens for the near field,
a negative story

The consequences of negative refraction
3. *Perfect* Focussing

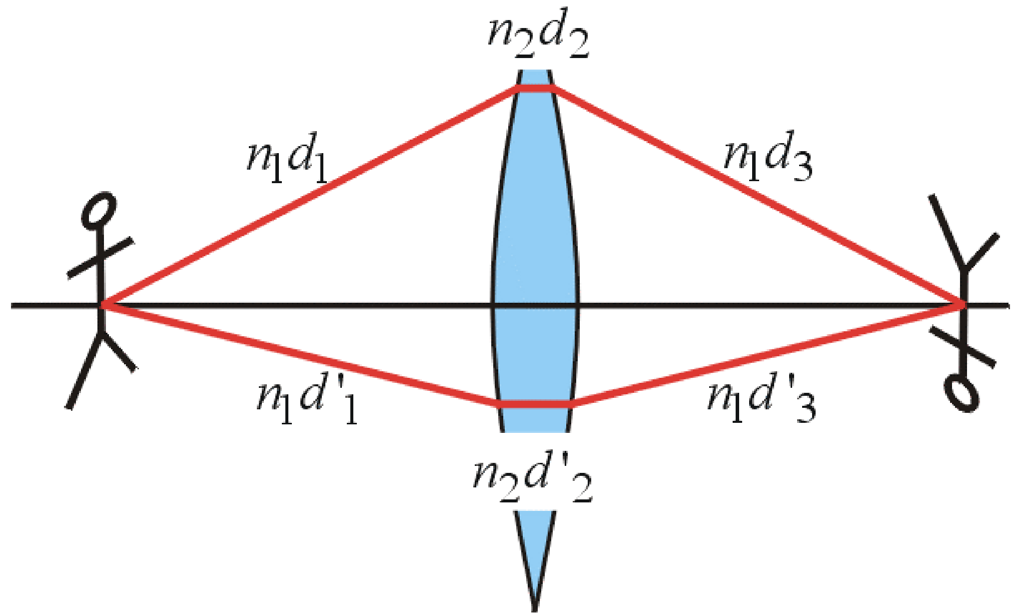
A conventional lens has resolution limited by the wavelength. The missing information resides in the near fields which are strongly localised near the object and cannot be focussed in the normal way.

The new lens based on negative refraction has *unlimited resolution* provided that the condition $n = -1$ is met exactly. This can happen only at one frequency. (Pendry 2000).

The secret of the new lens is that it can focus the near field and to do this it must *amplify* the highly localised near field to reproduce the correct amplitude at the image.

Fermat's Principle:

"Light takes the shortest optical path between two points"



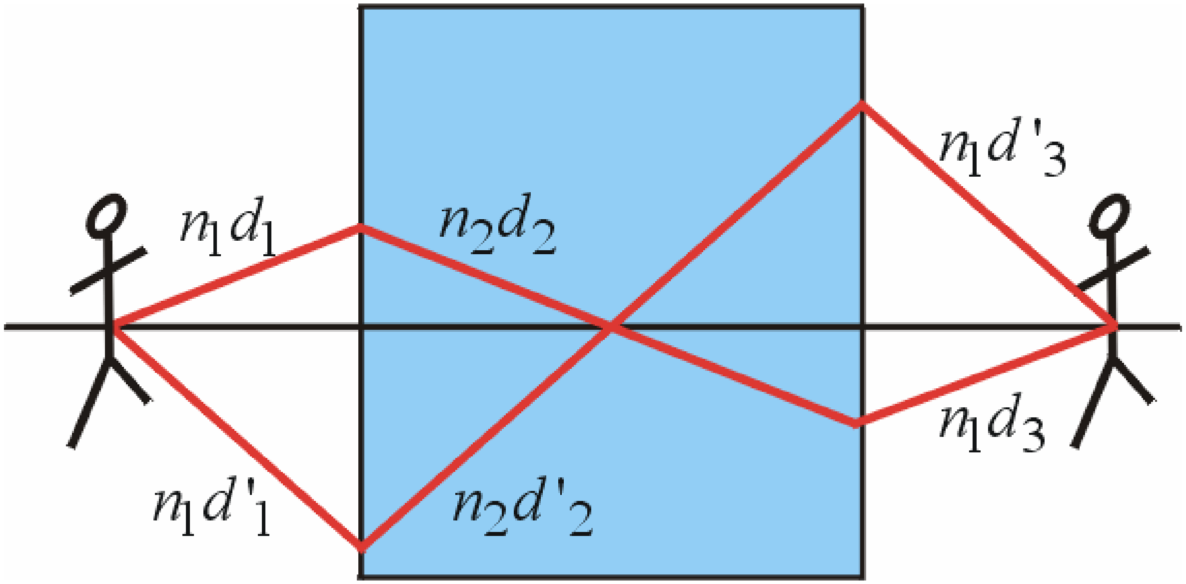
e.g. for a lens the shortest optical distance between object and image is:

$$n_1d_1 + n_2d_2 + n_1d_3 = n_1d'_1 + n_2d'_2 + n_1d'_3$$

both paths converge at the same point because both correspond to a minimum.

Fermat's Principle for Negative Refraction

If n_2 is negative the ray traverses **negative optical space**.



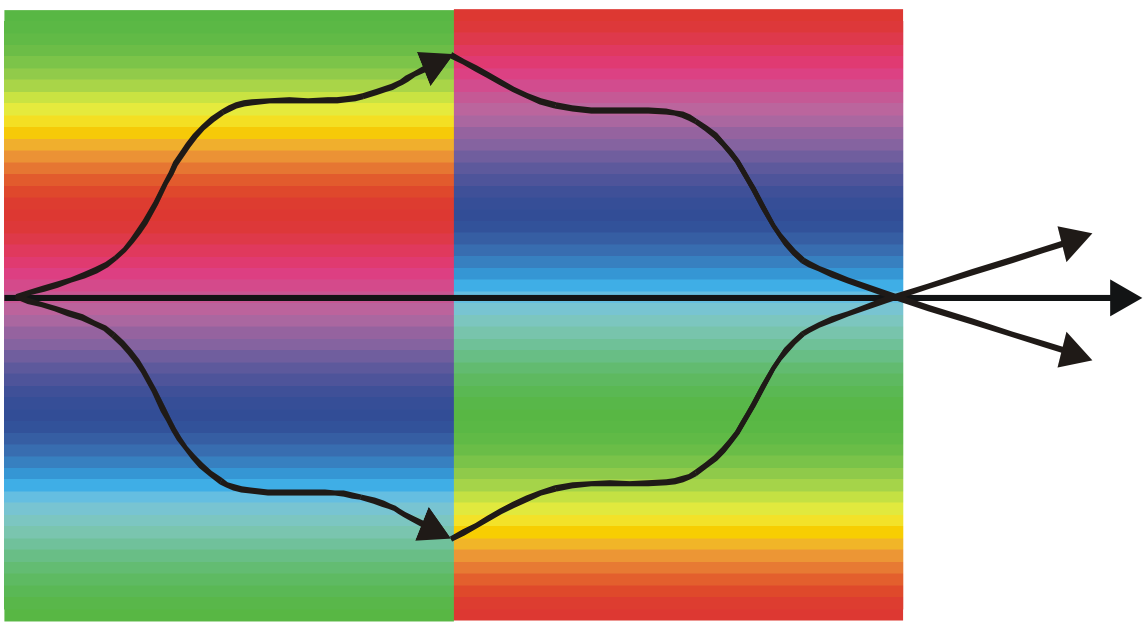
for a *perfect* lens ($n_2 = -n_1$) the shortest optical distance between object and image is **zero**:

$$
\begin{aligned}
0 &= n_1 d_1 + n_2 d_2 + n_1 d_3 \\
&= n_1 d'_1 + n_2 d'_2 + n_1 d'_3
\end{aligned}
$$

For a perfect lens the image *is* the object

Negative Space

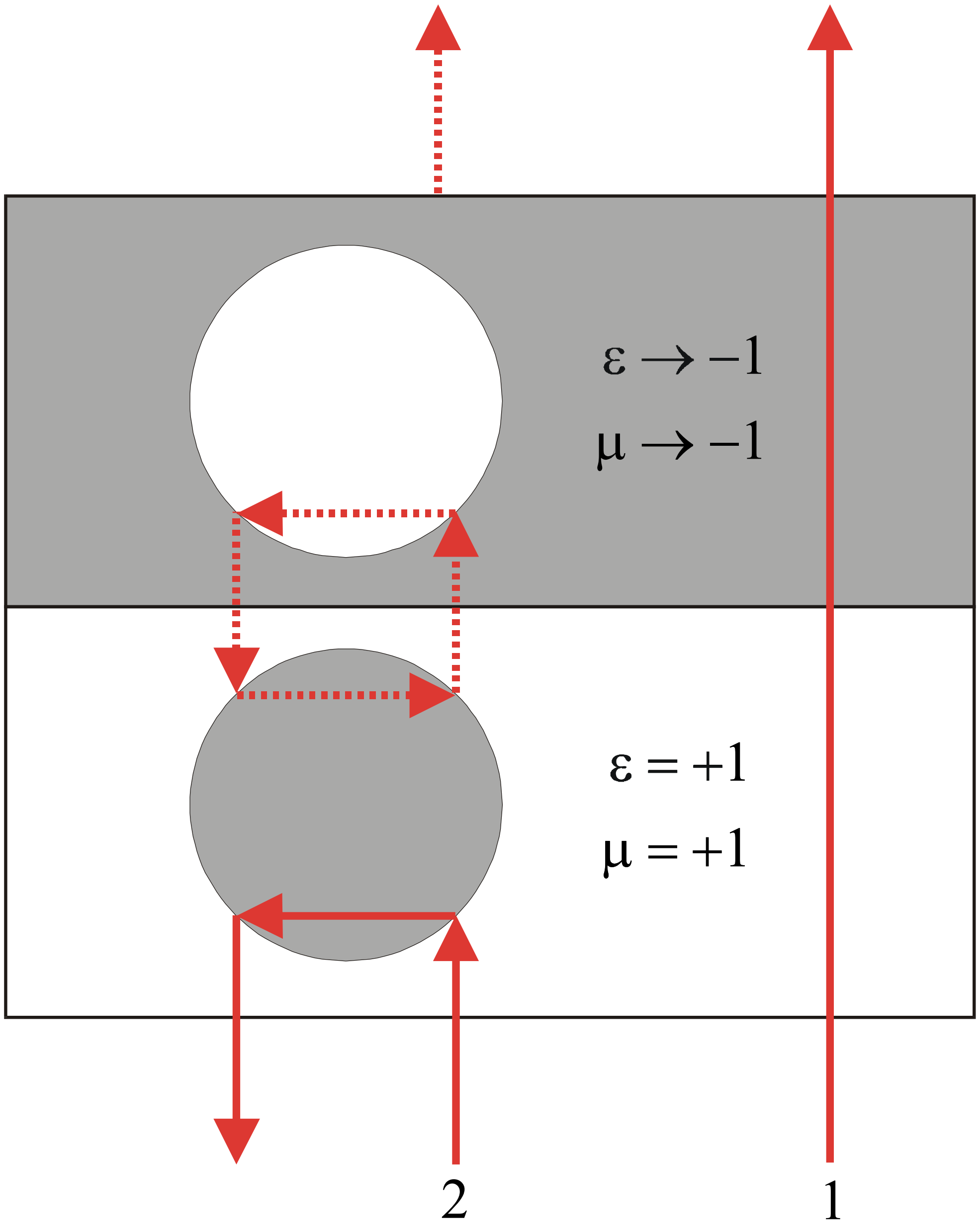
A slab of $n = -1$ material thickness d, cancels the effect of an equivalent thickness of free space. i.e. objects are focussed a distance $2d$ away. An alternative pair of complementary media, each cancelling the effect of the other. The light does not necessarily follow a straight line path in each medium:



General rule:
two regions of space optically cancel if in each region ε,μ are reversed mirror images.

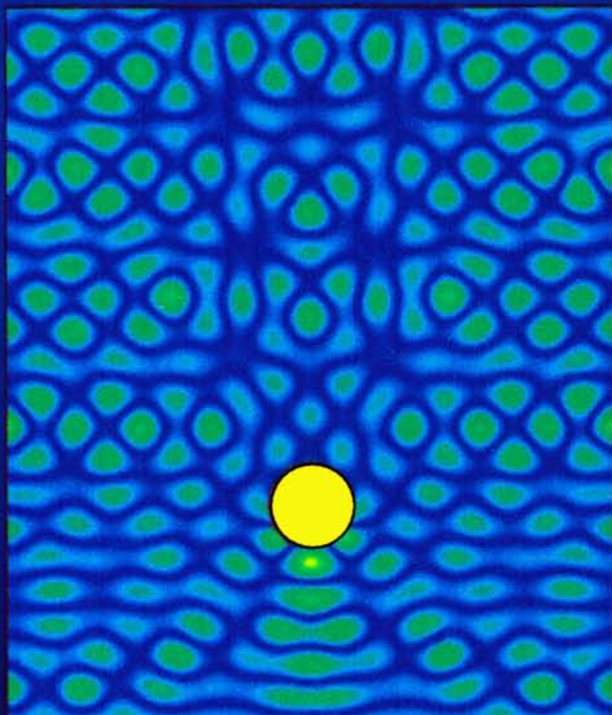
The overall effect is as if a section of space thickness $2d$ were removed from the experiment.

A Negative Paradox

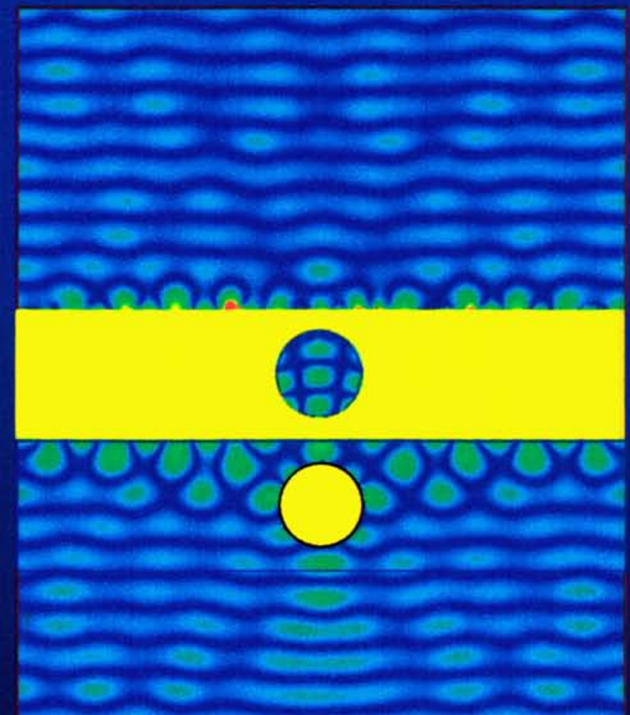


The left and right media in this 2D system are negative mirror images and therefore optically annihilate one another. However a ray construction appears to contradict this result. Nevertheless the theorem is correct and the ray construction erroneous. Note the closed loop of rays indicating the presence of resonances.

Compensation of inhomogeneous media



Scattering from a cylinder with n=-1.4



Compensation of the n=-1.4 cylinder

The ‘Poor Man’s Superlens’

The original prescription for a superlens: a slab of material with

$$\varepsilon = -1, \mu = -1$$

However if all relevant dimensions (the thickness of the lens, the size of the object etcetera) are much less than the wavelength of light, electric and magnetic fields are decoupled. An object that comprises a pure electric field can be imaged using a material with,

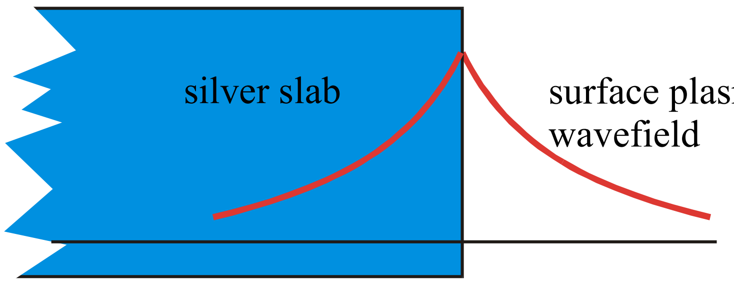
$$\varepsilon = -1, \mu = +1$$

because, in the absence of a magnetic field, μ is irrelevant.

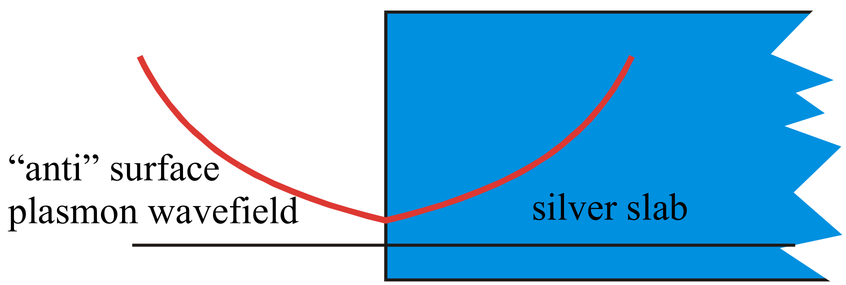
We can achieve this with a slab of silver which has $\varepsilon < 0$ at optical frequencies.

Anatomy of a Superlens

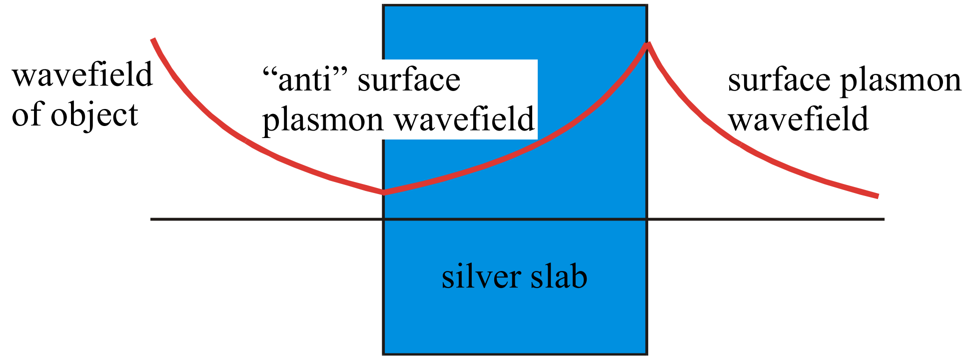
The superlens works by **resonant excitation of surface plasmons in the silver**,



At the same frequency as the surface plasmon there exists an unphysical "anti" surface plasmon - wrong boundary conditions at infinity,



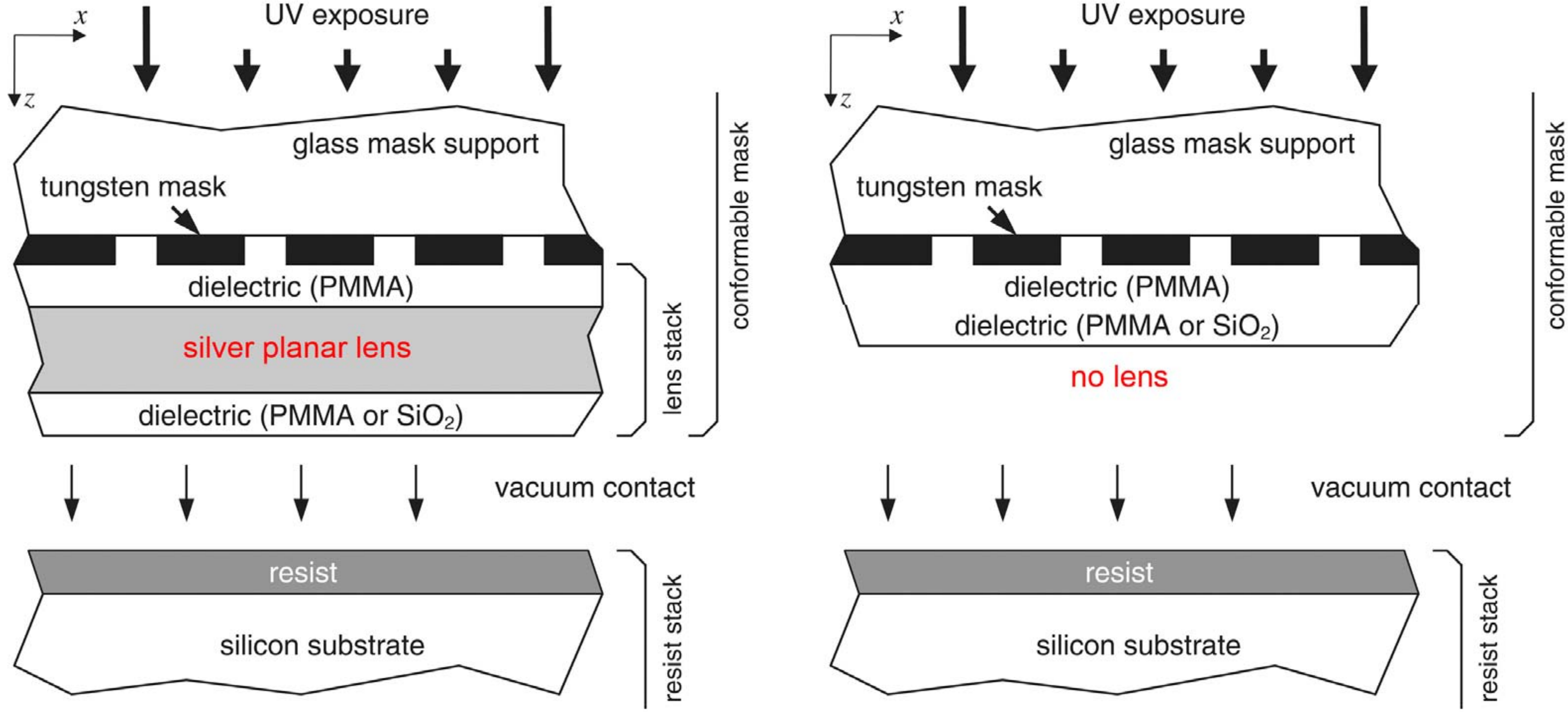
However,



Matching the fields at the boundaries selectively excites a surface plasmon on the far surface.

Near Field Superlensing Experiments

Richard Blaikie and David Melville, *J. Opt. A*, **7** S176 (2005)

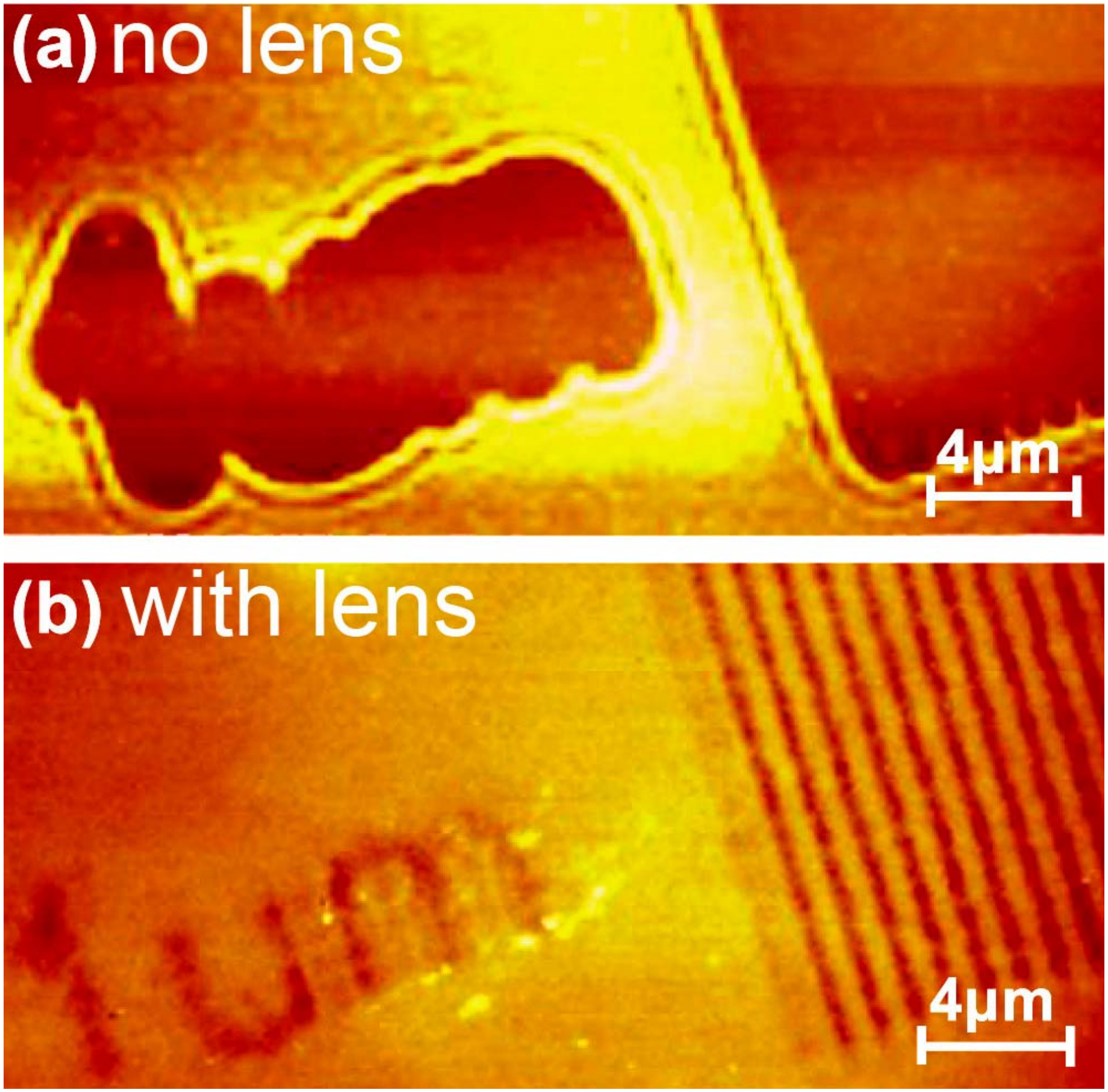


Left: experimental setup for a near field silver planar lens

Right: reference experiment excluding the lens

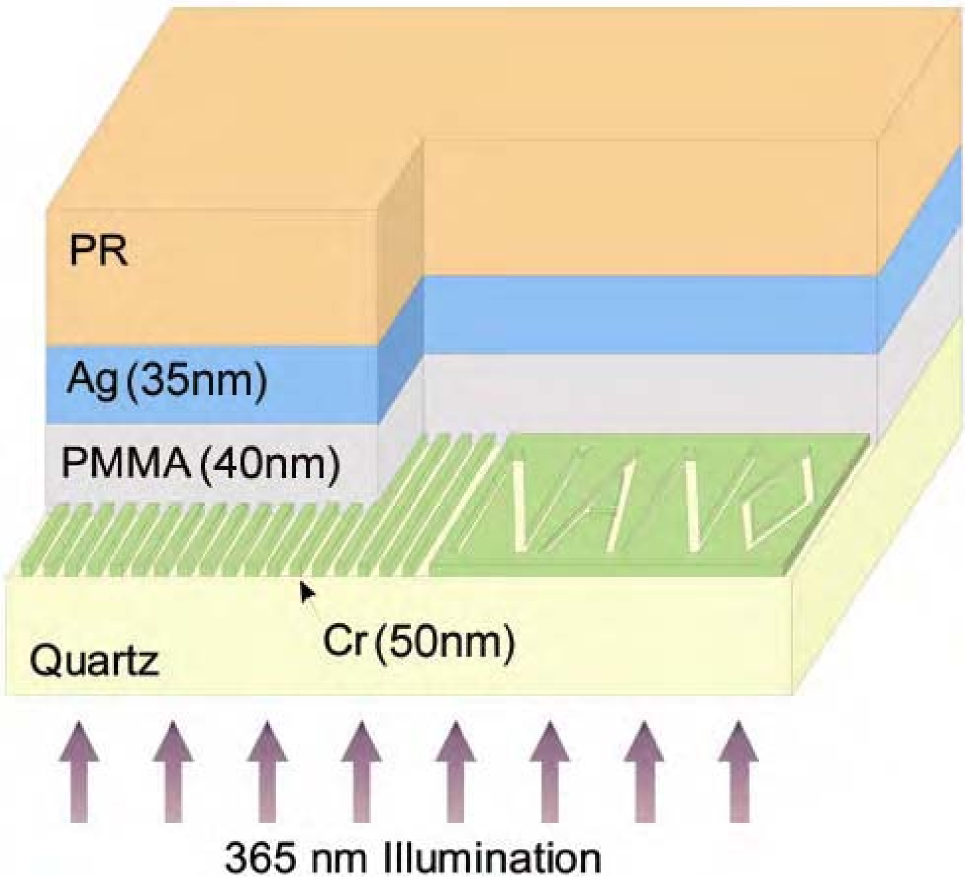
Near Field Superlensing Experiments

Richard Blaikie and David Melville, *J. Opt. A*, **7** S176 (2005)



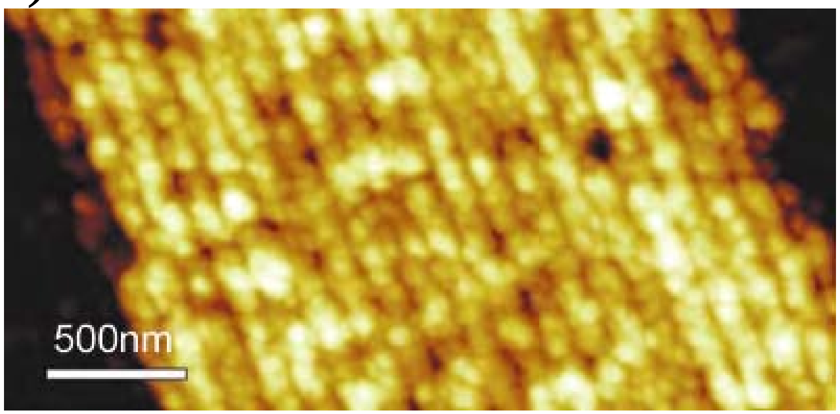
Near field superlensing experiment:

Nicholas Fang, Hyesog Lee, Cheng Sun and Xiang Zhan, UCB



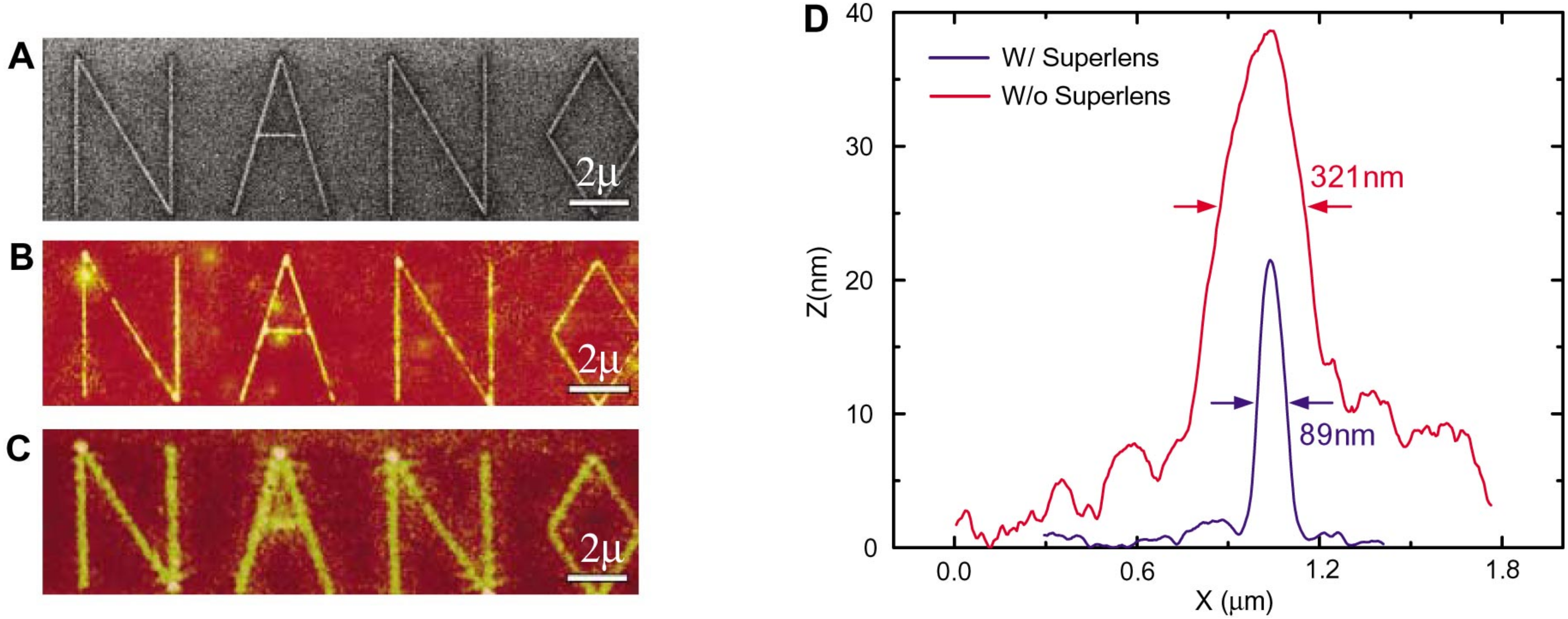
Left: the objects to be imaged are inscribed onto the chrome. Left is an array of 60nm wide slots of 120nm pitch. The image is recorded in the photoresist placed on another side of silver superlens.

Below: Atomic force microscopy of a developed image. This clearly shows a superlens imaging of a 60 nm object ($\lambda/6$).



Imaging by a Silver Superlens.

Nicholas Fang, Hyesog Lee, Cheng Sun, Xiang Zhang, *Science* 534 **308** (2005)



(A) FIB image of the object. The linewidth of the ‘‘NANO’’ object was 40 nm.

(B) AFM of the developed image on photoresist with a 35-nm-thick silver superlens.

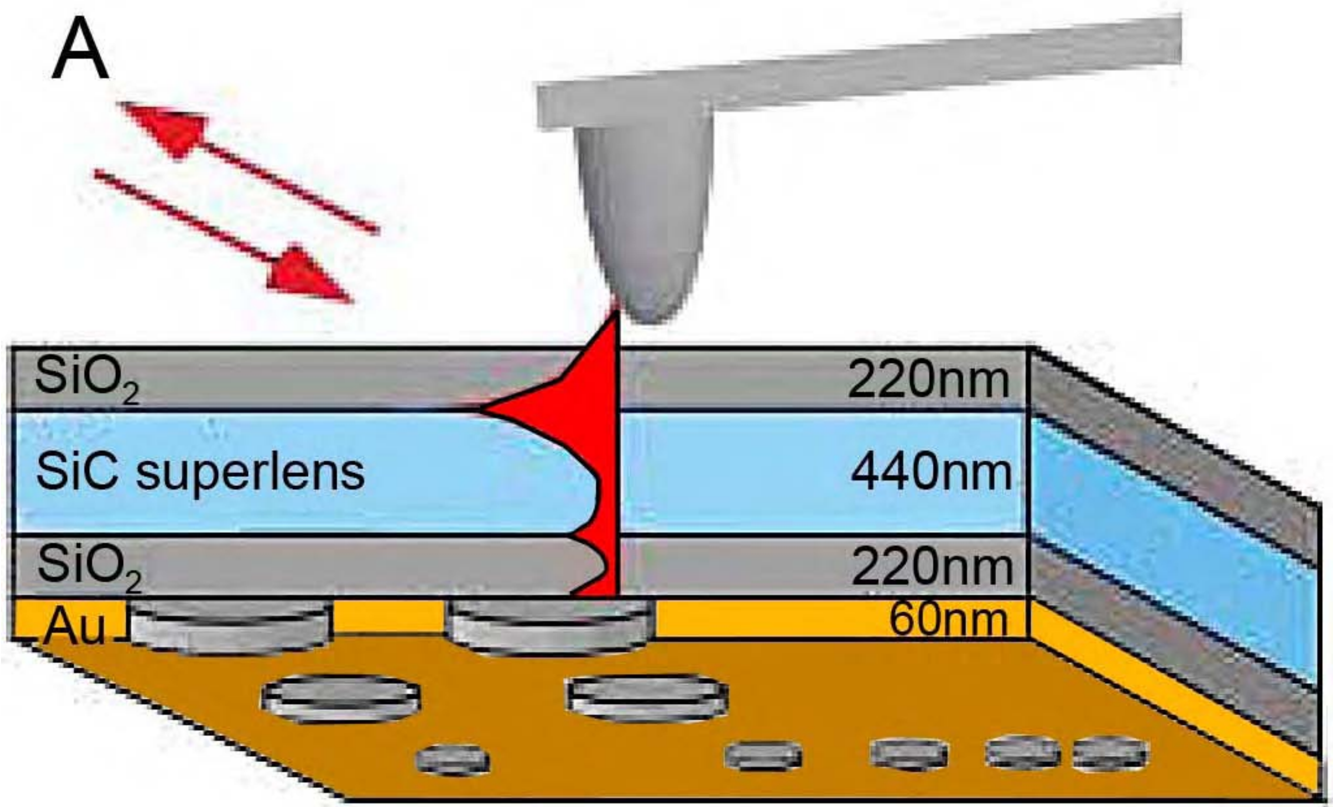
(C) AFM of the developed image on photoresist when the layer of silver was replaced by PMMA spacer as a control experiment.

(D) *blue line*: averaged cross section of letter ‘‘A’’ line width 89nm
red line: control experiment line width 321nm.

Near-Field Microscopy Through a SiC Superlens

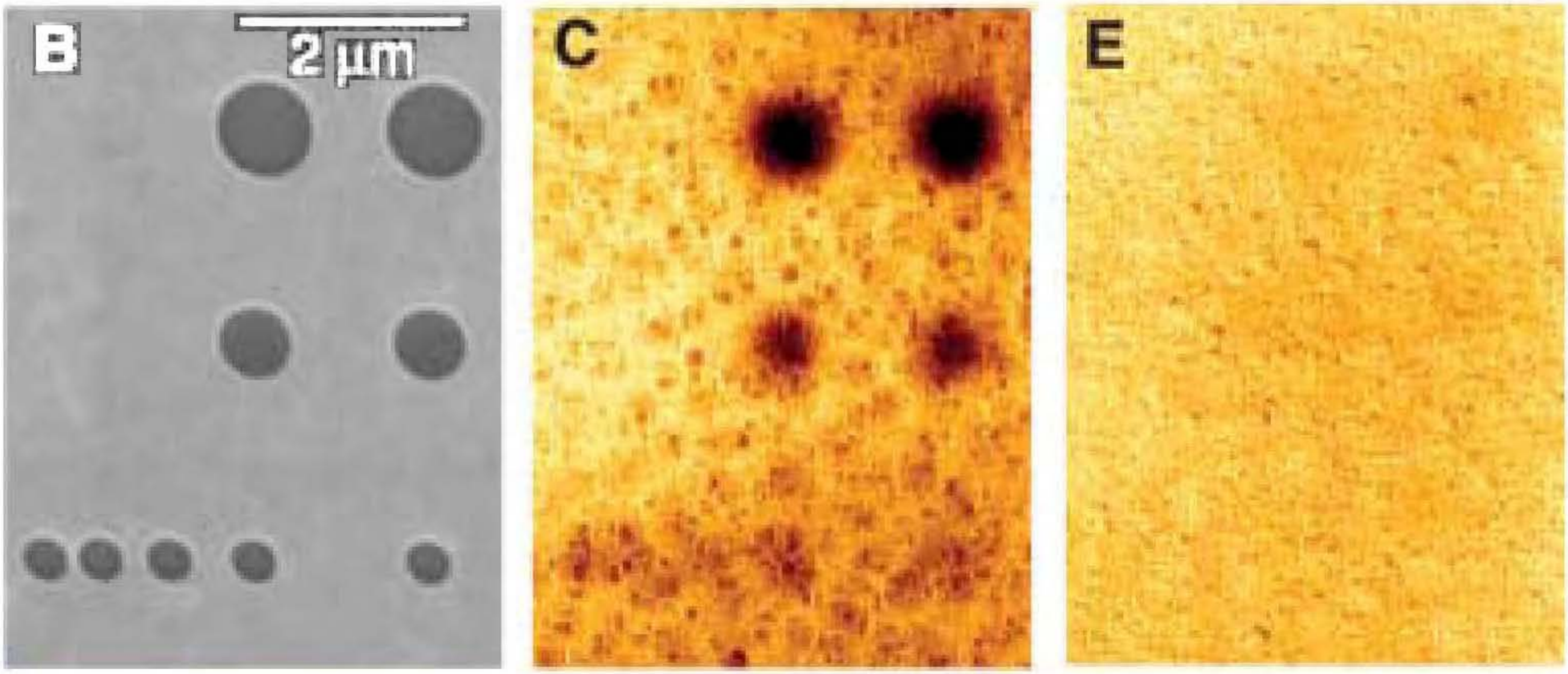
Science, **313** 1595 (2006)

Thomas Taubner, Dmitriy Korobkin, Yaroslav Urzhumov, Gennady Shvets, Rainer Hillenbrand



Near-field microscopy through a 880nm thick superlens structure: the superlens is a 440-nm-thick single-crystalline SiC membrane coated on both sides with 220-nm-thick SiO_2 layers. The two surfaces of the sandwich correspond to the object and the image planes of the lens, respectively. The object plane is covered by a Au film patterned with holes of different diameters

SiC Superlens: the Image



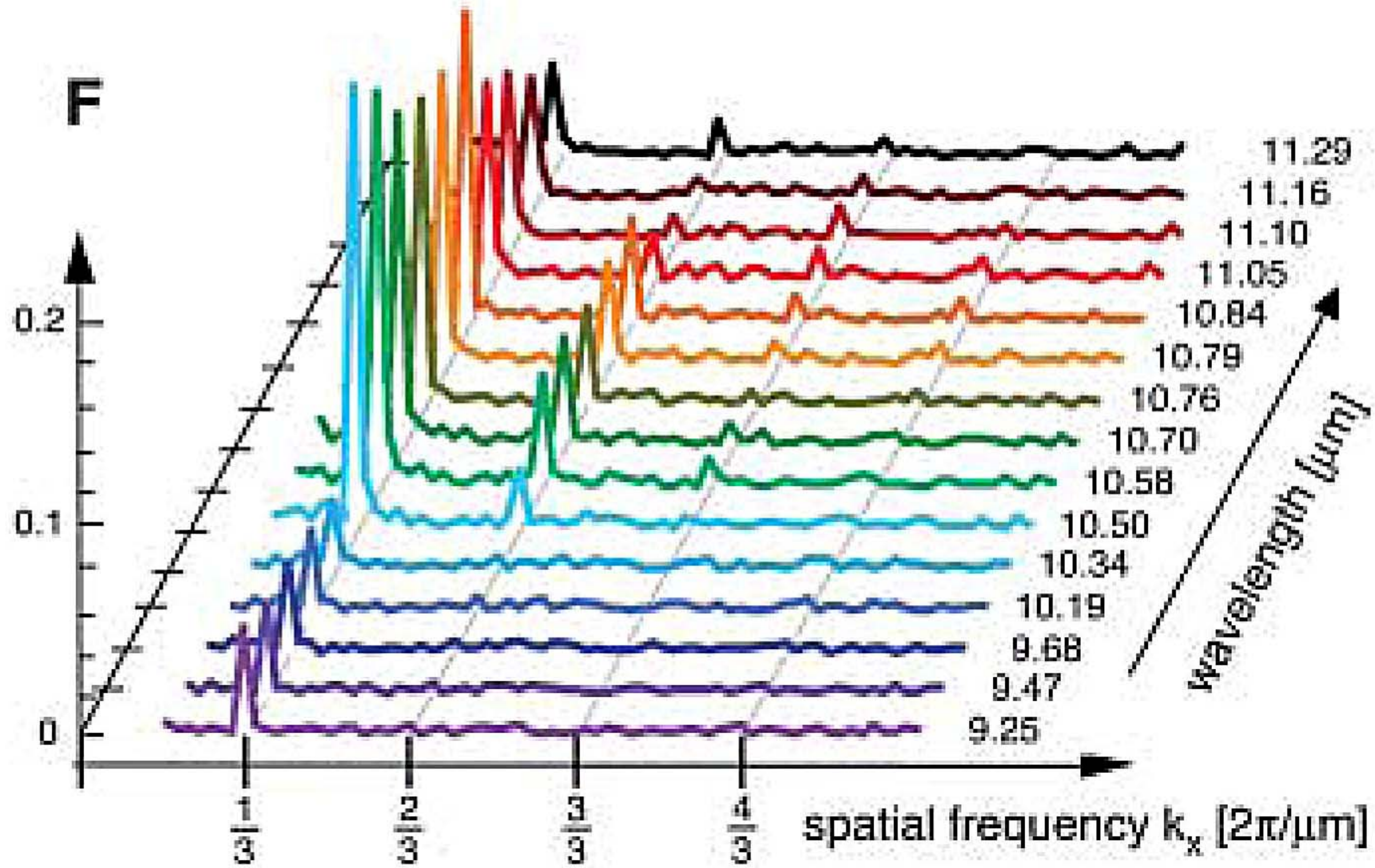
(B) Scanning electron microscope image of the object plane showing holes in a 60nm thick Au film.

(C) amplitude in the image plane at $\lambda = 10.85\mu$ where imaging is expected. NB the permittivity changes with frequency and hence imaging conditions are precisely met only at one frequency.

(E) Control image at $\lambda = 9.25\mu$ (no superlensing)

SiC Superlens:

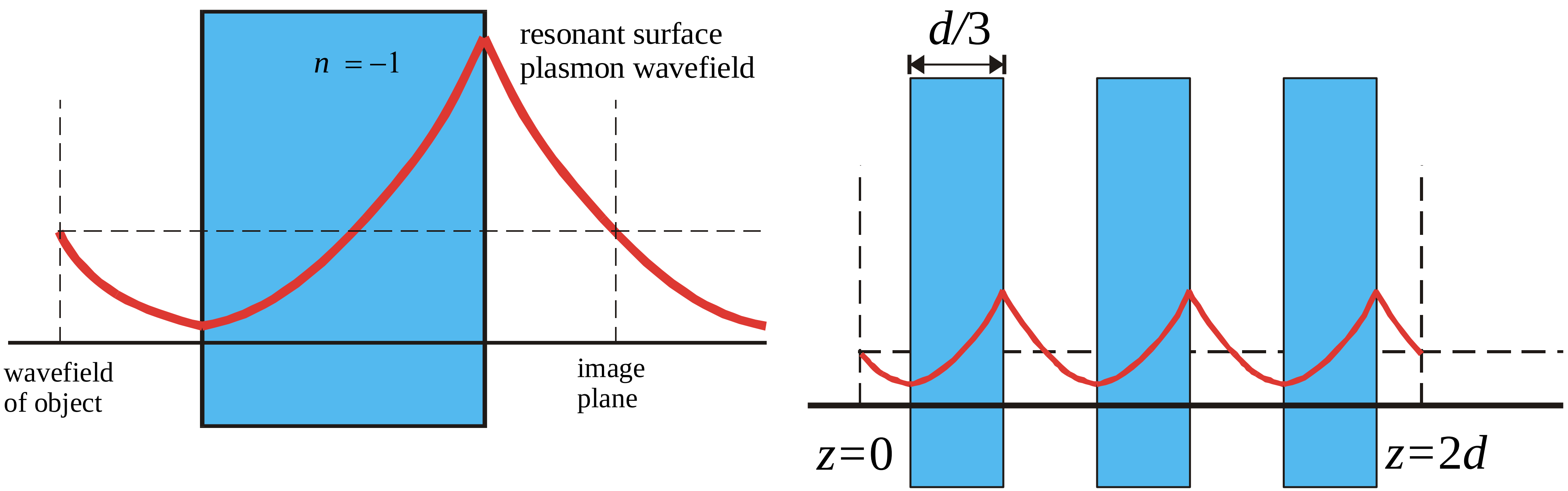
Fourier transforms of line scans taken from images of a grating, $\lambda \approx 3\mu$ period



High spatial frequencies, up to the grating's fourth harmonic, are imaged by the superlens around $\lambda \approx 10.84\mu$ where the SiC permittivity meets the superlensing condition.

Optimising Performance: the Layered Lens (1)

Absorption is a problem because of losses in the surface plasmon resonance. Cutting the lens into several mini lenses* reduces the maximum amplitude of the wave field and hence cuts the losses which in turn enhances the resolution.



* see also:

E. Shamonina, V.A. Kalinin, K.H. Ringhofer & L. Solymar, *Electron. Lett.* **37** 1243 (2001)

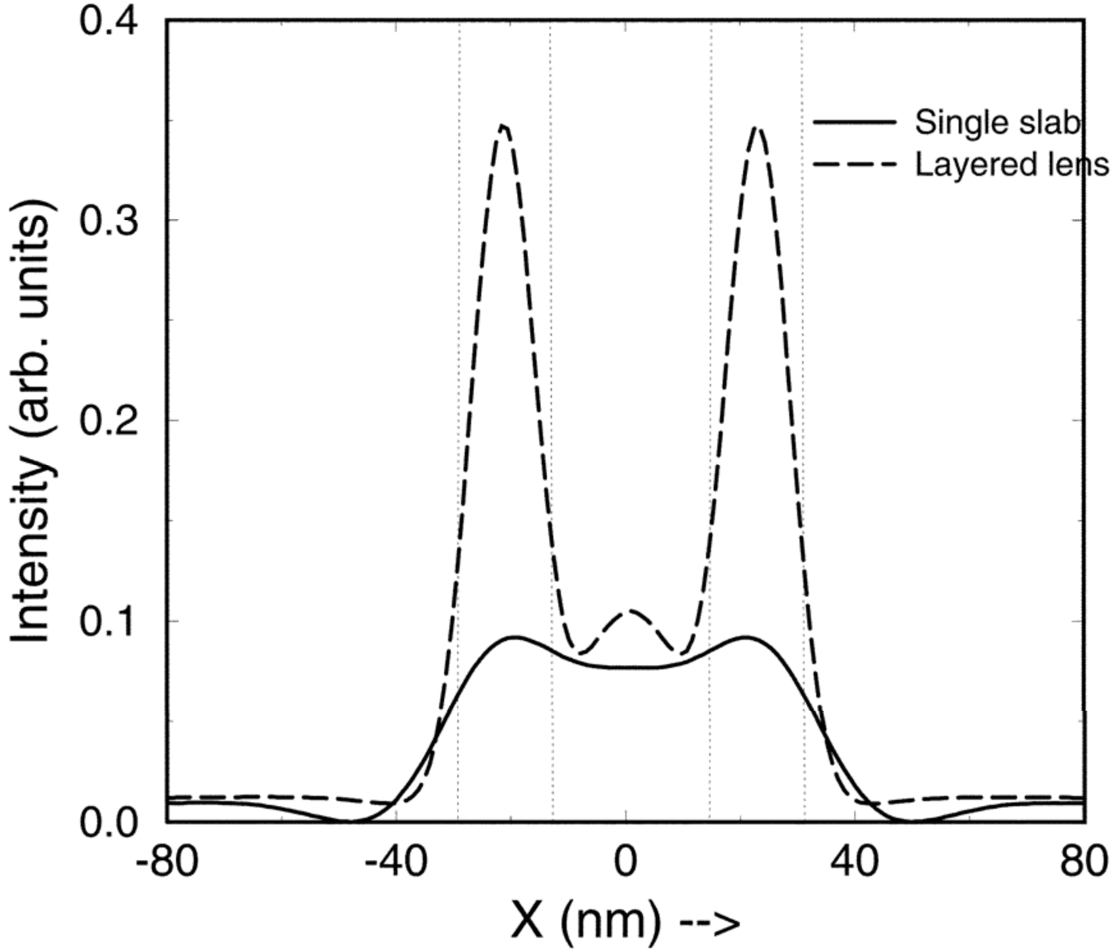
S. Anantha Ramakrishna and JB Pendry, *Phys. Rev.* **B67** 201101 (2003).

Optimising Performance: the Layered Lens (2)

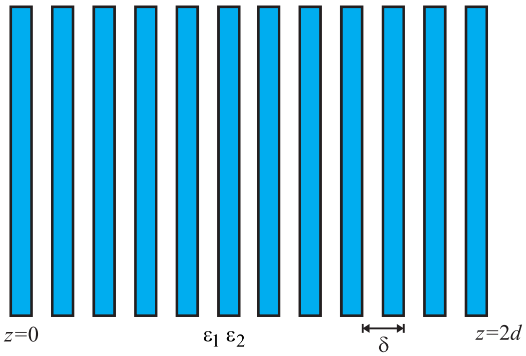
Reduced losses in the layered lens leads to enhanced resolution. The object comprises two slits of 5nm width and a peak-to-peak separation of 45 nm.

dashed curve: single slab of silver, $\varepsilon = -1 + 0.4i$, of thickness 40nm

full curve: layered stack comprising 8x5nm of silver (i.e. same total thickness).



Silver /dielectric layers as metamaterials



The near field optic fibre comprises alternate slices of positive and negative dielectric function material of equal thickness. This makes an effective medium. Averaging an electric field perpendicular to the layers gives an effective ε_z, and in the special case $\varepsilon_1 = +1, \quad \varepsilon_1 = -1,$

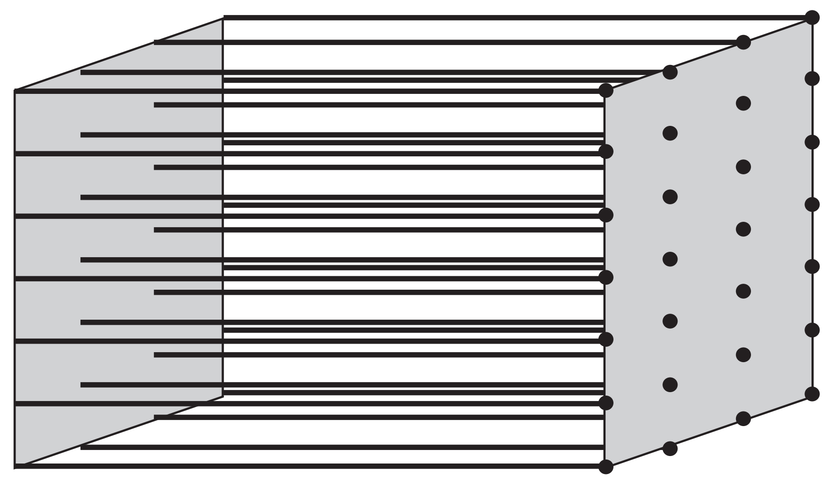
$$\varepsilon_z^{-1} = \tfrac{1}{2}\left(\varepsilon_1^{-1} + \varepsilon_2^{-1}\right) = \tfrac{1}{2}\left(\frac{1}{-1+i\delta} + 1\right) \approx -\tfrac{i}{2}\delta, \qquad \varepsilon_z \approx \infty$$

Averaging a displacement field parallel to the layers gives an effective ε_x,

$$\varepsilon_x = \tfrac{1}{2}\left(\varepsilon_1 + \varepsilon_2\right) = \tfrac{1}{2}\left(-1+i\delta+1\right) = \tfrac{i}{2}\delta \qquad \varepsilon_x \approx 0$$

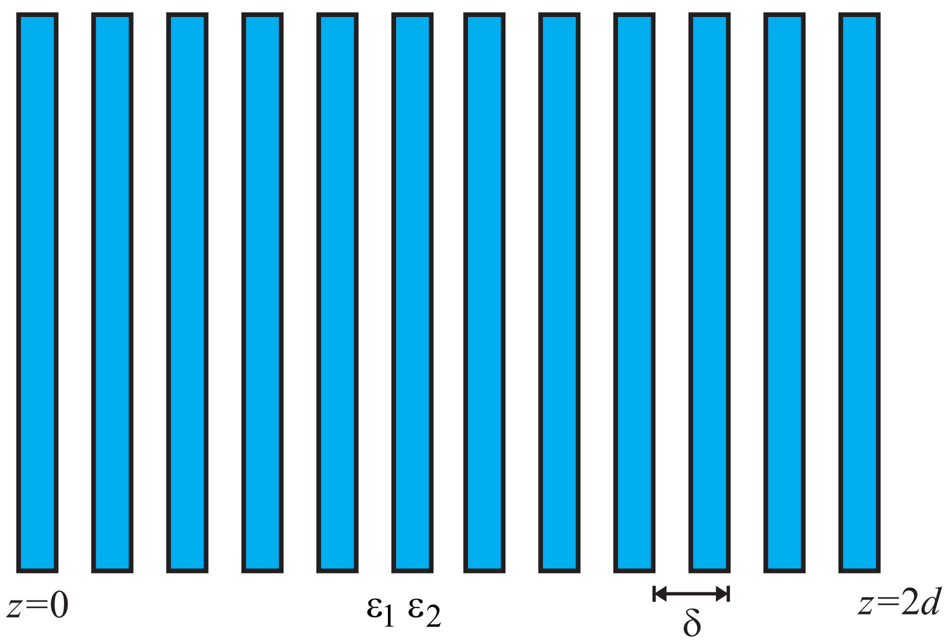
Therefore we have a *metamaterial* which resembles a set of infinitely fine, highly conducting wires aligned normal to the layers, separated by an almost perfect insulator.

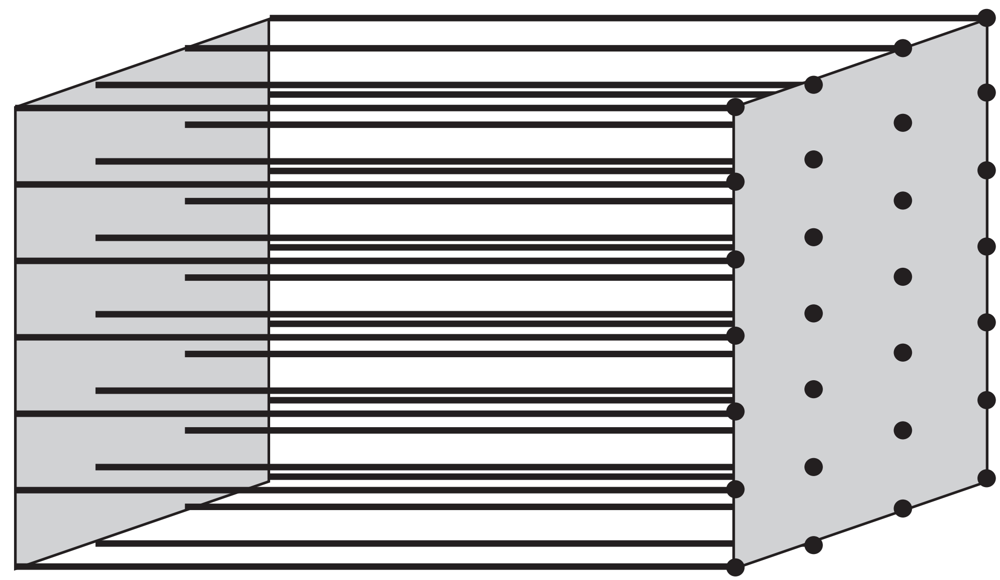
S. Anantha Ramakrishna and JB Pendry,
Phys. Rev. **B67** 201101 (2003).



Silver /dielectric layers as metamaterials

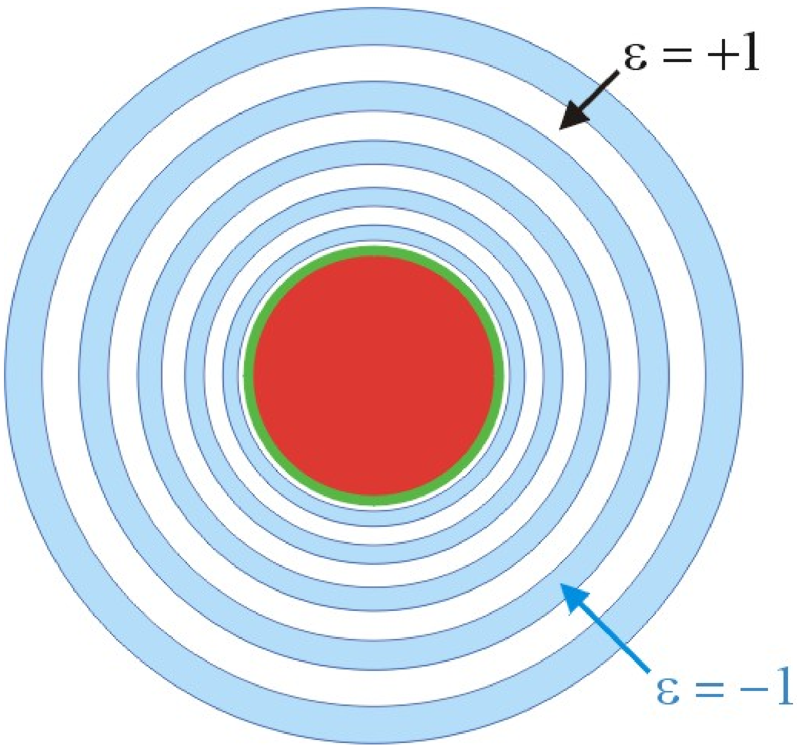
In the limit that the lens comprises many thin slices and $\varepsilon_1 = +1$, $\varepsilon_2 = -1$, a layered medium is effectively a fibre optic bundle with the unique capacity of guiding the near field. Electrical objects placed on one side of the layers are transmitted undistorted to the other side. The two sides are 'hard wired' together.



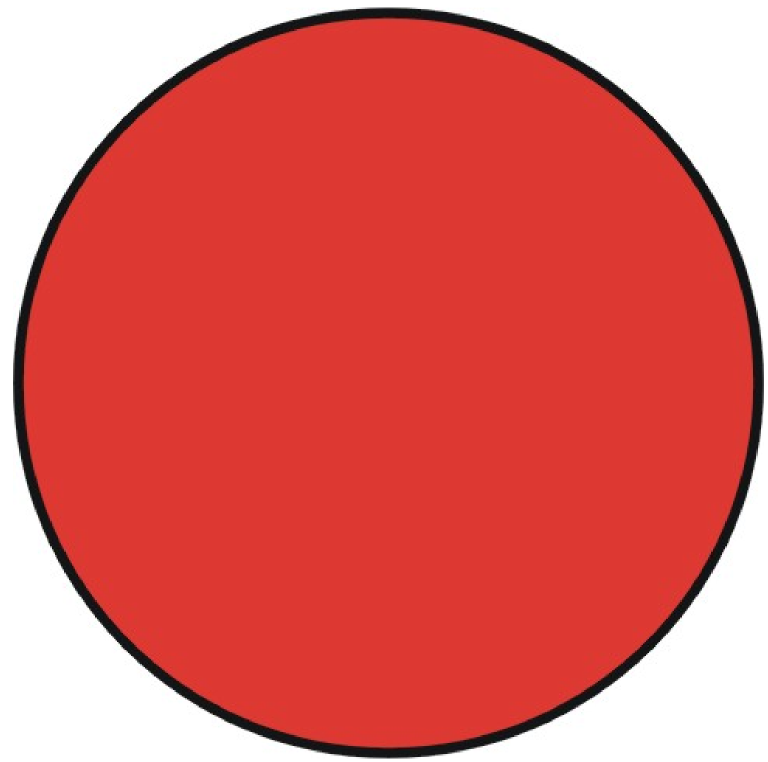


Spherical Layered Systems

Alternate flat layers of silver act like an endoscope, but the same is true of any curved surface. For example the contents of a small sphere can be magnified in this way.



looks like

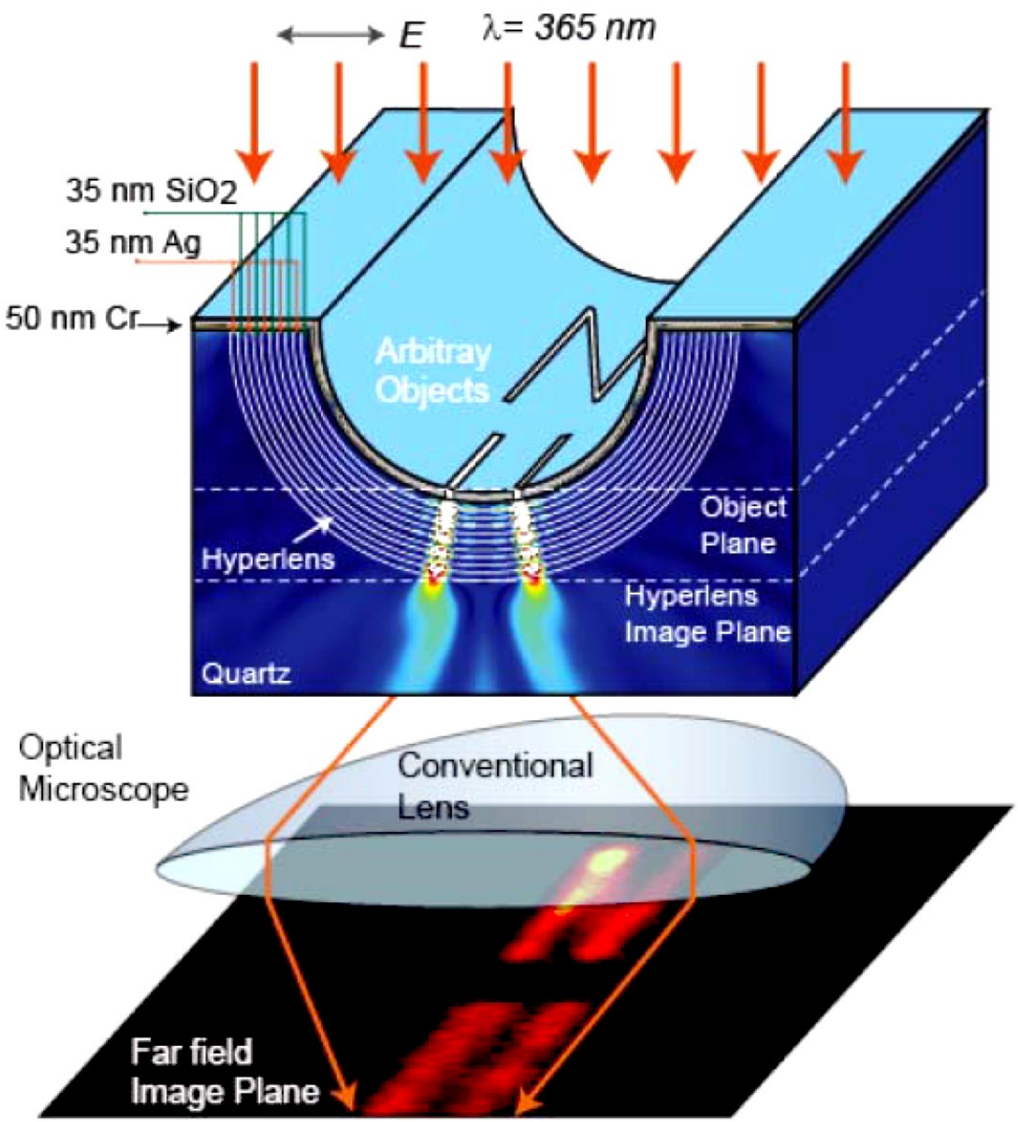


Theory: Z Jacob LV Alekseyev, E Narimanov *Optics Express* **14** 8247-8256 2006

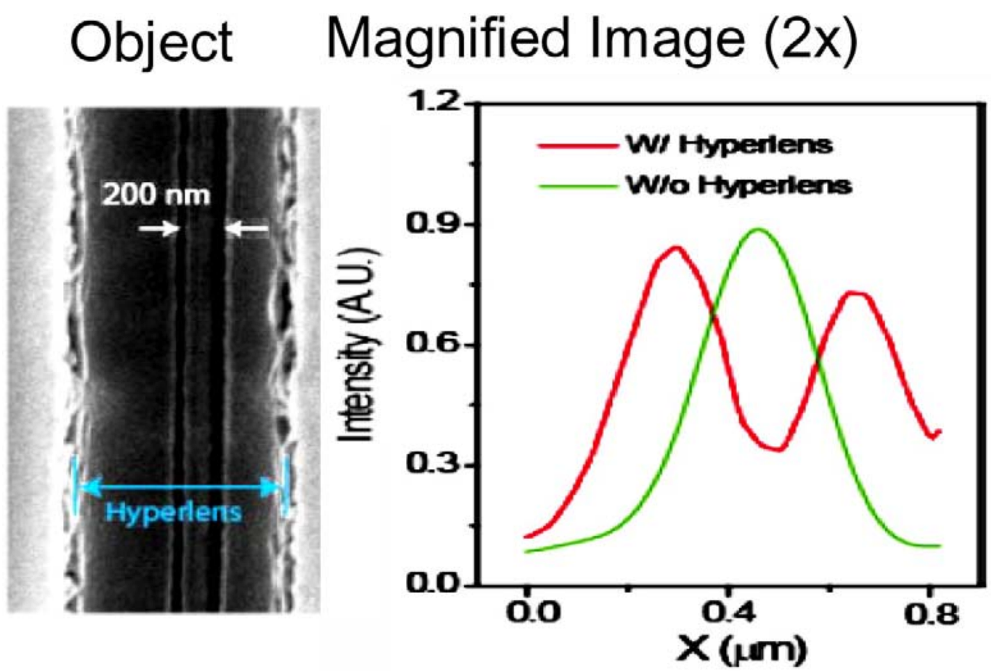
Experiment: II Smolyaninov, YJ Hung , and CC Davis, arxiv.org/pdf/physics/0610230

A magnifying optical hyperlens

Zhang Group at UC Berkeley (submitted)



Experimental schematic setup and numerical simulation for a hyperlens made of 16 layers of Ag/SiO2 imaging a line-pair object with **line width of 35 nm and spacing of 200 nm.** The diffraction limit is 260nm



Conclusions

- Negative refraction is a radical new concept in optics
- *Metamaterials* enable negative refraction to be achieved for the first time – it never occurs in natural materials
- Using negatively refracting materials it is possible to build a ‘perfect lens’ limited only by the quality of manufacture not by physical laws
- Version of this lens have been realised first in the microwave region and now at optical band THz frequencies

Metamaterials and Negative Refraction

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Some Reviews of Metamaterials

Not Just a Light Story
Nature Materials
5 755-64 (2006)

Negative Refraction
Contemporary Physics
45 191-202 (2004)

Metamaterials and Negative Refractive Index
Science
305 788-92 (2004)

Some Popular Articles

The Quest for the superlens
Scientific American
60- 67 July (2006).

Manipulating the near field with metamaterials
Optics & Photonics News
15 33-7 (2004)

Reversing Light with Negative Refraction
Physics Today
57 [6] 37-43 (June 2004)