Professor Walter Levin MIT

Lecture 21 Magnetic Materials

http://www.youtube.com/watch?v=SNDqAuxYOQ8
http://video.google.com/videoplay?docid=-2837189825550789110#
http://ocw.mit.edu/courses/physics/8-02-electricity-and-magnetism-spring-2002/video-lectures/lecture-21-magnetic-materials/

Lecture 22 Hysteresis and Electromagnets

http://www.youtube.com/watch?v=ddU6HBFlvEk
http://video.google.com/videoplay?docid=2266798550342930962#
http://ocw.mit.edu/courses/physics/8-02-electricity-and-magnetism-spring-2002/video-lectures/lecture-22-hysteresis-and-electromagnets/

Lecture 8 Polarization and Dielectrics

http://www.youtube.com/watch?v=E185G_JBd7U
http://video.google.com/videoplay?docid=-2012043130211460497#
http://ocw.mit.edu/courses/physics/8-02-electricity-and-magnetism-spring-2002/video-lectures/lecture-8-polarization-and-dielectrics/

$$K_M = 1 + X_M$$

Diamagnetic Materials $K_M < 1$

	X_M
Bi	-1.7×10^{-4}
Cu	-10^{-5}
H_2O	-10^{-5}
N_2 (1 atm)	-7×10^{-9}

Paramagnetic $K_M > 1$

	X_M	
Al	$+2 \times 10^{-5}$	$\sim 300K$
O_2 (1 atm)	$+2 \times 10^{-6}$	$\sim 300K$
O_2 (liquid)	$+3.5 \times 10^{-3}$	90 K

Ferromagnetism

$$X_M \simeq K_M$$
$$\simeq 10^2 \to 10^5 \ !$$

MIT OpenCourseWare
http://ocw.mit.edu

8.02 Electricity and Magnetism, Spring 2002
Transcript – Lecture 21

Yesterday we had 225 motors, and six of those motors went faster than 2000 RPM, which is a reasonable accomplishment.

And the elite is here.

These are the elite, the six highest.

The winner is, um, Jung Eun Lee, I talked to her on the phone last night.

If all goes well, she is here.

Are you here?

Where are you?

There you are.

Why don't you come up so that I can con- congratulate you in person.

I thought about the, the prize for a while, and I decided to give you something that is not particularly high tech, but come up here, give me a European kiss, and another one -- in Europe, we go three.

OK.

Um, the prize that I have for you is a thermometer which goes back to the days of Galileo Galilei -- come here.

Uh, it was designed in the early part of the, um, seventeenth century.

Uh, it doesn't, uh, require any knowledge of 8.02 to explain how it works.

If anything, you need 8.01.

It's not a digital thermometer.

But it's accurate to about 1 degree centigrade, and if you come here, you can tell, you look at these floaters, and the highest floater indicates the temperature.

It's now 72 degrees here.

And I suggest that you brush up on your knowledge of 8.01 so that perhaps next week you can explain to me how it works.

[laughter].

And of course tell your grandchildren about it.

You may want to leave it here.

It's very fragile.

Uh, there is also some package material here, so that you can take it home without breaking it.

So congratulations once more [applause] and of course -- [applause].

Terrific.

And you will join us for dinner on the thirteenth of April with the other five winners.

Thank you very much.

There are two other people who are very special who I want to mention.

And one is a person who is not enrolled in, uh, 8.02, but he did extremely well, and he was very generous.

He was not competing.

His name is Daniel Wendel.

His motor went 4900 RPM.

And then there was Tim Lo.

Is Tim Lo in the audience?

I hope he's going to be there at eleven o'clock.

Tim made a motor -- when I looked at it, I said to myself, it'll never run, but it's so beautiful.

It was so artistic that we introduced a new prize, a second prize, for the most artistic motor, and Tim Lo definitely is the one, by far the best, the most beautiful, the most terrific artistic design.

And so for him I bought a book on modern art -- what else can it be for someone who built such a beautiful motor?

It is here for those of you who want to see it later.

It's very hard to display it on television because it's so delicate.

It's like a birdcage that he built instead of having just -- looks like that it's a birdcage.

It's very nice.

The winning motor I have here, and I'm going to show you the winning motor, and I also want to teach you some, some physics by demonstrating the winning motor to you in a way that you may never have thought of.

So this is the winning motor.

And when we start this motor, the ohmic resistance of the current loop is extremely low.

So the moment that you connect it with your power supply, a very high current will run.

But the moment that the motor starts to rotate, you have a continuous magnetic flux change in these loops, and so now the system will fight itself, and it will immediately kill the current, which is another striking example of Faraday's Law.

I will show you the current of this motor when I block the rotor so that it cannot rotate.

It's about 1.6 amperes.

And you will see the moment that I run the motor that that current plunges by a huge amount.

Striking example of Faraday's Law.

So I now have to first show you this current, so here you see the 1.5 volts, and on the right side you see the current.

There is no current flowing now because the loop is hanging in such a way that the, that it makes no contact with the battery.

And I'm going to try to make it -- there it is.

Do you see the 1.6 amperes on the right?

The current is so high that due to the internal resistance of the power supply, the voltage also plunges.

But you saw the 1.6, right?

Now I'm going to run the motor.

See, the motor is running now, and now look at the current.

Current now, forty milliamperes, thirty milliamperes, fifty milliamperes.

It's forty times lower than when I blocked the rotor.

And so this is one of the reasons why when you have a, a motor, whichever motor it is, it could be just a drill, you try not to block it all of a sudden, because an enormous current will run, and it can actually damage the motors.

So you see here how the current goes down by a factor of forty between running and not running.

All right.

Electric fields can induce electric dipoles in materials, and in case that the, the molecules or the atoms themselves are permanent electric dipoles, an external electric field will make an attempt to align them.

We've discussed that in great detail before when we discussed dielectrics.

And the degree of success depends entirely on how strong the external electric field is and on the temperature.

If the temperature is low, you have very little thermal agitation, then it is easier to align those dipoles.

We have a similar situation with magnetic fields.

If I have an external magnetic field, this can induce in material magnetic dipoles.

And it, uh, induces magnetic dipoles at the atomic scale.

Now in case that the atoms or the molecules themselves have a permanent magnetic dipole moment, then this external field will make an attempt to align these dipoles, and the degree of success depends on the strength of the external field, and again on the temperature.

The lower the temperature, the easier it is to align them.

So the material modifies the external field.

This external field, today I will often call it the vacuum field.

So when you bring material into a vacuum field, the field changes.

The field inside is different from the external field, from the vacuum field.

I first want to remind you of our definition of a magnetic dipole moment.

It's actually very simple how it is defined.

If I have a current -- a loop, could be a rectangle, it doesn't have to be a circle -- and if the current is running in this direction, seen from below clockwise, and if this area is A, then the magnetic dipole moment is simply the current times the area A.

But we define A according to the, the vector A, according to the right-hand corkscrew rule.

If I come from below clockwise, then the vector A is perpendicular to the surface and is then pointing upwards.

And so the magnetic dipole moment, for which we normally write mu, is then also pointing upwards.

And so this is a vector A, which is this normal according to the right-hand corkscrew.

And if I have N of these loops, then the magnetic dipole moment will be N times larger.

Then they will support each other if they're all in the same direction.

I first want to discuss with you diamagnetism.

Diamagnetism.

All materials, when you expose them to an external magnetic field, will to some degree oppose that external field.

And they will generate, on an atomic scale, an EMF which is opposing the external field.

Now you will say, yes, of course, Lenz's Law.

Wrong.

It has nothing to do with Lenz's Law.

It has nothing to do with the free electrons in conductors which produce an eddy current when there is a changing magnetic field.

I'm not talking about a changing magnetic field, I'm talking about a permanent magnetic field.

So when I apply a permanent magnetic field, in all materials, a magnetic dipole moment is induced to oppose that field.

And there is no way that we can understand that with 8.02 It can only be understood with quantum mechanics.

So we'll make no attempts to do that, but we will accept it.

And so the magnetic field inside the material is always a little bit smaller than, than the external field, because the dipoles will oppose the external field.

Now I will talk about paramagnetism.

Paramagnetism.

There are many substances whereby the atoms and the molecules themselves have a magnetic dipole moment.

So the atoms themselves or the molecules, you can think of them as being little magnets.

If you have no external field, no vacuum field, then these dipoles are completely chaotically oriented, and so the net el- magnetic field is 0.

So they are not permanent magnets.

But the moment that you expose them to an external magnetic field, this magnetic field will try to align them.

And the degree of success depends on the strength of that field and on the temperature.

The lower the temperature, the easier it is.

And so if you had a magnetic field, say, like so -- this is your B field, this is your vacuum field -- and you bring in there paramagnetic material, then there is the tendency for the north pole to go a little bit in this direction.

And so these atomic magnets, then, would on average try to get the north pole a little bit in this direction.

Or, if I speak the language of magnetic dipole moments, then the magnetic dipole would try to go a little bit in this direction.

If you remove the external field of a paramagnetic material, immediately there is complete, total chaos, so there is no permanent magnetism left.

If you bring paramagnetic material in a non-uniform magnetic field, it will be pulled towards the strong side of the field.

And it is very easy to, to see how that works.

Suppose I have a magnet here, and let this be the north pole of the magnet and this the south pole.

And so the magnetic field is sort of like so.

Notice right here it's very non-uniform.

And I bring some paramagnetic material in there.

Let's say -- think of it as just one atom there.

It's not to scale, what I'm going to draw.

And here is that one atom, and this one atom now is paramagnetic, has its own magnetic dipole moment.

And this magnetic dipole moment, now, would like to align in this direction to support the field.

The field is trying to push it in that direction.

Let's suppose it is in this direction.

So if we look from above, the current then in this atom or in this molecule is running in this direction.

Seen from above, clockwise.

So that would be ideal alignment of this atom or this molecule in that external field.

This current loop will be attracted -- it wants to go towards the magnet.

Let's look at this point here.

That point, the current is going in the blackboard.

So here is that current I.

And the magnetic field is like so, the external magnetic field is like so.

So in what direction is the Lorentz force?

It's always in the direction I cross B.

And I cross B, I cross B is in this direction.

That's the direction of the Lorentz force.

So right here, there is a force on the loop in this direction.

So therefore right here, there is a force on the loop in this direction, on the current loop.

And so everywhere around this loop, there is a force that is pointing like this, and so there clearly is a net force up.

And so this metal wants to go towards the magnet.

Another way of looking at this is that this current loop is all by itself a little magnet, whereby the south pole is here and the north pole is there, because this is the direction of the magnetic dipole moment.

And the north pole attracts the south pole.

That's another way of looking at it.

That's the reason why magnets attract each other, why north and south pole attract each other, and why north and north poles repel each other.

That's exactly the reason.

It is the current that is flowing, it is the Lorentz force that causes the attraction or the repelling force.

So paramagnetic material is attracted by a magnet.

Essential is that this field is non-uniform.

And diamagnetic material, of course, will be repelled, will be pushed away from the strong field, because in paramagnetic -- in diamagnetic material, this current would be running in the opposite direction, because it opposes the external field whereas paramagnetism supports it.

We have a third form, and the third form of magnetism -- it's actually the most interesting -- is ferromagnetism.

In the case of ferromagnetism, we again have that the atoms have themselves permanent dipole moments.

But now, for very mysterious reasons which can only be understood with quantum mechanics, there are domains which have the dimensions of about 1/10 of a millimeter, maybe 3/10 of a millimeter, whereby the dipoles are hundred percent aligned.

And these dipoles, domains, which are in one direction, are uniformly distributed throughout the ferromagnetic material, and so there may not be any net magnetic field.

If I have here -- if I try to make a sketch of those domains, something like this, then perhaps here all these dipoles would all be hundred percent aligned in this direction, but for instance here, they will all be aligned in this direction.

And the number of atoms involved in such a domain is typically 10 to the 17, maybe up to 10 to the 21 atoms.

So if now I apply an external field, these domains will be forced to go in the direction of the magnetic field, and of course the degree of success depends on the strength of the external field, the strength of the vacuum field, and on the temperature.

The lower the temperature, the better it is, because then there is less thermal agitation, which of course adds a certain rando- randomness to the whole process.

So when I apply an external field, these domains as a whole can flip.

Inside the ferromagnetic material, the magnetic field can be thousands of times stronger than it is in the vacuum field.

And we will see some examples of that today.

If you remove the external field, in the case of paramagnetism, you have again complete chaos of the dipoles.

That's not necessarily the case with ferromagnetism.

Some of those domains may stay aligned in the direction that the external field was forcing them.

If you very carefully remove that external field, undoubtedly some domains will flip back, because of the temperature, there is always thermal agitation.

Some may remain oriented, and therefore the material, once it has been exposed to an external magnetic field, may have become permanently magnetic.

And the only way you can remove that permanent magnetism could be to bang on it with a hammer, and then of course these domains will then get very nervous, and then they will randomize themselves.

Or you can heat them up, and then you can also undo the orientation of the domains.

The domains themselves will remain, but then they average out not to produce any permanent magnetic field.

So for the same reason that paramagnetism is pulled towards the strong field, in case that we have a non-uniform magnetic field, ferromagnetism of course will also be pulled towards the strong field, except in the case of ferromagnetism, the forces with which ferromagnetic material is pulled towards the magnet, way larger than in case of paramagnetic material.

If I take a paperclip -- you can do that at home, you can hang a paperclip on the south pole of your magnet or the north pole of your magnet -- you all have gotten magnets in your motor kit, so you can try that at home.

Take a paperclip, hang it on the magnet.

Doesn't matter on which side you hang it, because ferromagnetic material is always pulled towards the strong field.

If you hang a few of those paperclips on there and you very carefully and slowly remove them -- don't hit them with a hammer yet -- you may actually notice that after you remove them that the paperclips themselves have become magnetic.

You can actually try to hang them on each other, make a little chain.

But drop them on the floor a few times and that magendas- magnetism will go away.

So what you have witnessed then is that some of those domains remained aligned due to your external field.

With paramagnetism, there is no way that you can hang paramagnetic material under most circumstances on a magnet.

There is one exception.

I will show you the exception later today.

And the reason is that the forces involved with paramagnetic material in general are only a few percent of the weight of the material itself.

So if you take a piece of aluminum and you have a magnet, aluminum will not stick to a magnet.

There is a force.

Aluminum will be attracted by the magnet, but the force is way smaller than the weight of the aluminum, so it won't be able to pick it up, unlike ferromagnetic material, which you can pick up with a magnet.

So what I could demonstrate to you, for one thing, I could take a bar magnet and show you that paperclips are hanging on this.

I could also show you that aluminum is not hanging on this.

But you won't find that very exciting.

And therefore I decided on a different demonstration, whereby my goal is to show you that ferromagnetic material is pulled with huge forces towards the strong magnetic field, provided that I have a magnetic field which is non-uniform.

And the way I will do that is with this piece of ferromagnetic material.

And this piece of ferromagnetic material is actually quite heavy.

And you are going to tell the class how heavy it is.

Be very careful.

What do you think?

Wow! Good for you! [laughter].

Do it again! Sounds go-- looks great.

[laughter].

It's fifteen kilograms.

Fifteen kilograms of ferromagnetic material.

It is not a permanent magnet.

There may be a little bit of permanent magnetism left, of course, because once you have exposed it to an external field, yes, there may be some permanent magnetism left.

So now I'm going to hold this -- let's first make sure that nothing happens to Galileo's thermometer.

So we're going to put this here.

See what the temperature is -- oh man, it's going up.

I must be sweating here.

74 degrees, yeah, 74 degrees now.

OK, so here is my magnet, 0:21:30.278 gau producing about 320 gauss.

But what counts is that the magnetic field is non-uniform here and also here.

And so I am going to turn on the magnet -- I believe I have to push a button here.

And the first thing I will do is now power this magnet.

So this is a solenoid.

I put my hand in here, my hand is paramagnetic, it's not being sucked in.

Really it isn't.

I feel nothing.

The force is -- I can't even feel anything.

But I'm not ferromagnetic, thank goodness.

Now this one.

Whssht, fifteen kilograms, just sucked in like that.

And I'm very lucky that when it's overshoots here that it wants to go back, because it always wants to go to the strongest field.

Doesn't matter whether you have it here or there.

The reason why that's lucky, because if that were not the case, this fifteen kilogram bar would go like a bullet coming out of here.

So the one thing you don't want to do when it goes in there, you don't want to break the current, because then it would come out as a bullet.

And I'm not going to do that, believe me.

But I want to show you that -- there it goes.

It's amazing, ferromagnetic material.

Aagh.

OK.

So ferromagnetic material, there's enormous force.

If you have a s- a field that is -- has a strong gradient, that is very non-uniform, is sucked, pulled towards the strong side.

That's why it hangs on magnets.

That's the basic idea.

I have another demonstration.

And another demonstration is to make you sort of see in a non-kosher way magnetic domains.

But I will tell you why it's non-kosher.

I have here an array of eight by eight magnetic needles, compass needles.

And you're going to see them there.

And I will change the situation so that you have better light.

And when I have an external magnetic field and I march over here a little, and I just let it go, and wait, you will see areas whereby these magnetic needles point in the same direction and you will see areas where they point in a different direction.

We'll just give it some chance.

And so that may make you think that this is the way that domains are formed in ferromagnetic material.

Oh, in fact we have now a situation that almost all are aligned in this direction, and there's only a group here that is pointing in this direction.

I can change that, of course, by changing the magnetic field.

Why is this not really a kosher demonstration to convince you that domains exist?

First of all, there is no thermal agitation, whereas in ferromagnetic material there is thermal agitation.

Some may be oriented like this and others like that, where here you only have two preferred directions.

You don't need quantum mechanics for that, simply a matter of minimum energy considerations.

And so they either are pointed like this or they are pointed like that, and so already that shows you that it's very different from ferromagnetism.

But the reason why we show it to you is it still gives you an interesting idea of the fact that you can have various orientations and that they come in groups.

That the groups stick together and are not all in the same direction.

But as I said, it is not really a good way to explain to you why there are domains in ferromagnetic material.

Ah, now you see again, you have some nicely aligned here and others are in very different direction here.

So the basic idea is there.

It's a nice demonstration, but it shows you something that really is not related to ferromagnetism.

The demonstration that is one of my favorites, one of my absolute favorites, is one whereby I can make you listen to the flip-over of these domains.

I have ferromagnetic material inside a coil.

I have here a coil and I'm going to put ferromagnetic material in here.

And I have here a loudspeaker -- an amplifier as well, call this an amplifier.

And this is a loudspeaker.

Let's first assume there is no ferromagnetic material in there.

That's the way I will start the demonstration.

And I approach this with a magnet, and I go very fast.

Whssht, what will happen?

Faraday will say, oh, there's a magnetic flux change, and there will be an EMF in this coil.

That means there will be a current in this coil, induced current.

And it will be amplified and you will hear some sissing noise.

And you will hear that.

If, however, I come in very slowly, you won't hear anything, because d phi/dt is then so low, because the time scale of my motion is so large, that you won't hear any current.

The induced current is insignificantly small.

Because remember the induced current is proportional to the induced EMF, and the induced EMF is proportional to the time change of the magnetic flux.

So I can make that flux change very, very small if I bring it in very slowly.

Now I will put in the ferromagnetic material, and I will approach it again very slowly.

And now, there comes a time that some of those domains go *cluk*, *cluk*.

But when the domains flip over, there is a magnetic flux change inside the material, and so the magnetic flux change means d phi/dt, and it's on an extremely short time scale.

And so now you get an EMF, you get a current going through the wire, and you hear a cracking noise over the loudspeaker.

And for every group of domains that flip, you can hear that.

And that's an amazing thing when you think about it, that some 10 to the 20 atoms go clunk and that you can hear that.

And so this is what we're going to do here, and I will do it then in, in several steps, so that you first can hear the noise if I don't have ferromagnetic material, and then -- so here is the, here's the coil.

This is a very small coil.

And here is a magnet.

And I'll come very fast towards the coil.

What you heard now is Faraday's Law.

You simply have a magnetic flux change in the coil -- oh, I shouldn't touch it.

Now I come in very slowly, and go away very slowly.

You hear nothing.

d phi/dt is just too low.

Now I put in the ferromagnetic material.

Put it inside the coil.

And now I approach it again, very slowly.

There they go.

You hear them?

Those are, those are domains that go.

I'll come in with the other side.

There it goes, the domains.

Isn't that amazing?

You hear atoms switch, groups of atoms.

I'll turn it over again.

Now they flip back.

They don't like it that that's their problem.

This is called the Barkhausen effect.

I find it truly amazing that you hear groups of atoms, 10 to the 20 atoms at the time, they flip over, and when they do, there is a magnetic flux change inside the ferromagnetic material, is sensed by the coil, and you hear a current.

And if I do it fast, uh, these, these, these, these domains go haywire.

They go nuts now.

Imagine that you were a domain and I would treat you that way.

You'd go *cluk*, *cluk*, *cluk*, *cluk*, *cluk*.

But the fact that you can hear it is absolutely amazing, isn't it.

So that's actually a nice way of demonstrating that these domains exist.

If you did that with faramagnetic- paramagnetic material, you wouldn't hear that.

So in all cases, whether we have diamagnetic material or paramagnetic material or ferromagnetic material, uh, the magnetic field inside is different from what the field would be without the material.

And what the field would be without the material we've called external field.

I've called it vacuum field.

And in many cases, but not all -- next lecture I will discuss the issues of not all -- in many cases but not all cases, is the field inside the material proportional to the vacuum field.

And if that is the case, then you can write down that the field inside is linearly proportional -- so this is the field inside the material, regardless of whether it's diamagnetic or paramagnetic or ferromagnetic, is proportional to the vacuum field.

I will write down vacuum for this.

And this proportionality constant I call kappa of M.

I -- our book calls it K of M.

And it's called the relative permeability.

And so now we can look at these values for the relative permeability and we can immediately understand now the difference between diamagnetic material, paramagnetic material, and ferromagnetic material.

Since in the case of diamagnetic material and paramagnetic material, the B field inside is only slightly different from the vacuum field, it is common to express kappa of M in terms of 1 + something which we call the magnetic susceptibility, which is chi of M.

Because if it is very close to one, then it is easier to simply list chi of M.

And let's look at diamagnetic material.

Notice that these values for chi of M are all negative -- of course, they have to be negative, otherwise it wouldn't be diamagnetic.

It means that the field inside is slightly, a hair smaller than the vacuum field, because these induced dipoles oppose the external field, remember.

It has nothing to do with Lenz's Law, but they oppose it nevertheless.

And so you express it in terms of the, um, magnetic susceptibility, and so you have to take 1 - 1.7 times 10 to the -4 to get kappa of M, which is very close to one.

If now you go to paramagnetic materials, the minus signs become plus.

Again, the numbers are small.

But the fact that it is plus means that inside paramagnetic material, the magnetic field is a little, a hair larger than the vacuum field.

But now if you go to ferromagnetic material, it is really absurd to ever list the value for chi of M, because chi of M is so large that you can forget about the one, and so chi of M is about the same as kappa of M.

And so you deal there with numbers that are 100, 1000, 10000, and even larger than 10000.

That means that if kappa of M is 10000, you would have a field inside ferromagnetic material that is 10000 times larger than your vacuum field.

Next lecture I will tell you that there is a limit to as far as you can go, but for now we will, we will leave it with this.

So paramagnetic and ferromagnetic properties depend on the temperature.

Diamagnetic properties do not depend on the temperature.

So at very low temperatures, there is very little thermal agitation, and so you can then easier align these dipoles, and so the values for kappa of M will then be different.

For ferromagnetic material, if you cool it, you expect the kappa of M going up, so you got a stronger field inside.

So it's temperature-dependent.

If you make the material very hot, then it can lose completely its ferromagnetic properties.

What happens at a certain temperature, that these dosmain- domains fall apart, so the domains themselves no longer exist.

They annihilate.

And that happens at a very precise temperature.

It's very strange.

That's also something that is very difficult to understand, and you need quantum mechanics for that too.

But at a certain temperature, which we call the Curie temperature, which for iron is 1043 degrees Kelvin, which is 770 degrees centigrade, all of a sudden the domains disappear and the material becomes paramagnetic.

In other words, if ferromagnetic material would be hanging on a magnet and you would heat it up above the Curie point, it would fall off.

It would become paramagnetic, but paramagnetic material in general doesn't hang on a magnet, because the forces involved are quite small.

And the change is very abrupt, and I am going to show that to you with a demonstration.

I have a ferromagnetic nut.

It's right there.

You will see it very shortly.

And this nut, or washer, hanging on a steel cable, and there is here a magnet.

I don't know whether this is north or south.

It doesn't matter.

And here we have a thermal shield.

And so this washer is against the thermal shield, because it's being attracted.

It wants to go towards the strong magnetic field.

It's ferromagnetic.

So it will be sitting here.

And now I'm going to heat this up above the Curie point, 770 degrees centigrade, and you will see it fall off.

And when it cools again, it goes back on again.

So I can make you see ferromagnetic properties disappear.

And let me make sure I have the proper settings.

I see nothing.

I see nothing.

But there it is.

So here is this nut, and here is this shield, and the magnet is behind it, you can't see it, but it's right there.

And so it goes against it, right, it goes just towards the magnetic poles.

It goes into the strong magnetic field.

The magnetic field is non-uniform outside a magnet, and it goes towards it.

And so now I'm going to heat it.

It will take a while, because, um, 770 degrees centigrade is not so easy to achieve.

The three most common ferromagnetic materials are cobalt, nickel, and iron.

Nickel has a Curie point of only 358 degrees centigrade, so if this were nickel -- ooh.

If this were nickel -- uh-uh.

[laughter].

Oh, you like that, huh.

I think I need strong hands.

A strong hand is coming.

OK.

I think I fixed it.

I'm a big boy, I did it myself today.

I lost my pen, but that's a detail.

OK, let's try again.

So I'm going to heat it up, and I was mentioning that, um, nickel has a Curie point of 358 degrees centigrade.

So that's quite low.

This is 770.

Cobalt is 1400 degrees Kelvin Curie point.

Gadolinium is a very special material.

Gadolinium is ferromagnetic in the winter, when the temperature is below 16 degrees centigrade, but it is paramagnetic in the summer, when the temperature is above 16 degrees centigrade.

It's beginning to be red-hot now.

770 degrees centigrade, you expect some visible light in the form of red light -- there it goes.

And I will keep it heating, I will keep the torch on it, so that you can see that indeed it's no longer attracted by the magnet.

And the moment that I stop heating it, it will very quickly cool.

It will become ferromagnetic again, and it will go back.

Just watch it.

There it goes.

So now it's again ferromagnetic.

So the transition is extremely sharp.

All right.

Uh, OK.

So paramagnetic materials, as I mentioned several times, in general cannot hang on a magnet.

The attractive force is -- there's not enough.

To hang on a magnet, the force has to be larger than its own weight.

And diamagn- diamagnetic materials is of course completely out because diamagnetic materials are always pushed towards the weak part of the field.

It's only paramagnetic materials and ferromagnetic materials that experience a force towards the strong part of the field if the field itself is non-uniform.

Now there is one very interesting exception.

And I want to draw your attention to this, um, transparency here.

Look here at oxygen at one atmosphere.

Oxygen at one atmosphere and 300 degrees Kelvin has a value for chi of M which is 2 times 10 to the -6.

But now look at liquid oxygen at 90 degrees Kelvin.

That value is 1800 times larger than this value.

Why is that so much higher?

Well, liquid, in general, is about thousand times denser than gas at one atmosphere.

So you have thousand times more dipoles per cubic meter that in principle can align.

And so clearly you expect an immediate one-to-one correspondence between the density, how many dipoles you have per cubic meter, and the value for kappa M -- for chi of M.

And so you see indeed that this value is substantially larger.

The reason why it is more than a factor of thousand higher is that the temperature is also lower.

You go from 300 degrees to 90 degrees, and that gives you another factor of two, because when the temperature is lower, there is less thermal agitation, and so the external field can align the dipoles more easily.

And so that's why you end up with a factor of 1800.

Even though this value for chi of M is extraordinarily high for a paramagnetic material, notice that the field inside would only be 0.35% higher than the vacuum field, because if chi of M is 3.5 times 10 to the -3, that means that the field inside is only 0.35% higher than the vacuum field.

But that is enough for liquid oxygen to be attracted by a very strong magnet, provided that it also has a very non-uniform field outside the magnet.

And so the force with which liquid oxygen is pulled towards a magnet can be made larger than the weight of the liquid oxygen.

And so I can make you see today that I can have liquid oxygen hanging from a magnet.

And that's what we are going to do here.

Make sure I have the right setting.

Ah, this is it.

Now we're going to have some changes in the lights.

So there you see the two magnetic poles.

It's a electromagnet.

And so we can turn the magnetic field on at will.

So here are the poles of the magnet.

And the first thing I will do is very boring.

I will throw some, uh, liquid nitrogen between the poles.

Now I don't have the value for liquid nitrogen there, but nitrogen is diamagnetic, so it's not even an issue.

Diamagnetic material is pushed away from the strong field.

So even though the value for chi of M will be very different for liquid nitrogen than it is for gaseous nitrogen, it doesn't matter.

So certainly it will be pushed out.

So that's the first thing I want to do, just to bore you a little bit.

Because I have to keep you on the edge of your seat before you're going to see this oxygen, which will be hanging in there.

So let's first power this magnet -- I hope I did that -- yes, I think I did.

And here comes the liquid nitrogen.

Boring like hell, just falls through.

Now comes the oxygen.

Liquid oxygen.

It's hanging in there.

I challenge you, you've never in your life seen liquid hanging on a magnet.

You can tell your parents about it -- and of course your grandchildren.

It's hanging there.

I'll put some more in -- make sure I have the right stuff, yeah.

Put some more in.

There is liquid oxygen.

When I break the current, it's no longer a magnet, it will fall of course.

Don't worry, you'll get more.

Who has ever in his life seen a liquid hang on a magnet?

It's paramagnetic, it's not ferromagnetic, but because the density is so high and because it's so cold, the value for chi of M is high enough that the force on it is larger than its own weight.

If you do this with aluminum, not a chance in the world.

Aluminum will not hang in there, even though aluminum, as you can see there, is paramagnetic.

But the value 2 times 10 to the -5 is way too small, and it will not stick to a magnet.

OK.

You have something to think about.

I will see you Friday.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{free}}{\epsilon_o \kappa}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \kappa_m (I + \epsilon_o \kappa \frac{d\phi_E}{dt})$$

MIT OpenCourseWare
http://ocw.mit.edu

8.02 Electricity and Magnetism, Spring 2002
Transcript – Lecture 22

When I expose material to an external magnetic field, then we learned last time that the field inside that material is modified.

And we expressed that in terms of an equation, that the field inside the material is kappa of M, which is called the relative permeability, times the external field, and I will refer to that all the time as the vacuum field.

And when we have diamagnetic material, kappa M is just a hair smaller than one; with paramagnetic material, it is a hair larger than one; but when we have ferromagnetic material it can be huge.

It can be thousands, 10 thousands, and even higher.

Now in the case of para- and ferromagnetic material, the kappa of M is the result of the fact that the intrinsic dipoles of the atoms and the molecules are going to be aligned by the external field.

And today I want to raise the question, how large can the magnetic dipole moment of a single atom be?

And then comes the logical question, how strong can we actually, then, have a field inside ferromagnetic material?

That means if we were able to align all the dipole moments of all the atoms, what is the maximum that we can achieve?

To calculate the magnetic dipole moment of an atom, you have to do some quantum mechanics, and that's beyond the scope of this course.

And so I will derive it in a classical way, and then at the very end I will add a little pepper and salt, which is quantum mechanics, just to make the result right.

But it can be done in a classical way, and it can give you a very good, good idea.

If I have a hydrogen atom, which has a proton at the center, has a charge plus e -- e is the charge of the electron but this is the plus charge.

And let this have an orbit R, circular orbit.

And the electron here e, I'll give it the minus sign to make sure that you know that it's negative.

Say the electron goes around in this direction.

This is the velocity of the electron.

That means that of course the current around the proton would then be in this direction.

If an electron goes like this, the current goes like that, that's just by convention.

The mass of an electron -- you should know that by now -- is approximately 9.1 times 10 to the -31 kilograms.

The charge of the electron, 1.6 times 10 to the -19 coulombs.

And the radius of the orbit in a hydrogen atom -- it's often called the Bohr radius, by the way -- is approximately 5 times 10 to the -11 meters.

We're going to need these numbers.

That's why I write them down for you.

If you look at this current running around the proton, it's really a current which the current, say, goes in this direction.

And here is that proton -- trying to make you see three dimensionally.

Then it creates a magnetic field in this direction, and so the magnetic dipole moment mu is up.

And the magnitude of that magnetic dipole moment, as we learned last time, is simply this current I times the area A of this current loop.

Now the area A is trivial to calculate.

That's pi R square, R being the radius of the orbit.

And so A, that's the easiest, is pi R squared.

And if I use my 5 times 10 to the minus 11, then I find that this area is 8 times 10 to the -21 square meters.

So that's easy.

But now comes the question, what is I?

What is the current?

So now we have to do a little bit more work.

And we have to combine our knowledge of, uh 8.02 with our knowledge of 8.01.

If this electron goes around, the reason why it goes around is that the proton and the electron attract each other.

And so there is a force in this direction.

And we know that force, that's the Coulomb force.

It's an electric force.

That force is this charge times this charge, so that's e squared, divided by our famous 4 pi epsilon 0, and then we have to divide it by the radius squared.

So that's Coulomb's Law.

But from 8.01, from Newtonian mechanics, we know that this is what we call the centripetal force that holds it in orbit, so to speak, and that is M V squared, M being the mass of the electron, V being the speed of the electron, and V squared divided by R.

And so this allows me to calculate as a first step, before we get into the current, what the velocity of this electron is.

It's phenomenal.

It's an incredible speed.

So V then becomes -- I lose one R -- so I get the square root, I get an e squared upstairs here, my M goes downstairs, I have 4 pi epsilon 0, and I have here an R.

And I know all these numbers.

I know what e is, I know what capital R is, I know what 4 pi epsilon 0 is -- one over 4 pi epsilon 0 is the famous 9 to the power -- 9 times 10 to the power 9.

And so I can calculate what V is.

And if I stick in the numbers and if I did not make a mistake, then I find about 2.3 times 10 to the 6 meters per second.

It's an immensely high speed, 5 million miles per hour.

If this were a straight line, you would make it to the moon in three minutes.

5 million miles per hour this electron goes around the proton.

Now I have to go to the current.

I have to find out what the current is.

So the question that I'm going to ask now is how long does it take for this electron to go around.

Well, that time, capital T, is of course the circumference of my circle divided by the speed of the electron.

Trivial.

Even the high school students in my audience will understand that one, I hope.

And so I know what 2 pi R is, because I know R and I know V and so I can calculate that time, just by sticking in the numbers.

And I find that it is about 1.14 times 10 to the -16 seconds.

Just imagine how small that time is.

You cannot even -- we cannot even imagine what it's like.

It goes 10 to the 16 times per second around, because it has this huge speed.

The 1.14 times 10 to the -16 really should have been 1.4 times 10 to the -16.

Of course, it doesn't make much difference, but in case you substitute in the numbers, it is 1.4 times 10 to the -16.

Now, we still haven't found the current, but we're almost there.

Because when you look here, there is this electron going by, and every 1.14 10 to the -16 seconds, that electron goes by.

So the current I, that's the definition of current, is the charge per unit time.

And so every capital T seconds, the charge e goes by, and so this is per definition the current.

And so this current, then, that you have, which is simply due to the electron going around the proton, is about 1.1 times 10 to the -3 amperes.

And that is mind-boggling.

A milliampere.

One electron going around a proton represents a current of a milliampere.

And now of course I have the magnetic moment mu, that is I times A.

We already calculated A, and now we also have the current I, and so we now get that mu is approximately 9.3, if you put in all the decimals correctly, times 10 to the -24, and the unit is of course amperes square meters.

This is area, and this is current.

This A has nothing to do with that A, hey.

This is amperes.

Be careful.

And this is square meters.

But these are the units.

And this has a name.

This is called the Bohr magneton.

Bohr magneton.

What we cannot understand with our knowledge now, but you can if you ever take quantum mechanics, that the magnetic moment of all electrons in orbit can only be a multiple of this number, nothing in between.

Quantum mechanics, the word says it is quantization.

It's not in between.

It's either or.

It includes even 0, which is even harder to understand, that it can even be 0.

In addition to a dipole moment due to the electron going around the proton, the electron itself is a charge which spins about its own axis, and that also means that a charge is going around on the spinning scale of the electron.

And that magnetic dipole moment is always this value.

And so the net magnetic dipole moment of an atom or a molecule is now the vectorial sum of all these dipole moments, all these electrons going around, means orbital dipole moments, and you have to add the spin dipoles.

Some of these pair each other out.

One electron would have its dipole moment in this direction and the other in this direction, and then the vectorial sum is 0.

The net result is that most atoms and molecules have dipole moments which are either one Bohr magneton or 2 Bohr magnetons.

That is very common.

And that's what I will need today to discuss with you how strong a field we can create if we align all those magnetic dipoles.

The magnetic field that is produced inside a material when I expose it to an external field, that magnetic field B is the vacuum field that I can create with a solenoid -- we will discuss that further today -- plus the field which I will call B prime which is that magnetic field that is the result of the fact that we're going to align these dipoles.

The external field wants to align these dipoles, and the degree of success depends on the strength of the external field and of course on the temperature.

If the temperature is low, it's easier to align them, because there is less thermal agitation.

If, and that's a big if -- today you will see why it's a big if -- if B prime is linearly proportional to B vacuum, if that is the case -- today you will see that there are situations where that's not the case -- then I can write down that B prime equals chi of M -- we called that last lecture the magnetic susceptibility -- times B vacuum.

The linear proportionality constant.

If I can do that, then of course B is also proportional to B vacuum because now I can write down that B is one plus chi M times B vacuum.

And that, for that we write kappa of M times B vacuum, which is the equation that I started out with today.

And so that is only a meaningful equation if the sum of the alignment of all these dipoles can be written as being linearly proportional with the external field.

And this is what I want to explore today in more detail.

With paramagnetic material, there is never any worry that the linearity doesn't hold.

But with ferromagnetic material, that is not the case, because with ferromagnetic material, it is relatively easy to align these dipoles, because they already group in domains, as we discussed last time, and the domains flip in unison.

And so with ferromagnetic material, as you will see today, we can actually go into what we call saturation, that all the dipoles are aligned in the same direction.

And now the question is, how strong would that field be?

I'm going to make a rough calculation that gives you a pretty good feeling for the numbers.

It depends on what material you have.

I will choose a material whereby the magnetic dipole moment is 2 Bohr magnetons, so this is -- I told you it's either one or two or three, I pick one for which it is two.

And I have them all aligned.

So I take the situation that they're all aligned.

So here is the current going around the nucleus, here's another one, here's another one, here's another one.

This is a solid material, so these atoms or these molecules are nicely packed.

And here we see all these currents going around, and all these magnetic dipole moments are nicely aligned.

And so these magnetic fields are supporting each other.

And the question now is, what is the magnetic field inside here?

Well, that's an easy calculation, because this really looks like a solenoid, like you have windings, and you have a current going around.

And you remember, or should remember, that if we have a solenoid and we run a current through a solenoid that the magnetic field in the solenoid is mu 0 -- this mu 0 is not this mu, this mu 0 is the same one that -- oh no, it's no- we don't have it on the blackboard.

You notice the famous 4 pi times 10 to the -7.

And then we have the current I, and then we have N, if that's the number of windings that we have in the solenoid, and then we have L, which is the length of the solenoid.

So this is the number of windings of the solenoid per unit length, the number of windings per meter.

So if we could figure out, for this arrangement, what this quantity is, then we're in business.

I take a material, which is not unreasonable, whereby the number density of atoms, I call that capital N, written in the subscript way, is about 10 to the 29.

So this is atoms or molecules, whatever may be the case, per cubic meter.

That's not unreasonable.

And now I have to somehow manipulate, massage the mathematics, so that I get in here this magnetic moment, this Bohr magneton.

The 2 Bohr magnetons.

And there are several ways of doing that.

I have chosen one way, and that's the following.

I take here a length of 1 meter.

So this is a solenoid, and I take only 1 meter.

Could have taken 3 or 5 meters, makes no difference.

I take 1 meter.

And each one of these loops here has an area A.

So we would agree, I hope, that the area -- that the volume, the volume of this solenoid -- this has a length 1 meter -- that that volume is A square meters times 1.

And so the volume is A cubic meters.

A times 1 meter is A cubic meters.

But the number of atoms per cubic meter is 10 to the 29th, and so the number of atoms that I have in this solenoid per meter is this A times that N.

So this is the number of windings, if I call this one winding, the number of windings per meter.

Or you can think of it the number of atoms per meter, the way they're lined up.

And so now I am in business, because this now is my N divided by L.

And so I can write now mu 0 times the current I times that area A times N, which is 10 to the 29.

But look now.

Now you see why I did it this way, because I times A is the magnetic dipole moment of my atoms.

And that was 2 Bohr magnetons.

And so this now also equals mu 0 times twice mu Bohr times N.

And I'm finished, because I know what mu 0 is, and I know what moo- mu Bohr is -- we calculated that, it's still here, this number -- and I know what my capital subscript N is, 10 to the 29 atoms per cubic meter.

And so I just shove those numbers in my equation, and I find that this is approximately 2.3 tesla for the numbers that I have chosen.

It's not for all materials this way, because I have adopted 2 Bohr magnetons for the magnetic dipole moment of each atom, and I have adopted a density of 10 to the 29 atoms per cubic meter.

And for that situation, you get 2.3 tesla.

Now I want to use this number and understand what's going to happen in ferromagnetic material.

I take ferromagnetic material and I expose it to an external field, I call it a vacuum field.

So I stick it in a solenoid and I can choose the current through my solenoid.

And I'm going to plot for you the vacuum field.

This vacuum field is linearly proportional to the current through my solenoid.

This is an actual physical solenoid.

I have a wire, it goes around like this.

I don't have one here now.

And I run a current through there.

And that vacuum B field equals mu 0 times I times N divided by L, except that this N divided by L is now the number of windings of my current wire, and this is the length of my solenoid.

So don't confuse it with the one we had there, because that was on an atomic scale and this is on a macroscopic scale.

How many windings you have could be -- where you had a big one here in class, 2800 windings, and we had 60 centimeters long, and so that's what this number is all about.

So the moment that I know what the current is through my solenoid, I immediately know what my vacuum field is.

There's a one-to-one correspondence.

And now I'm going to measure inside the ferromagnetic material, which I stick in the solenoid.

I'm going to measure there the magnetic field.

And what's going to happen now?

Well, first of all I cannot at all plot this curve on a one-to-one scale, the reason being that kappa of M for ferromagnetic material is so large -- let us adopt for now a number of 1000, say.

Or it could be larger, even.

Let's say kappa of M is 1000.

That means that if the magnetic field in terms of a vector is this big, 1 centimeter in the length of the vector, that the field inside the ferromagnetic material is a thousand times higher.

If this is 1 centimeter in length, a thousand times higher is 10 meters.

So this is the length of the vector of the magnetic field inside the ferromagnetic material.

That's why I cannot do it to scale.

So when I draw this line here, keep in mind that if I did it to scale, if I did, but I can't, then the tangent of alpha would be kappa of M, which in this case would be 10 to the third.

And so this angle alpha is something like 89.9 something degrees.

So I cannot do it to scale.

Keep that in mind.

So in the beginning I get a nice linear curve, but now slowly I'm beginning to reach saturation, that all these dipoles are going to be aligned, and what you're going to see is that this curve bends over and bends over and bends over, and the magnetic field that you finally achieve here is the famous 2.3 tesla, which I calculated for that imaginary material, plus B vacuum.

The 2.3 is now the field which I will call B prime.

This is the field that is the result of the alignment of all those dipoles.

And so when I increase the vacuum field, this goes into saturation, and settles for 2.3 and can no longer increase, because I have aligned all these magnetic dipoles.

And so if your vacuum field is this strong, then this field is no longer thousand times stronger than the vacuum field.

You're no longer in the linear part.

So you could also think of it as kappa of M being smaller than a thousand.

Whichever way you prefer is fine.

But it's no longer proportional to the value of 1000.

If the temperature is lower of the material, it's easier to align them, and so you will achieve saturation earlier, and so your curve would go like this.

So the curve is also a function of temperature.

So this is if the temperature is low relative to this one.

So these curves depend on, on temperature as well.

The lower the temperature, the easier it is to align them.

If I reach this point here, when my B prime goes into saturation, I can only increase the field, the B field in the material, by increasing the vacuum field, because B prime is not going to go up again.

And so I can only get a higher field by increasing this current so that this B, this B vacuum goes up.

And that goes up very slowly, because this huge magnification factor of 1000 is gone now.

So the slow, it's very slow, the growth, and that's why you see that I drew it like this, that it increases very slowly.

But my plot is not to scale anyhow.

Now I want to discuss with you what happens with the material once I have driven it into saturation.

What happens if now I change the current and I make my vacuum field 0 again?

And now you get a very unusual behavior.

Let me do that here on the blackboard.

So I'll make a new drawing.

I could have continued with that one, but let me make a new one.

So I'm going to do the following experiment in my head.

I have a solenoid and I run a current through that solenoid.

And if the current is in clockwise direction, my vacuum field will be in this direction.

And when the current is in counterclockwise direction, I will assume that my vacuum field is in this direction.

So there is ferromagnetic material in here.

If the current flows in clockwise direction, the vacuum field is in this direction.

If I run it in counterclockwise direction, the vacuum field is in that direction.

And here is going to be my vacuum field.

It's easy for me to know what that is, because if I know the current through my solenoid, this equation will immediately tell me what the vacuum field is.

So I have never any problems with the vacuum field.

I stick a probe in here and I measure the magnetic field inside that material.

How we do that is not so easy, but we can do it.

There are a few things that I can't tell you.

This is one of them.

So here is the magnetic field inside the material.

All right, so there we start.

We do the same thing that we did here.

So we approach the saturation.

But now when I'm here, I am reducing the current and go back to 0.

Remember that when we are here, all these domains that we discussed last time have all flipped in the direction of the vacuum field.

So this field is enormous.

But now I make the current go back to 0.

And what happens now is I end up here, at this point P here.

The current now is 0.

There is no current going through the solenoid.

Notice the vacuum field is 0.

I can take the material out, the ferromagnetic material.

The material itself is now magnetic.

And you see there is a magnetic field inside it.

Why is that?

Because some of those domains remain aligned, they don't go back.

And so we have created permanent magnetism.

And so in the location, at the location P, we have B vacuum is 0, but B prime, which is the result of those aligned magnetic moments, is still in this direction.

Nothing is to scale here, of course.

And so you still have a magnetic field.

Now I reverse the current.

I go counterclockwise.

So I'm creating a magnetic field vacuum now in this direction.

And so now what will happen with this curve?

I come up here.

And look now here, at this location Q.

What do I have now?

I have something very bizarre.

I have now a situation whereby the vacuum field is in this direction but there is no magnetic field inside the material.

The magnetic field inside is 0.

So when we have point Q, so we have B vacuum is in this direction, but B inside, B prime -- oh no, it's not B prime, it's B.

It's the total field inside, is 0.

The reason being that B prime is still in this direction.

The reason being that the domains are still aligned in this direction, and so the vacuum field plus the B prime field, which has to be vectorially added, adds up with a net field 0.

Quite bizarre, isn't it?

Now I increase the current, but I keep going counterclockwise, and so the magnetic field of the vacuum remains in this direction.

I go into saturation again, in a similar way that I went into saturation here.

And now I stop here -- oh, I don't want to lose my brooch.

And now I stop here and I say to the current, go back to 0 again.

So my current now goes back to 0.

There we go.

And now I arrive here, point S.

And again, I have a situation that my vacuum field is 0.

I could take the material out of the solenoid, just walk around with it on the street.

It will be a permanent magnet.

But now the magnetic field inside this material in point P it was in this direction.

If I take it out here, then it is in this direction.

Now some domains stay aligned in this direction.

The reason was that I had counterclockwise current, and so those domains flipped over.

And they're not all willing to flip back again.

So I've also made a permanent magnet here.

Vacuum field 0, but B prime is now in the opposite direction.

And then if I continue now to go clockwise with current again and increase the current, I end up there.

And this is a bizarre curve.

We call this the hysteresis curve.

If you look at this curve, it's really amazing.

It's actually hard to, to digest this.

You, you have to give it a little bit of thought.

Because for one particular value of the current, for instance here, once I have a particular value of the current, I know that B vacuum is a given, I have two possibilities here for the magnetic field.

And for the current here, I have two possibilities for the magnetic field.

And so I cannot even know when I take this material and I expose it to an external field, I can't even calculate what the magnetic field inside will be.

It depends on the history of this material.

Look at this point here and at this point there.

If I asked you what is kappa of M, [pff] it's almost a ridiculous question.

Because what is kappa of M?

I have, um, I have a vacuum field, but I have no field inside.

So the field inside, which is B, is 0, and the vacuum field is not 0.

So you would have to answer, kappa M is 0.

That's the only thing you could say.

Quite bizarre, right.

But right here, there is a vacuum field but no field inside.

So kappa M is 0 here and kappa M is 0 here.

And remember, here it was 1000.

Look at the situation, take this point of the curve and this point of the curve.

Kappa M is less than 0, is negative, because here the vacuum field is in this direction, but B prime is in that direction, so they are in opposite directions, so the net field is in that direction, but B vacuum is in this direction.

And here it's also reversed.

So there's a bizarre situation that you are effectively having situations whereby kappa of M is 0 and kappa of M can also be negative for those s- points that I have there.

I can show you this hysteresis curve, and I do it exactly the way that I explained to you, except that I will not be able to run this current very slowly up and down.

I do it with 60 hertz alternating current, just get it out of the, the wall.

And so I run through this solenoid n- 60 hertz alternating current.

That means we go through this curve very quickly back and forth.

Between this point maximum current and this point maximum current.

The current clockwise, counterclockwise, clockwise, counterclockwise, and we change that 60 times per second.

And then I will show you this curve, again, highly distorted.

I cannot plot it one-to-one, for the reasons that I explained to you.

And you will see then the hysteresis curve.

That's called the hysteresis curve.

And for that, I have to do several things.

And I always forget what I have to do, but just don't worry about it, I will find out.

My TV goes on.

This light goes off and this light goes off.

Look there on the screen, and you will see within seconds there is the hysteresis curve.

And as I said to you earlier, I cannot start here, unfortunately, because we switch it so fast back and forth that this part of the curve, by the way, which is called the virginal curve -- virginal, because it's once and never again.

Once you have reached that point, from that moment on, you always stick to this -- I know, you should know about that.

[laughter].

And so here you see a striking example of a hysteresis curve.

And so you can ask yourself now the question, can we make this material virginal again?

And then the answer is yes.

There are various ways you can do that.

One way is you could take the material out, so that means you take it out, for instance, when there is remnant magnetism here, so it's a magnet, and now you heat it up above the Curie point, as we did last time in my lecture, and then the domains completely fall apart, and then you cool it again below the Curie point, and then it is virginal material again.

And then you could start here again.

That's one way you could do it.

There is another way you could try to do it.

You take a hammer and you just bang on it.

So you take it out and you have permanent magnetism, either here or there, and you bang on it, and you hope for the best.

And maybe you can get it back to here.

There is another way, which we call demagnetization, and I think that's what happens when you steal a book in the library and you try to get away with it, and the alarm goes off.

Someone hasn't demagnetized the magnetic strip in the book.

You may have noticed that when you take it out, that someone under the table goes like this.

And what they are doing then is they are demagnetizing that strip.

And the way you do that is as follows.

Here the current goes back and forth between this value and this value.

That means B vacuum and I are directly coupled to each other.

So I think of this as being current.

And now I go up to here and then back and then here and here and here and here and here and here and here and here and here and I end up there again.

And I can show you that.

So again you have alternating current, but the amplitude of the current you decrease and decrease and decrease, and you're going to see that there.

And you see that I can turn this material -- see the hysteresis curve changes.

So the amplitude of the current is not as large, so the amplitude of the B vacuum is not as large, and I slowly go back.

And I can change a non-virgin back into a virgin.

And that's the way we do it as physicists.

Demagnetization.

Did I turn that off?

Yes, I did.

I have here a, a coil.

You can't see that there is a coil inside here, but there is.

I can, uh, power this coil, putting a current through it.

Ferromagnetic material, I don't know what kappa is, but, uh, at least a thousand.

I get an enormously strong field inside here.

And this field is so strong that this piece of ferromagnetic material will be attracted.

The field is non-uniform outside.

We discussed last time that it will go clunk, it will stick there.

So let's do that.

So I power this electromagnet -- you can't see that I did, you have to take my word for it, but you believe it now.

There it goes.

Oh boy.

The force is so large that I would ask two people to come and see whether they can pull it apart.

The force that the two are together is so large that you may not even be able to separate it.

Do we have two strong people?

One strong woman, one strong man.

You look very strong to me.

[laughter].

Come on.

It's not going to be a tug of war between the two of you.

That's not my plan.

Be very careful, because if you touch this, you get electrocuted.

So don't do that.

[laughter].

But make sure that everyone can see you.

Because there is a current running through this solenoid now, yeah?

OK.

Now just in case that you succeed, I don't want you to get hurt.

Student: OK.

So make sure that you secure yourself.

[laughter].

Because suppose the current stopped running all of a sudden.

It's possible, it's MIT [laughter] right, anything could happen.

Then, of course, it's no longer a strong magnet.

It's only a strong magnet as long as the current is running through the solenoid.

So you secure yourself too.

OK, three, two, one, zero, go.

[laughter].

Don't worry.

I knew that in advance.

But thank you very much.

[laughter].

Very kind of you.

[applause].

Now comes something that you will understand.

If I make the current go to 0, then the vacuum field goes to 0.

That means the field that is generated by the solenoid goes to 0.

I will do that now in front of your own eyes.

No more current, right?

Why is this still hanging there?

Yeah?

The solenoid [inaudible] magnetize the ferrous [inaudible].

You hear?

You've taken the vacuum field out, but the domains to a certain degree are still aligned, and so the whole thing is still a magnet.

Not as strong as magnet -- if I were to invite you to come now and take it apart, you could.

But it still takes sub- substantial force.

And I will show you how large that force is.

I'm going to load it down now.

There's now 1 kilogram hanging on it.

Ooh, let me make sure I secure that, otherwise I can get killed for a change.

OK, 3 kilograms is hanging on it now.

5 kilograms is hanging on it now.

7 kilograms is hanging on it now.

9 kilograms is hanging on it now.

Oh boy, we may never make it.

10 kilograms is -- there it goes.

[laughter].

10 kilograms.

Now the show is not over yet.

What is very interesting, and I want you to think about it, that if now I take these two pieces of -- [laughter].

If I take these two pieces of ferromagnetic material, that -- nothing.

Do you know why?

I dropped it on the floor.

[laughter].

That's really why.

I dropped it on the floor, and that is like banging it with a hammer, and then the domains go away.

Had I not dropped it on the floor [laughter], there would have been something left, but very little, which is interesting by itself.

The shock of the separation, when that happened, already makes many of the domains flip back, and there would be very little left.

Not enough to carry this weight anymore, but that's largely because I dropped it on the floor.

I did that purposely so you can see when you drop [laughter] things on the floor.

If I bring ferromagnetic material in the vicinity of a magnet, I change the magnetic field configuration, and that's very easy to understand now.

Suppose I have here a magnet, north pole, south pole, and the magnetic field, magnetic dipole field is sort of like this.

And now I bring in the vicinity here a piece of ferromagnetic material.

Could be a wrench.

What happens now is that this ferromagnetic material will see this vacuum field -- this is called the vacuum field, is an external field.

And so these domains in there trying to align a little bit.

Degree of success depends on how strong the field is, depends on the temperature, depends on the kappa M of that material.

But certainly this will become sort of a south pole, this will become a north pole.

That's the way that these dipoles are going to align themselves.

They are going to create themselves a field in this direction.

They're going to support that field.

And so the net result is that the field inside here becomes very strong.

And so what happens with these field lines, they go like this.

They're being sucked into this ferromagnetic material.

It's very hard to know exactly how they go.

And the field here will weaken.

And I'm going to demonstrate that to you.

This is actually very easy to demonstrate.

And the way I'm going to demonstrate that is as follows.

I have there a setup whereby we have a, a magnet and we have a nail and we have a string.

And the nail wants to go to the magnet.

The mail its- the nail itself is, uh, ferromagnetic.

So the nail would love to go in this direction, but it can't.

So it just sits there, hangs there in space.

First of all, what I want to show you is that if I bring paramagnetic material in the vicinity, that that magnetic field configuration here is not going to change at all.

Paramagnetic material has a kappa of M so close to one that nothing is going to happen.

But the moment that I bring ferromagnetic material, for instance, here, then you get a field configuration change, and if I do that just the right way, then the nail will fall.

In other words, there is not enough magnetic field here in order to hold the nail in that direction.

And I'm going to show that for you, to you there.

Need some power here, I believe, and we have to make it dark here.

I'm going to shadow project it for you.

And the shadow projection you will see coming up very shortly.

This is a carbon arc.

You have to give it a little bit of time to start.

There is the carbon arc.

So there you see the nail.

And here you see the magnet.

You see that?

That's exactly the way I drew the picture.

And here, you hav- I have here a piece of aluminum, which is paramagnetic.

I can bring that through the field here.

Nothing happens.

My hands, believe it or not, are definitely not ferromagnetic, so I can also bring my hands here.

Nothing.

Nothing.

So magnetic field is not disturbed in any way, in any serious way, either by paramagnetic material, aluminum, or my hands, which I think are also paramagnetic, but I'm not sure.

I'm not sure whether I'm diamagnetic or paramagnetic, but it doesn't make any difference, because in both cases there is no significant change of the magnetic field.

But now I have a wrench here.

Here, there's a wrench.

You see it?

[laughter].

OK.

And now I'll bring the wrench close to that magnet.

My major worry is that magnetic field is so strong that once the wrench go- there's no way I can get it off again.

So I get only one shot at it.

And there goes the nail.

So what I -- what you saw now in front of your own eyes is that I changed the magnetic field configuration in such a way that the field was not strong enough to pull in the, the nail.

Now comes an important question, a big moment in our life.

And that is, what now is the effect of magnetic material on Maxwell's equations?

And let's take a look at Maxwell's equations.

Here we have Maxwell's equations the way we know them.

[laughter].

And let's first look at number one.

That's Gauss's Law.

Gauss's Law says that the closed surface integral of E dot dA -- that's the electric flux through a closed surface -- E- is equal to all the charge inside divided by epsilon 0, but you have to allow for the kappa, for the electric, dielectric constant.

The kappa, by the way, always lowers the field inside the material when we deal with electric fields.

It never increases it like magnetic fields.

It always lowers it.

Kappa is normally a few -- except for water, it is 80, it's quite large, and there are some ridiculous substances whereby kappa can be as large as 300.

I think strontium titanate -- I just looked it up this morning -- has a ridiculous value of kappa of 300.

So that's Gauss's Law.

Nothing's going to change there as far as I can see.

And then we have the second one.

The closed surface integral of B dot dL equals 0.

Oh, it says an, it says an L.

That shouldn't even be an L.

Ho, I hope you caught that.

This is an A, of course.

How could I?

This is the closed surface integral of B dot dA is 0.

What this is telling me is that magnetic ma- magnetic monopoles don't exist.

At least we think they don't exist.

Don't think that people are not trying to find them.

And if you find a magnetic monopole, and if you put that inside a box, then the closed surface integral of the magnetic flux through that box would not be 0.

And so then this is not true.

But as far as we know, it's always true, because we don't think that magnetic poles- monopoles exist.

And so then we come to the Faraday's Law.

Faraday's Law runs our economy.

Faraday's Law tells you when you move conducting loops in magnetic fields that you create electricity.

This equation runs our economy.

And now we come -- none of these, by the way, require any adjustment in terms of kappa of M.

But now we come to Ampere's Law.

Ampere's Law, which was amended by Maxwell himself, tells me what the magnetic field is, and all these results are for vacuum.

But now we know that that's not true anymore.

So this has to be adjusted now by a factor of kappa of M, which is the relative permeability.

And kappa of M is perfectly kosher for paramagnetic and diamagnetic materials.

There's never any problem there.

So here it comes.

For diamagnetic materials, it's a little less than one.

For paramagnetic materials, a little larger than one.

But when we deal with ferromagnetic materials, you have to be very careful, because we have seen today this hysteresis phenomenon, that there are even situations whereby kappa of M is negative, whereby kappa M is 0, and whereby kappa M can be huge, can be 10 to the 3.

So there you have to be very, very careful when you apply this equation without thinking.

Maxwell's equations are so important that I'm sure you want to see more of them.

So you see them there again.

And maybe that's not enough.

[laughter].

Maybe you want to see even more of them.

So look at them.

Inhale them.

Let them penetrate your brains.

[laughter].

I don't care in which direction you look now.

It's hard not to see them.

Today is therefore very special, because today we have all four Maxwell's equations in place.

And this was one of the main objectives of 8.02.

So we have completed a long journey, and on April 5, we have reached the summit.

Now I realize that the view is not spectacular for all of you yet, because often at the summit there is some fog.

But the fog will clear.

And I can assure you that from here on on, it's climbing downhill.

I think this moment is worth celebrating, and therefore I bought 600 daffodils for this occasion.

[laughter].

And I would like you to come at the, at the end of the lecture and pick up one of these daffodils and take it back to your dormitory.

I don't know whether I have enough for all these high school students and all their parents, but why not, give it a shot, and take one.

And when you look at it tonight at home and tomorrow, remember that you only once in your life go through this experience, that for the first time you see all four Maxwell's equations complete, hold it, and that you're capable of appreciating them, at least in principle.

This will never happen again.

You will never be the same.

[laughter].

To put it in simple terms, as far as 8.02 is concerned, you are no longer unspoiled virginal material [laughter].

You've lost your virginity.

[laughter].

Congratulations! [laughter].

Massachusetts Institute of Technology - Physics Department

Physics 8.02 February 22, 2002.

Dielectrics & Polarization

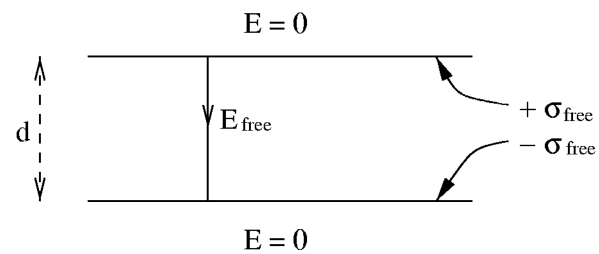
Some notes from Lecture #8 on Friday February 22.

These notes do not stand on their own; they should be "consumed" in combination with my lecture.

We discussed twice before that non-conductors can become *polarized* due to an external electric field. The field can *induce* dipoles in atoms and molecules; the stronger the field, the stronger the degree of polarization. We call these materials *dielectrics*[1]. We exclude all ideal conductors in which charge ***flows*** in response to the field. What I will discuss now is notoriously confusing.

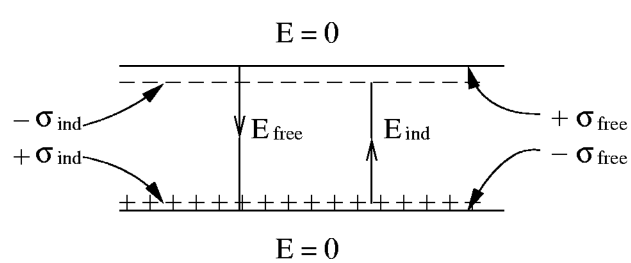
Symbols without arrows represent magnitudes.

Take a plate capacitor (conducting plates). We connect it to a battery (of voltage V) and consequently charge the plates with the same amount of charge but of opposite sign. I will call the charge on ***each*** plate Q_{free}, and the resulting surface charge density σ_{free} (this is Q_{free} divided by the surface area of one plate), and I call the electric field inside the capacitor due to this charge, E_{free}. Notice, Giancoli, on page 625 mentions the words "free charge", but uses the symbols Q and σ (without the subscript "free").



$$E_{free} = \frac{\sigma_{free}}{\varepsilon_0} \tag{1}$$

After the capacitor is charged up, *I disconnect the battery* that supplied the potential difference V between the plates. The charge on the plates is now "trapped". We now shove a dielectric between the plates. It produces two layers of induced charge on the dielectric (remember the "+ -" transparency shown in lectures). I will call this induced surface charge density σ_{ind}; it is the result of polarization of the dielectric under influence of the electric field. The induced charge is often called "bound charge", as opposed to the "free charge".



[1] *The molecules of some dielectrics have an intrinsic dipole moment even in the absence of an external electric field. When they are "exposed" to an E field, they try to line up with the field. The degree of success depends on the strength of the applied field. They are included here.*

The induced charges produce "their own" electric field, $\vec{E}_{ind}$, which ***opposes*** the field $\vec{E}_{free}$.

$$E_{ind} = \frac{\sigma_{ind}}{\varepsilon_0} \tag{2}$$

and the resulting ***net*** field is

$$\overrightarrow{E} = \overrightarrow{E}_{free} + \overrightarrow{E}_{ind} \tag{3}$$

The magnitude of E is thus less than that of E_{free} (the electric field has become weaker).

$$E = E_{free} - E_{ind} \tag{4}$$

Under "normal" circumstances, $E_{ind} \propto E_{free}$, thus $E_{ind} = bE_{free}$, where b is a constant which depends only on the dielectric substance. Thus, $\overrightarrow{E}_{ind} = -b\overrightarrow{E}_{free}$. We substitute this in eq. (3), and find $\overrightarrow{E} = (1-b)\overrightarrow{E}_{free}$. The constant $(1-b)$ is called $1/\kappa$ (κ is the dielectric constant ; it depends only on the material that is inserted between the plates). Thus,

$$\overrightarrow{E} = \frac{\overrightarrow{E}_{free}}{\kappa} \tag{5}$$

This is a key equation.

In our experiment where the free charge was trapped (the battery was disconnected before we inserted the dielectric), the dielectric material ***lowered*** the field strength by a factor κ. Note, the potential difference between the plates, V, must go down by the same factor as d remains unchanged (V ***always*** equals the product of the ***total*** E field (E_{tot}) between the plates and the distance d between the plates). *However, if I keep the battery connected while I stick the dielectric in, the potential difference between the plates remains unchanged; thus E can* ***not*** *go down, therefore Q_{free}, and thus E_{free}, must go* ***up***, consistent with eq. (5); the additional charge that will then flow to the plates will, of course, be delivered by the battery.

As an example, if I use glass, with $\kappa \approx 5$ (see Table 24.1, page 622 of Giancoli), the E field will be reduced by a factor of ≈ 5. For water, $\kappa \approx 80$ (this is enormous), the E field will be reduced by a factor of 80 (for ice at $-40°C$, $\kappa \approx 100$).

In vacuum, per definition, $\kappa = 1.0000000$ exactly. For most gases, κ is only a "hair" larger than 1.000 which means that b is very small, and for most applications in air, unless stated otherwise, we use for κ exactly 1.

It follows from the above relation between κ and b that

$$E_{ind} = \left(1 - \frac{1}{\kappa}\right) E_{free} \tag{6}$$

Since $E_{ind} = \frac{\sigma_{ind}}{\varepsilon_0}$, and $E_{free} = \frac{\sigma_{free}}{\varepsilon_0}$ (see eqs. 1 and 2), it follows that

$$\sigma_{ind} = b\sigma_{free} = \left(1 - \frac{1}{\kappa}\right)\sigma_{free} \tag{7}$$

Eq. 7 gives us the magnitude of the induced surface charge density, σ_{ind}, if σ_{free}, and κ are known. Keep in mind that the induced charge is negative (*positive*) near the plate where the free charge is positive (*negative*); see the figure above and below. If you feel the need to express the difference in sign, you could add a minus sign to eq. (7), but I don't advise that. In all my equations σ_{ind} and σ_{free} represent the magnitude of the charge; that is why my figures indicate $-\sigma_{ind}$ and $-\sigma_{free}$ where needed.

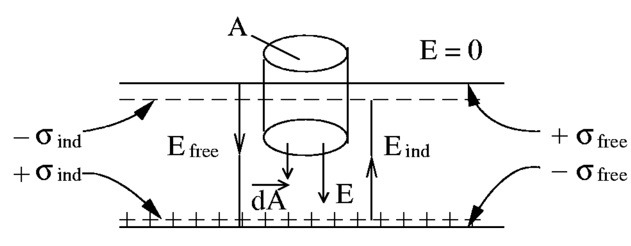
Notice that for very high values of κ ($b \approx 1$), $\sigma_{ind} \approx \sigma_{free}$, and the *net* surface charge density at either plate is then ≈ 0, and there is ***almost no field between the plates!***

Does Gauss's law still hold? Of course! Observe the pill box below; the flat top and bottom (each with area A) are parallel to the plates.

The integral of the electric flux over the entire surface of the pill box (***closed surface integral***):

$$\oint \vec{E} \cdot \vec{dA} = \frac{1}{\varepsilon_0} \sum Q_{enclosed} \tag{8}$$

and $Q_{encl} = A(+\sigma_{free} - \sigma_{ind})$. The pill box contains the "free" charge on the conducting plate and the "induced" charge on the surface of the dielectric; both have to be taken into account.



The electric field is assumed to be zero outside the capacitor, and uniform in the dielectric. At the curved cylindrical surface, $\vec{E}$ and $\vec{dA}$ are perpendicular to each other, but at the bottom flat surface of the pill box (area A), $\vec{E}$ and $\vec{dA}$ have the same direction. Thus, eq. (8) becomes:

$$EA = \frac{1}{\varepsilon_0}(\sigma_{free} - \sigma_{ind})\,A \tag{9}$$

Using eq. (7), I can eliminate σ_{ind}, and find:

$$E = \frac{\sigma_{free}}{\kappa\varepsilon_0} = \frac{E_{free}}{\kappa} \tag{10}$$

This is exactly what we found above (eq. 5).

It simplifies matters to modify eq. (8) and replace the ***enclosed*** charge (which includes both the induced and the free charge) by the ***free charge*** only:

$$\oint \overrightarrow{E} \cdot \overrightarrow{dA} = \frac{1}{\kappa \varepsilon_0} \sum Q_{enclosed,free} \tag{11}$$

which is **Gauss's law for dielectrics**.

Notice that eqs. (5), (10) and (11) do not explicitly contain the induced surface charge density σ_{ind} (i.e. the polarization charge); they only contain the surface charge density on the conducting plate (i.e., the "free" charge).

The κ takes the induced charge *automatically* into account.

Walter Lewin

MIT OpenCourseWare
http://ocw.mit.edu

8.02 Electricity and Magnetism, Spring 2002
Transcript – Lecture 8

Electric fields can induce dipoles in insulators.

Electrons in insulators are bound to the atoms and to the molecules, unlike conductors, where they can freely move, and when I apply an external field -- for instance, a field in this direction, then even though the molecules or the atoms may be completely spherical, they will become a little bit elongated in the sense that the electrons will spend a little bit more time there than they used to, and so this part becomes negatively charged and this part becomes positively charged, and that creates a dipole.

I discussed that with you, already, during the first lecture, because there's something quite remarkable about this, that if you have an insulator -- notice the pluses and the minuses indicate neutral atoms -- and if now, I apply an electric field, which comes down from the top, then, you see a slight shift of the electrons, they spend a little bit more time up than down, and what you see now is, you see a layer of negative charge being created at the top, and a layer of positive charge being created at the bottom.

That's the result of induction, we call that also, sometimes, polarization.

You are polarizing, in a way, the electric charge.

Uh, substances that do this, we call them dielectrics, and today, we will talk quite a bit about dielectrics.

The first part of my lecture is on the web, uh, if you go to 8.02 web, you will see there a document which describes, in great detail, what I'm going to tell you right now.

Suppose we have a plane capacitor -- two planes which I charge with a certain potential, and I have on here, say, a charge plus sigma and here I have a charge minus sigma.

I'm going to call this free -- you will see, very shortly why I call this free -- and this is minus free.

So there's a potential difference between the plate, charge flows on there, it has an area A, and sigma free is the charge density, how much charge per unit area.

So we're going to get an electric field, which runs in this direction, and I call that E free.

And the distance between the plates, say, is D.

So this is given.

I now remove the power supply that I used to give it a certain potential difference.

I completely take it away.

So that means that this charge here is trapped, can not change.

But now I move in a dielectric.

I move in one of those substances.

And what you're going to see here, now, at the top, you're going to see a negative-induced layer, and at the bottom, you're going to see a positive-induced layer.

I called it plus sigma induced, and I call this minus sigma induced.

And the only reason why I call the other free, is to distinguish them from the induced charge.

This induced charge, which I have in green, will produce an electric field which is in the opposite I- direction, and I call that E-induced.

And clearly, E free is, of course, the surface charge density divided by epsilon 0, and E induced is the induced surface charge density, divided by epsilon 0.

And so the net E field is the vectorial sum of the two, so E net -- I gave it a vector -- is E free plus E induced, vectorially added.

Since I'm interested -- I know the direction already -- since I'm interested in magnitudes, therefore the strength of the net E field is going to be the strength of the E fields created by the so-called free charge, minus the strength of the E fields created by the induced charge, minus -- because this E vector is down, and this one is in the up direction.

And so, if I now make the assumption that a certain fraction of the free charge is induced, so I make the assumption that sigma induced is some fraction B times sigma free, I just write, now, and I for induced and an F for free.

B is smaller than 1.

If B were .1, it means that sigma-induced would be 10% of sigma free, that's the meaning of B.

So clearly, if this is the case, then, also, E of I must also be B times E of F.

You can tell immediately, they are connected.

And so now I can write down, for E net, I can also write down E free times 1 minus B, and that 1 minus B, now, we call 1 over kappa.

I call it 1 over kappa, our book calls it 1 over K.

But I'm so used to kappa that I decided to still hold on to kappa.

And that K, or that kappa, whichever you want to call it, is called the dielectric constant.

It's a dimensionless number.

And so I can write down, now, in general, that E -- and I drop the word net, now, from now on, whenever I write E, throughout this lecture, it's always the net electric field, takes both into account.

So you can write down, now, that E equals the free electric fields, divided by kappa, because 1 minus B is 1 over kappa.

And so you see, in this experiment that I did in my head, first, bringing charge on the plate, certain potential difference, removing the power supply, shoving in the dielectric that an E field will go down by a factor kappa.

Kappa, for glass, is about 5.

That will be a major reduction, I will show you that later.

If the electric field goes down, in this particular experiment, it is clear that the potential difference between the plates will also go down, because the potential difference between the plates, V is always the electric field between the plates times D.

And so, if this one goes down, by a factor of kappa, if I just shove in the dielectric, not changing D, then, of course, the potential between the plates is also going down.

None of this is so intuitive, but I will demonstrate that later.

The question now arises, does Gauss's Law still hold?

And the answer is, yes, of course, Gauss's Law will still hold.

Gauss's Law tells me that the closed loop -- closed surface, I should say, not closed loop -- the closed surface integral of E dot dA is 1 over epsilon times the sum of all the charges inside my box.

All the charges! The net charges, that must take into account both the induced charge, as well as the free charge.

And so let me write down here, net, to remind you that.

But Q net is, of course, Q free plus Q induced.

And I want to remind you that this is minus, and this was plus.

The free charge, positive there, is plus, and at that same plate, if you have your Gaussian surface at the top, you have the negative charged Q induced.

And so therefore, Gauss's Law simply means that you have to take both into account, and so, therefore, you can write down 1 over epsilon 0, times the sum of Q free, but now you have to make sure that you take the induced charge into account, and therefore, you divide the whole thing by kappa.

Then you have automatically taken the induced charge into account.

So you can amend Gauss's Law very easily by this factor of kappa.

Dielectric constant is dimensionless, as I mentioned already, it is 1, in vacuum, by definition.

1 atmosphere gases typically have dielectric constant just a hair larger than 1.

We will, most of the time, assume that it is 1.

Plastic has a dielectric constant of 3, and glass, which is an extremely good insulator, has a dielectric constant of 5.

If you have an external field, that can induce dipoles in molecules -- but there are substances, however, which themselves are already dipoles, even in the absence of an electric field.

If you apply, now, an external field, these dipoles will start to align along the electric field, we did an experiment once, with some grass seeds, perhaps you remember that.

And as they align in the direction of the electric field, they will strengthen the electric field.

On the other hand, because of the temperature of the substance, these dipoles, these molecules which are now dipoles by themselves, through chaotic motion, will try to disalign, temperature is trying to disalign them.

So it is going to be a competition, on the one hand, between the electric field which tries to align them and the temperature which tries to disalign them.

But if the electric field is strong, you can get a substantial amount of alignment.

Uh, permanent dipoles, as a rule, are way stronger than any dipole that you can induce by ordinary means in a laboratory, and so the substances which are natural dipoles, they have a much higher value for kappa, a much higher dielectric constant that the substances that I just discussed, which themselves, do not have dipoles.

Water is an example, extremely good example.

The electrons spend a little bit more time near the oxygen than near the hydrogen, and water has a dielectric constant of 80.

That's enormous.

And if you go down to lower temperature, if you take ice of minus 40 degrees, it is even higher, then the dielectric constant is 100.

I'm now going to massage you through four demonstrations, four experiments.

One of them, you have already seen.

And try to follow them as closely as you can, because if you miss one small step, then you miss, perhaps, a lot.

I have two parallel plates which are on this table, as you have seen last time, and I have, here, a current meter, I put it -- an A on there, that means amp meter.

And the plates have a certain separation D.

I'm going to charge this capacitor up by connecting these ends to a power supply, and I'm going to connect them to 1500 volts.

I'm -- I'm already going to set my light, because that's where you're going to see it very shortly.

I'm going to start off with a distance D -- so this is going to be my experiment one -- with a distance D of 1 millimeter.

And the voltage V always means the voltage the -- the -- the potential difference between the plates is going to be 1500 volts.

Forgive me for the two Vs, I can't help that.

This means, here, the potential difference, and this is the unit in volts.

Once I have charged them, I disconnect -- this is very important -- I disconnect the power supply, for which I write PS.

That's it.

So the charge is now trapped.

As I charge it, as you saw last time, you will see that the amp meter shows a short surge of current, because, as I put charge on the plates, the charge has to go from the power supply to the plates, and you will see a short surge of current which will make the handle -- the hand of the power su- of the amp meter, as you will see on the -- on the wall there -- go to the right side, just briefly, and then come back.

This indicates that you are charging the plates.

Now, I'm going to open up the gap -- so this is my initial condition, there is no dielectric -- and now I'm going to go D to 7 millimeters.

And this is what I did last time.

The reason why I do it again, because I need this for my next demonstration.

If I make the distance 7 millimeters, then the charge, which I call now, Q free, but it is really the charge on the plates, is not going to be -- is not going to change, it is trapped.

So there can be no change when I open up the gap.

That means the amp meter will do nothing, you will not see any charge flow.

The electric field E is unchanged, because E is sigma divided by epsilon 0.

If sig- if Q free is not changing, sigma cannot change.

So, no change in the electric field.

But the potential V is now going to go up by a factor of 7, because V equals E times D.

E remains constant, D goes up, V has to go up.

And this is what I want to show you first, even though you have already seen this.

And I need the new conditions for my demonstration that comes afterwards.

I'm going from 1500 volts to about 10000 volts, it goes up by a factor of 7.

And you're going to see that there.

There you see your amp meter.

I'm going to -- you see the, um, this is this propeller volt meter that we discussed last time, and here you see the -- the plates.

They're 1 millimeter apart now, very close.

And I'm going to charge the plates, I will count down, so you keep your eye on the amp meter, three, two, one, zero, and you saw a current surge.

So I charged the capacitor.

It is charged now.

The volt meter doesn't show very much, 1500 volts.

Maybe it went up a little, but not very much, but now I'm going to increase the gap to 10 -- to 7 millimeters, and look that the amp meter is not doing anything, the charge is trapped, so there is no charge going to the plates, but look what the volt meter is doing.

It's increasing the voltage, it's not approaching almost 10000 volts, although this is not very quantitative, and now I have a gap of about 7 millimeters, and that's what I wanted.

We've seen that the plates on the left side here are now farther apart than they were before.

So that is my demonstration number one, a repeat of what we did last time.

So now comes number two.

So now my initial conditions are that V is now 10 kilovolts, so that's the potential difference between the plates that I have now, and D is now 7 millimeters, and I'm not going to change that.

At this moment, kappa is 1.

But now, I'm going to insert the dielectric.

So I take a piece of glass, and I'll just put it into that gap.

Q free cannot go anywhere, because I have disconnected the power supply.

So Q free, no change.

If there is no chee- no change in the free charge, the amp meter will do nothing.

So as I plunge in this dielectric, you will not see any reading on the amp meter.

But, as we discussed at length now, the electric field, which is the net electric field, will go down by that factor kappa.

That's what the whole discussion was all about.

That's going to be a factor of 5.

And since the potential equals electric field times D -- but I keep D at 7 millimeters, I'm not going to change it -- if E goes down by a factor kappa, then clearly, the potential will also go down by a factor kappa.

So now you're going to see the second part, and that is I'm going -- as it is now, I'm going to plunge in this glass, the 7 millimeters thick, I put it in there, you expect to see no change on the amp meter, but you expect the voltage difference over the plates to go down by a factor of 5, so you will see that -- that the propeller volt meter will have a smaller deflection.

You ready for this?

There we go.

Now you have a smaller potential difference, but there was no current flowing to the plates or from the plates.

When I take it out again, the potential difference comes back to the 10000 volts.

So that's demonstration number two.

Now we go to number three.

But before we go to number three, I want to ask myself the question, what actually happened with the capacitance when I bring the dielectric between those plates?

Well, the capacitance is defined as the free charge divided by the potential difference over the plates.

That's the definition of capacitance.

And since, in this experiment, as you have seen, the voltage went down by a factor of kappa, the capacitance goes up by a factor of kappa, because Q free was not changing.

And so, since the capacitance, as we derived this last time for plane -- plate capacitors, I still remember, it was the area times epsilon 0 divided by the separation D -- since we now know that with the glass in place, that's -- the capacitance is higher by a factor of kappa, this is now the amendment we have to make.

To calculate capacitance, we simply have to multiply, now, by the dielectric constant of the thin layer that separates the two conductors, the layer that has thickness D that is in between the two plates.

In our case, I brought in glass.

I could write down a few equations now that you can always hold on to in your life, and you can also use them in the two demonstrations that follow.

And one is that E -- which is always the net E, when I write E it's always the net one -- equals sigma free divided by epsilon 0 times kappa.

There comes that kappa that we discussed today.

Let's call that equation number one.

The second one is that the potential difference over the plates is always the electric field between the plates times D, because the integral of E dot dL over a certain pass, is the potential difference.

That's not going to change.

And then the third one that may come in handy is the one that I have already there, C equals Q free divided by the potential difference, which, in terms of the plate area, is A times epsilon 0, divided by D, times kappa.

Let's call this equation number three.

Now comes my third experiment.

In the third demonstration, I am not going to disconnect my power supply.

So now, in number three, I start out with 1500 volts, just like we did with number one, but the power supply will stay in there throughout, never take it off.

We start with D equals 1 millimeter, just like we did in experiment one.

No glass.

I'm going to charge it up, just like I did with number one, and, of course, I will see that the amp meter will show this charge.

[clk].

See a surge of current.

Now I'm going to increase D to 7 millimeters.

Now something very different will happen from what we saw in the first experiment.

The reason is that the potential difference is going to be fixed, because the power supply is not disconnected, the power supply stays in place.

Look, now, at equation number two.

If that V cannot change, and if I increase D by a factor of 7, now the electric field must come down by a factor of 7.

And so now the electric field will come down by that factor of 7, because I go from 1 millimeter to 7 millimeters.

So now the electric field changes, because D goes up.

In case you were interested in the capacitance, the capacitance will also go down by a factor of 7, because, if you look at this equation, kappa is 1.

If I make D go up by a factor of 7, C goes down by a factor of 7.

Just look at this, simple as that.

So C must also go down by a factor of 7.

Nothing to do with dielectric.

Nothing.

And so Q free must now also go down by a factor of 7, because if the potential difference doesn't change, but if Q free goes down a factor of 7 -- or by -- if C goes down by a factor of 7, Q free must go down by a factor of 7.

This goes down by a factor of 7, this doesn't change.

So the free charge goes down by a factor of 7.

And what does that mean?

That means charge will flow from the plates, away from the plates, and so my amp meter will now -- will tell me that charge is flowing from the plates, and so that handle -- that hand there will go [wssshhht] to the left.

And so, as I open up, depending upon how fast I can do that, charge will flow from the plates, in the other direction, it -- the charge will flow off the plates, and that current meter will show you, every time that I open it a little bit [klk], it will go to this direction.

So let's do that first, no dielectric involved, simply keeping the power supply connected.

So I have to go back, first, to 1 millimeter, which is what I'm doing now, I have here this thin sheet to make sure that I don't short them out, it's about 1 millimeter, and I am going to now connect the 1500 volts, and keep it on, and as I charge it, you will see the current meter surge to the right, right?

That always means we charge the plates.

So there we go, did you see it?

I didn't see it because I had to concentrate.

Did it go like this?

Good.

So now it's charged.

We don't take this connection off, it's connected with the power supply all the time.

And now I'm going to open up, and as I'm going to open up, the potential remains the same, so this volt meter doesn't give a damn, it will stay exactly where it is, because 1500 volts remains 1500 volts, but now, we go -- as we open up, we're going to take charge off the plates and so this, I expect to go to the left.

Every time that I give it a little jerk, I do it now, it went to the left.

I go it now, again, I go to 2 millimeters, go to 3 millimeters, go to 4 millimeters, make it 5 millimeters, 5 millimeters, 6 millimeters, and I finally end up at 7 millimeters.

And every time that I made it larger, you saw the hand go to the left.

Every time I took some charge off.

So that is demonstration number three.

Why did I go to 7 millimeters?

You've guessed it! Now I want to plunge in the dielectric.

So my experiment number four, I start with 1500 volts, I start with D equals 7 millimeters, and I'm not going to change that.

There's no dielectric in place, but now, I put a dielectric in.

So kappa goes in.

What now is going to happen?

Well, for sure, V is unchanged, because it's connected with the power supply, so that cannot change.

What happens with Q free?

Look at this equation.

When I put in the dielectric, I know that the capacitance goes up by a factor of kappa.

C will go up by a factor of kappa.

If C goes up with a factor of kappa, and if V is not changing, then Q free must go up by a factor of kappa.

Follows immediately from equation three.

So this must go up by a factor of kappa.

What does that mean?

That the charge will flow through the plates.

I increase the charge on the plates, and so my amp meter will tell me that.

And so my amp meter will say, "Aha! I have to put charge on the plates," and so my amp meter will now do this.

And that's what I want to show you.

The remarkable thing, now, is that the electric field E, the net electric field E, will not change.

And you may say, "But you put in a dielectric!" Sure, I put in a dielectric.

But I kept the potential difference constant, and I kept the D constant.

And since V is always E times D, if I keep this at 1500 volts, and I keep the 7 millimeter 7 millimeters, then the net electric field cannot change, it's exactly what it was before.

That is the reason why Q free has to change, think about that.

Because you do introduce -- induce charges on the dielectric, and you have to compensate for that to keep the E field constant, and the only way that nature can com- compensate for that is to increase the charge on the plates, the free charge.

And so that's what I want to show you now, which is the last part.

So I'm going now to put in the dielectric, and what you will see, then, is that current will flow onto the plates, so the propeller will do nothing, will sit there, and you will see this one go klunk when I bring in the glass.

And then it goes back, of course.

There's only a little charge that comes off, and then it will go back.

So as I plunge it in, you will see charge flowing onto the plates.

There we go, you're ready for it?

Three, two, one, zero.

And you saw a charge flowing onto the plates.

When I remove the glass, of course, then the charge goes off the plates again, and you see that now.

I've shown you four demonstrations.

None of this is intuitive.

Not for you, and not for me.

Whenever I do these things, I have to very carefully sit down and think, what actually is changing and what is not changing?

I have no gut feeling for that.

There is not something in me that says, "Oh yes, of course that's going to happen.

Not at all.

And I don't expect that from you, either.

Then only advice I have for you, when you're dealing with these cases whereby dielectric goes in, dielectric goes in, plates separate, plates not separate, power supply connected, power supply not connected, approach it in a very cold-blooded way, a real classic MIT way, very cold-blooded.

Think about what is not changing, and then pick it up from there, and see what the consequences would be.

How can I build a very large capacitor, one that has a very large capacitance?

Well, capacitance, C, is the area, times epsilon 0, divided by D, times kappa, which your book calls K.

So give K -- make K large, make A large, and make D as small as you possibly can.

Ah, but you have a limit for D.

If you make D too small, you may get sparks between the conductors, because you may exceed the electric field, the breakdown electric field.

So you must always stay below that breakdown field, which in air, it would be 3 million volts per meter.

If you want a very large kappa, you would say, "Well, why don't you make the layer water, in between, that has a kappa of 80." Ah, the problem is that water has a very low breakdown electric field, so you don't want water.

If you take polyethylene -- I'll just call it poly here, just as abbreviation -- polyethylene has a breakdown electric field of 18 million volts per meter, and it has a kappa, I believe of 3.

Many capacitors are made whereby the layer in between is polyethylene, although mica would be really superior.

Be that as it may, I want to evaluate, now, with you, two capacitors, which each have the same capacitance of 100 microfarads.

But one of them, the manufacturer says, that you could put a maximum potential difference of 4000 volts over it, that's this baby.

And the other, I go to Radio Shack, and it says you cannot exceed the potential difference, not more than 40 volts.

Well, if I have polyethylene in between the layers of the conductors, then I can calculate what the thickness D should be before I get breakdown.

That's very easy, because V equals E D, and so I put in here, 18 million volts per meter, and I go to 4000 volts, and then I see what I am for D.

And it turns out that the minimum value for D, you cannot go any thinner, is then 220 microns, and so for this one, it is only 2.2 microns.

You can make it much thinner, because the potential difference is 100 times lower.

So you can make the layer 100 times thinner before you get electric breakdown.

I want the two capacitors to have the same capacitance.

That means, since they have the same kappa, and they have the same epsilon 0, it means that A over D has to be the same for both capacitors.

So A divided by D, for this one, must be the same as A divided by D for that one.

But if D here is 100 times larger than this one, then this A must also be 100 times larger, because A over D is constant.

So if A here is 100, then A is here 1.

But now, think about it.

What determines the volume of a capacitor?

That's really the area of the plates, times the thickness.

And if I ignore, for now, the thickness of the conducting plates, then the volume of a capacitor clearly is the product between the area and the thickness, and so it tells me, then, that this capacitor, which has 100 times larger area, is 100 times thicker, will have a 10000 times larger volume than this capacitor.

And this baby is 4000 volts, 100 microfarads, it has a length of about 30 centimeters, 10 centimeters like this, 20 centimeters high, that is about 10000 cubic centimeters.

10000 cubic centimeters.

You go to Radio Shack, and you buy yourself a 40 volt capacitor, 100 microfarads, which will be 10000 times smaller in volume.

It will be only 1 cubic centimeter.

And if I had one of them behind my ear, you wouldn't even notice that, would you?

Could you tell me what it says here?

100 microfarad.

How many volts?

40.

40 volts.

That's small.

Compared to this one, which can handle 4000 volts.

But the capacitance is the same.

So you see now, the connection with area and with thickness, by no means trivial.

All this has been very rough on you.

I realize that.

It takes time to digest it, and you have to go over your notes.

And therefore, for the remaining time -- we have quite some time left -- I will try to entertain you with something which is a little bit easier.

A little nicer to digest.

Professor Musschenbroek in the Netherlands, invented -- yes, you can say he invented the -- the capacitor.

It was an accidental discovery.

He called them a Leyden jar, because he worked in Leyden.

And a Leyden jar is the following.

This is a glass bottle, so all this is glass, that's an insulator, and he has outside the insulator, he has two conducting plates, so that's a beaker outside, and there's a beaker inside, conducting.

That's a capacitor.

Although he didn't call it a capacitor.

And so he charged these up, and so you can have plus charge here, and minus Q on the inside, and he did experiments with that.

The, um, the energy stored in a capacitor -- we discussed that last time -- equals one-half times the free charge times the potential difference, if you prefer one-half C V squared, that's the same thing, I have no problem with that, because the C is Q free divided by V, so it's the same thing.

What I'm going to do, I'm going to put a certain potential difference over a Leyden jar, I will show you the Leyden jar that we have -- you'll see there -- and once I have put in -- put on some potential difference, put on some charge on the outer surface and on the inner surface -- you can see the outer surface there, the inner one is harder to see, but I will show that later to you.

So here you see the glass, and here you see the outer conductor, and there's an inner one, too, which you can't see very well.

Once I have done that, I will disassemble it.

So I first charge it up so there is energy in there, this much energy.

And then I will take the glass out, I will put the, um, the outside conductor here the inside conductor here, I will discharge them completely.

I will hold them in my hands, I will touch them with my face, I will lick them, I will do anything to get all the charge off.

And then I will reassemble them.

Well, if I get all the charge off, all this Q free [wssshhh] goes away, there's no longer any potential difference.

When I reassemble that baby, then, clearly, there couldn't be any energy left.

And the best way to demonstrate that, then, to you, is, to take these prongs, which I have here, conducting prongs, and see whether I can still draw a spark by connecting the inner part with the outer part.

And you would not expect to see anything.

So it is something that is not going to be too exciting.

But let's do it anyhow.

So here is this Leyden jar, and I'm turning the Windhurst to charge it up.

I'm going to remove this connection, remove this connection, take this out, take this out, come on -- believe me, no charge on it any more.

This one.

It's all gone.

Believe me.

There we go.

And now let's see what happens when I short out the outer conductor with the inner conductor.

Watch it.

That is amazing.

There shouldn't be any energy on that capacitor.

Nothing.

And I saw a huge spark, not even a small one.

When I saw this first, and I'm not joking, I was totally baffled.

And I was thinking about it, and I couldn't sleep all night.

I couldn't think of any reasonable explanation.

And so my charter for you is, to also have a few sleepless nights, and to try to come up, why this is happening.

How is it possible that I first bring charge on these two plates, disassemble them, totally take all the charge off, and nevertheless, when I reassembled them again, there is a huge potential difference between the two plates, otherwise, you wouldn't have seen the spark.

So give that some thought, and later in the course, I will make an attempt to explain this.

At least, that's the explanation that I came up with, it may not be the best one, but it's the only one that I could come up with.

In the remaining 8 minutes, I want to tell you the last secret, which I owe you, of the Van de Graaff.

And that has to do with the potential that we can achieve.

Remember the large Van de Graaff?

We could get it up to about 300000 volts.

How do we charge a conducting sphere?

Well, let's start off with a -- with this hollow sphere, which is what the con- the Van de Graaff is -- and suppose I have here a voltage supply, with a few kilovolts.

I can buy that.

And I have a sphere, and I touch with this sphere, with an insulating rod, I touch the output of the kilo- the few kilovolt supply, and I bring this -- so there's positive charge on here, say -- and I bring it close to the Van de Graaff, there will be an electric field between this charged object and the Van de Graaff, and the closer I get, the stronger that electric field will be.

And when I touch the outer shell, then the charge will flow in the Van de Graaff.

I go back to my power supply, I touch again the few thousand volts, and I keep spooning charge on the Van de Graaff.

Will I be able to get the Van de Graaff up to 300000 volts?

No way, because there comes a time that the potential of this object -- which comes from my power supply -- is the same electric potential as the Van de Graaff, and then you can no longer exchange charge.

What it comes down to is that when you come with this conductor and you approach the Van de Graaff, there will be no longer any electric fields between the two.

So there will be no longer any potential difference.

So you can't transfer any more charge.

So you run very quickly into a situation which will freeze.

You cannot get it above a few thousand volts.

So now what do you do?

And here comes the breakthrough by Professor Van de Graaff from MIT, who now said, "Ah.

I don't have to bring the charge on this way, but I can bring the charge in this way." So now you go to your power supply, a few thousand volt, and you bring it inside this sphere, where there was no electric field to start with.

When you charge the outside, there's going to be an electric field from this object, and there's going to be an electric field from this object, the net result will be 0 in between.

There was no electric field inside.

If I now bring the positively charged sphere there, I'm going to get E field lines like this, problem 2-1, and so now there is a potential difference between this object and the sphere.

What I have done by moving it from here to the inside, I have done positive work without having realized it, and therefore, I have brought this potential higher than the sphere.

Now I touch the inside of the Van de Graaff, and now the charge will run on the outer shell.

And I can keep doing that.

Inside, touch.

Inside, touch.

Inside, touch.

And every time I come in here, there is no electric field in there.

So I can do that until I'm green in the face.

Well, there comes a time that I can no longer increase the potential of the Van de Graaff, and that is when the Van de Graaff goes into electric breakdown.

When I reach my 300000 volts, it's all over.

I can try to bring the potential up, but it's going to lose charge to the air.

And so that is the -- ultimately the limit of the potential of the Van de Graaff.

So how does the Van de Graaff work?

Uh, we have a belt, which is run by a motor -- here is the Van de Graaff -- and right here, through corona discharge, we put charge on the belt.

They're very sharp points, and we get a corona discharge at a relatively low potential difference, it goes on the belt, the belt goes here, and right here, there are two sharp points, which through corona discharge take the charge off.

On the inside, that's the key.

And then it goes through the dome, and then it charges up, up to the point that you begin to hear the sparks, and that you have breakdown.

And I can demonstrate that to you.

I built my own Van de Graaff.

And the Van de Graaff that I built to you is this paint can.

I'm going to charge that paint can by touching it repeatedly with a conductor, and the conductor has a -- is going to be -- yes, I'm going to touch the conductor with a few thousand volt power supply every time -- this is the power supply, turning it on now -- and you're going to see the potential of the Van de Graaff there.

Uh, that is a very crude measure for the potential on the Van de Graaff, but very crudely, when it reads 1, I have about 10000 volts -- this is the probe that I'm using for that -- 2, it's 20000 volts.

My power supply is only a few thousand volts.

But that's not very good.

Well, I will first start charging it on the outside to demonstrate to you that I very quickly run into the wall that I just described.

That if they have the same potential, then I can no longer transfer a charge.

But then I'm going to change my tactics and then I go inside.

And then you will see that it will go up further.

So let's first see what happens if I now bring charge on the outside.

There it goes.

It's about 1000 volts, about 2000 volts, 2000 volts, keep an eye on it, 3000 volts, it's heading for 3000 volts, 3000 volts, 3000 volts, 3000 volts, 3000 volts, not getting anywhere, I'm beginning to reach the saturation, maybe 3500 volts, 3.5, it's slowly going to 4, let's see whether we can get it much higher than 4, I don't think we can.

So this is the end of the story before Professor Van de Graaff.

But then came Professor Van de Graaff.

And he said, "Look, man, you've got to go inside.

Now watch it.

Now I have to concentrate on this scooping, so I would like you to tell me when we reach 5000, you just scream.

Oh, man, we already passed the 5000, you dummies! 10000, scream when you see 10000.

[crowd roars].

Scream when you see 15000.

Scream when you see 15000.

[crowd roars].

Very good, keep an eye on it, tell me when you see 20000.

[noise] I don't hear anything! [crowd roars] Now I want you tell me every 1000, because I think we're going to run into the wall very quickly.

21?

I want to hear 22.

[crowd roars].

Already at 23.

So I expect that very s- very quickly now -- [crowd roars] -- the can will go into discharge, you won't see that, but you get corona discharge, and then, no matter how hard I work, I will not be able to bring the potential up.

But let's keep going.

Are we already at 2500?

25000, sorry, 25000?

25000 volts.

25 - 6.

27.

27.

28.

28.

It looks like we are beginning to get into the corona discharge.

28! Boy, 28! That's a record.

28, keep an eye on it.

29?

29?

Whew.

You realize I'm doing all this work.

Well, I get paid for it, I -- I think I've reached the limit.

I've reached my own limit and I've reached the limit of the charging.

Now, we have 30000 volts, and we started off with only a few thousand volts.

Originally, it wasn't a very dangerous object.

But now, 30000 volts -- shall I?

OK, see you next week.